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A TEXT BOOK
OF THE
PRINCIPLES OF PHYSICS

A TEXT BOOK

OF THE

PRINCIPLES OF PHYSICS

BY

ALFRED DANIELL, M.A., LL.B., D.Sc., F.R.S.E.

LATE LECTURER ON PHYSICS IN THE SCHOOL OF MEDICINE,
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PREFACE TO THE SECOND EDITION.

I HAVE been encouraged, by the kindness with which this work has been received, to take some pains in revising it.

A. D.

PREFACE TO THE FIRST EDITION.

In the following pages I have endeavoured to give, in terms as simple as the nature of the subject will permit, a connected account of the leading principles of modern physical science.

My aim has not been to build up a mere compendium of physical facts, but rather to put the reader in possession of such principles as will enable him with small difficulty to apprehend and to appreciate those facts.

I am regretfully aware of many material omissions. The subject of Natural Philosophy is so vast that many things which in themselves are by no means devoid of importance—but to which different writers would perhaps be inclined to attribute different degrees of importance—must necessarily be laid aside in the course of the preparation of a text-book of limited size. One of these omissions, which my own love of the developmental history of science made me decide upon with extreme unwillingness, is that of the history and the personal aspect of scientific discovery. As a general rule, the names of discoverers, even where they are mentioned, play a very subordinate part in the text.

At the same time, I trust that the reader of this work will find that, after assimilating its contents, he is in some measure prepared for the reception of further information in the course of that wider reading and practical study to which I hope the following pages will be found fitted to serve as an elementary introduction.

It is wholly beyond question that to him who desires to become a physicist, Practical Laboratory Work is absolutely essential. Thorough knowledge must be drunk in by the eyes and the ears, and absorbed by the finger-tips; and the true use of a book of this kind is, I take it, not to replace practical work but to economise the labours of the student. This it may do by so furnishing his mind with a store of general principles, that when he comes to enter a physical laboratory he may there find around him, in the concrete form, a collection of pieces of apparatus, the construction and the action of which he is able, by the application of the principles already familiar to him, promptly and intelligently to comprehend. Bearing the necessary limitations of the usefulness of any mere book steadily before me, I have endeavoured, as far as possible, to simplify and generalise all descriptions of apparatus, and in the same way to simplify and generalise the accompanying diagrams; and thus I have tried not only to adapt the work to the requirements of those who may use it as a stepping-stone to further attainments, but also to render it a suitable text-book for that larger circle of readers who, having no distinct desire to follow out the special study of physics, may yet wish to possess an elementary acquaintance with the modern aspect of natural philosophy.

This book was primarily designed as a contribution to Medical Education, and as such I hope it may be found useful. That arrangement, which still prevails in some of our Universities, under which a student of medicine may even proceed to the degree of M.D. without any adequate knowledge of physics, is self-evidently opposed to common sense, and to the exigencies of physiological study and of medical practice. Such an anomaly cannot, it may be anticipated, endure much longer. Before many years are over it will be universally acknowledged in practice, as it already is in theory, that knowledge of natural philosophy is an essential part of the mental equipment of the medical student and of the properly-trained medical man. The needs of the intelligent student of physiology have been kept constantly before my mind, as I hope those of my readers who are already physiologists will recognise; but I have been careful not to make the book one suited for admission only into a medical class-room; my aim has been to produce a work useful at once to the Student of Medicine, the Student of Science, and the General Reader.

The plan of the work is that of a gradual progression from the simplest to the more complex. No preliminary knowledge of physical principles is assumed, and every effort has been made to attain to absolute lucidity of expression, even though this be found occasionally to necessitate the frequent repetition of a single word in the course of a single sentence. While the reader is expected as he reads each page to remember the contents of the preceding pages, I trust that I have sufficiently carried out my intention of nowhere setting before him anything of the nature of an unsolved riddle, so far as that could be guarded against by my own efforts on his behalf.

I have endeavoured to secure intelligible continuity throughout the paragraphs printed in larger type, and thereby to enable the reader on his first perusal to confine his attention to the more prominent portions of the text.

However imperfectly its design may have been executed, I shall be glad if this work be found to contribute in any degree to the extension of that mode of teaching Natural Philosophy for the establishment of which we have come, directly and indirectly, to owe so much to the advocacy and example of Professors Thomson and Tait—a mode of teaching under which the whole of Natural Philosophy is regarded as substantially a single science, in which scattered facts are connected and co-ordinated by reference to the principles of Dynamics and the great experimental Law of the Conservation of Energy.

20*th February* 1884.

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SYMBOLS, &c., USED IN THIS WORK.

$+$, $-$, $\times$, $\div$, $\sqrt{}$, as usual ; $\pm$ "plus or minus."

/, "divided by." $a/b = \frac{a}{b}$; $a \sin b/c = a \sin \left(\frac{b}{c}\right)$; $a \sin b./c = \frac{a \sin b}{c}$; $ab/\overline{c + d + c} = \frac{ab}{c + d + e}$; $a + b/\overline{c + d + e} = a + \frac{b}{c + d + e}$; $\overline{a + b}/\overline{c + d + e}$ or $a + b./\overline{c + d + c} = \frac{a + b}{c + d + c}$. This notation is due to Professors De Morgan and Stokes.

∞ "infinity," a quantity exceeding any numerical value which can be stated. $1/\infty = 0$.

$\propto$ "is proportional to" or "varies as." If $a \propto bc$, a is *equal* to some constant number $\times\, bc$.

$a > b$, "a is greater than b;" $a < b$, "a is less than b."

$\backsim$ "the numerical difference between." $a \backsim b$ is either $(a - b)$ or $(b - a)$, but is itself always positive.

The "numerical sum" of $+ 10$ and $- 10$ is 20 ; the "algebraical sum" is 0.

ΣH or Σ(H) ; "algebraic sum of all the H's."

"Arithmetical mean" of a and b is $\frac{1}{2}(a + b)$; of a, b, c, d, e, is $\frac{1}{5}(a + b + c + d + e)$, etc.

"Geometrical mean" of a and b is $\sqrt{ab}$; of a, b, c, d, e, is $\sqrt[5]{abcde}$, etc.

$10^9 = 1000{,}000000$; 1 followed by nine cyphers.

$10^{-9} = 1/10^9 = 0{\cdot}000000{,}001$; nine cyphers in all, or *eight* after the point.

$a^{\frac{1}{2}} = \sqrt{a}$; $a^{\frac{3}{2}} = \sqrt{a^3}$; $a^{-\frac{1}{2}} = 1/\sqrt{a}$, etc.

$\angle$, "angle."

Sin, cos, tan. Refer to Fig. 12, page 60 ; the triangle ACD is right-angled ; AD is the "hypotenuse" ; the angle at A is "a." Then CD/AD = "sin a" ; AC/AD = "cos a" ; and CD/AC = "tan a." If we call AD, the hypotenuse, "r," then we have, in the right-angled triangle, the side CD = $r.\sin a$ and the side AC = $r.\cos a$. Similarly CD = $r.\cos b$ and AC = $r.\sin b$.

Sin $0° = \cos 90° = 0$; $\sin 30° = \cos 60° = \frac{1}{2}$; $\sin 45° = \cos 45° = 1/\sqrt{2}$; $\sin 60° = \cos 30° = \sqrt{3}/2$; $\sin 90° = \cos 0° = 1$. Tan $0° = 0$; $\tan 30° = 1/\sqrt{3}$; $\tan 45° = 1$; $\tan 60° = \sqrt{3}$; $\tan 90° = \infty$.

REMARKS, ETC.

Page 70, problem 10, answers; to avoid ambiguity, delete the points.

Page 77, line 1; AB′ should be A′B′.

Page 95, last paragraph; "AB and CD" should be "OA and OC," twice.

Page 150, Radii of Gyration in particular cases: It should be explained that in No. 1 the point of suspension is the extremity of the rod.

Page 307, line 9 from the bottom; carbonic *acid*, should be *oxide*.

Page 350, line 3 from the bottom; the divisor l, should be l_0.

Page 611, Secondary Cells. Observe that PbO_2 and Pb in a secondary cell have the same relation to one another as Cu and Zn in an ordinary cell. The current flows in the connecting wire from PbO_2 to Pb, though the former is electro-negative to the latter.

Page 623, line 13; π should be 2π.

The author takes with pleasure the opportunity of remarking that, on revision of the machined sheets as sent to the binders, he has not succeeded in lighting upon a single error purely due to misprinting.

THE PRINCIPLES OF PHYSICS.

INTRODUCTORY.

NATURAL PHILOSOPHY or PHYSICS may be briefly defined as the **Science of Matter and Energy.** This definition is one which is obviously comprehensive enough to include within its range the whole of Chemistry and of Biology as well as of Chemical and Physiological Physics. Chemistry is in truth but a colony of facts closely related to one another, and classified by us on principles which depend almost entirely upon our ignorance of the fundamental nature of the relation between those apparently different forms of Matter which we know as the various Chemical Elements; and the consummation of Chemistry, a full and accurate knowledge of the inner mechanism of all chemical reactions, would probably result in the absorption of all Chemistry in the wider Science of Molecular Physics. In the meantime the fundamental unity of the two nominally distinct sciences, Chemistry and Physics, is shown by the extent to which they overlap one another in the field of Chemical Physics.

Physiology, again, or in a wider sense Biology, is concerned with the matter and the energy of living beings; and if it ever come to attain its highest ideal, even Biology must thereupon necessarily merge in Natural Philosophy. Already we see that while physiological research is steadily conquering the unknown, that which it succeeds in thoroughly explaining falls out of its grasp and comes to form a part of ordinary physical or, it may in the meantime be, of ordinary chemical knowledge.

We may now more amply define Natural Philosophy or Physics as the systematic exposition of the Phenomena and Properties of Matter and Energy in so far as these phenomena and properties can be stated in terms of definite Measurement and explained by reference to mechanical principles or Laws.

Here, again, we must admit that our definition is, in the present state of our knowledge, too ideal. A perfect and accurate physical knowledge even of the simplest actual phenomenon would imply absolute omniscience. Often we find that we can measure but not explain the phenomena of Nature; and we find at the very outset of our exposition that we are compelled to confess entire ignorance as to the very nature of our subject-matter; for we do not know what Matter is.

To us the question, What is Matter?—What is the essential basis of the phenomena with which we may as physicists make ourselves acquainted?—appears absolutely insoluble. Even if we became perfectly and certainly acquainted with the intimate structure of Matter, we would but have made a further step in the study of its properties; and as physicists we are forced to say that while somewhat has been learned as to the properties of Matter, its essential nature is quite unknown to us.

As little able are we to give any full and satisfactory answer to the question, What is Energy? As a provisional statement we may say that Energy is the Power of doing Work; a rifle-bullet in motion, a coiled watch-spring, possesses the power of doing work upon other bodies suitably arranged; but plainly this power depends upon the relation into which the matter which is said to possess it is brought with reference to other matter, and it ultimately depends upon the position of one set of particles of matter with reference to other sets. Since Energy depends, then, upon the relative position of particles of Matter, we are not able to explain its own essential nature, though we may acquire a considerable amount of information as to its very remarkable properties.

These properties of Energy, those of Matter, their mutual relations, and the laws which govern these, constitute the subject-matter of Natural Philosophy; and these properties and laws have been ascertained by observation, by measurement, and by judicious reasoning upon the data supplied by investigation.

In the investigations upon which Natural Philosophy is founded, the guiding principle is a belief, based on the recorded experience of the human race, in the **Constancy of the order of Nature.** This does not mean that things are to continue for ever as they are at present. If a closed boiler containing water be heated to a certain temperature, the Constancy of Nature would not be interfered with by the consequent explosion of the boiler; it would be seriously infringed if the boiler did not burst. So, again, if volcanic eruptions thrust up mountain ranges through a flat plain, as in the case of the Rocky Mountains, or if a crack in the earth's crust allow a flood of lava to flow over a wide region, as in the geological history of Idaho, such a cataclysm would appear to be an awful break in the uniformity of Nature; yet if the earth's crust be so pressed upwards that it can resist no further pressure, the Constancy of Nature is confirmed by its giving way. On this belief in the Constancy of Nature are based all rational calculations of eventualities, and all our arrangements from day to day, which are subject to the transpiry of facts unknown or unforeseen at the time when these arrangements were made.

This belief finds formulated expression in the **Law of Causality,** which affirms that every effect has a sufficient cause. If we observe any given phenomenon, we conceive ourselves entitled as the result of all experience to enquire into its cause, and conversely, to affirm that if there be no cause tending to produce change in any particular respect in the present condition of things, there will in that respect be no change. It is scarcely necessary here to investigate the meaning of the word Cause itself; it will be quite sufficient to point out that for us the relation of Cause and Effect is one of Sequence, found to be invariable if not interfered with by the intervention of circumstances which render cases dissimilar. In similar cases, the same causes are observed to be followed by the same effects. It is plain, however, that the same effects are not always and necessarily the results of the same causes; and when different causes are found to produce the same effects they are equivalent in effectiveness to, and may be substituted for, one another.

Again, the principle may be stated that **the cause** is **equivalent** and in proper terms of measurement **numerically equal** to the **effect** produced by it. Apparent exceptions to this statement arise only when the problem is not of the extremely simple form in which one cause, and one cause alone, is brought into play. It is not, except in a loose popular sense, the heat of the spark which causes the explosion of a magazine and consequent destruction of property; it is not drawing the trigger which is *the* cause of the bullet's leaving the gun. The heat of the spark, the drawing of the trigger, is necessary as one cause out of several; but the problem is not here so simple that these can be adduced as cases in which the effect is greater than the cause. They only point out an extended statement that the **total effect** produced is

equivalent to the effective sum of the causes acting; and when one of the causes acting is an arrangement of matter which is explosive or in unstable equilibrium, ready to topple over so as to assume a stable position, the effect produced, though apparently greater than the small disturbance which disarranged the unstably-balanced matter, must be traced not to it only, but to all the conditions and circumstances involved, including the unstable equilibrium, the antecedent cause of which may itself be sought for.

If several causes act simultaneously, each produces only a part of the aggregate effect, and the total effect is equal to the sum of the acting causes. Under the name of **Galileo's principle** this is one of the fundamental truths of physics, and is thus enunciated:—If a body be acted on by two or more Forces (*force* being meanwhile defined as *any cause of motion*), each of these forces acts independently, and produces its own effect without reference to the others, the total effect produced being ascertained by finding, in any appropriate way, the sum of the effects due to the several forces. A cannon-ball, for instance, fired from a height, is, as it passes through the air, under the influence of at least two forces or causes of motion: the force exerted upon the ball during the explosion sends the ball forwards, that of gravity continuously draws it downwards. If, for the sake of convenience, we neglect the resistance of the air, and enquire what would be the path pursued by a shot travelling *in vacuo*, we would find by making use of this principle that the position of the shot at any moment would be found by enquiring (1) How far outwards the shot would have been projected had there been no tendency to fall; and (2) How far the ball would have fallen if gravity had alone acted on it. For any specified instant a point may in this way be found, which, being both so far outwards and so far downwards, must be the position of the ball at the instant in question; and by thus finding the position of the ball at several separate succeeding instants of time, we may find the curved path which a ball fired *in vacuo* would traverse. This principle of the independence of simultaneously-acting causes was an experimental discovery of Galileo's: before his time it was held as self-evident truth that one cause must cease to act before another can commence to do so; and it was accordingly believed that when a projectile was shot into the air, the force of projection must be expended and dissipated before any tendency to fall to the earth could assert itself.

Experimentation.—When we learn that a certain phenomenon is due to a congeries of causes we may arrange matters so as to prevent one of the ordinarily-acting causes from producing its effect, and then we may observe in what respect the resultant phenomenon now produced differs from that usually seen. Thus we may find the way in which a given cause acts. Again, we may directly arrange matters so that a given cause, and, as far as possible, that cause alone, shall act, and we may then observe what happens. The principle of the Constancy of Nature shows us that like causes will always produce like results; and if we find that by ingeniously varied interrogations of Nature we have obtained as reply an assurance that certain causes are allied to certain effects, we feel assured that the same causes and the same effects will continue to be so allied. This assurance is the only basis of the art of Experimentation. By this art we become acquainted with the constant modes in which events follow one another in the material world, these modes being the Laws of Nature arbitrarily appointed, and only to be learned by us through the instrumentality of our own experimental enquiry, or else through attentive consideration of the varying phenomena of the Universe, "experiments made at Nature's own hand."

Newton's Laws of Motion or Axioms.—If a body be at rest it will remain at rest: if in motion it will continue to move until stopped by friction or some external force. Here we find the word Force meaning not only that which causes motion, but also that which arrests motion. Experiment shows us that this is true in reference to bodies which are at rest, for they remain at rest if undisturbed; but it also shows that among bodies which are in motion, it is only those which are moving in a Straight Line that retain their course unaffected when allowed to move unexposed to the action of any disturbing cause. Bodies which are moving in curved paths, such as sling-stones, do not retain their curved paths when liberated, but continue their course in a straight line in that direction in which they happened to be moving at the instant of release from constraint. Hence Newton, in his First Law of Motion, says, "Every body tends to persevere in its state of Rest or of Uniform Motion in a Straight Line unless in so far as it is acted on by impressed Force," and this is tersely expressed by saying that **"Matter has Inertia."**

If a single force act upon a body which is at rest, the body will begin to move in a straight line; and further, the greater the force, the more rapid will be the motion of the body acted upon. If the body be already in motion, the force acting upon it will cause it to move more rapidly or more slowly in the same straight line or else in a deflected course. Experiment shows that every force has a definite direction in which it tends to cause a body to move, whether that body be already under the action of other forces or not. Thus the words of Newton, in his Second Law of Motion, are: "Change of Motion is proportional to the impressed Force, and takes place in the direction of the Straight Line in which the force acts."

The word Motion in this law is now rendered Momentum (p. 19).

The third of the Laws of Motion which Newton formulated as axiomatic is the following—"To every Action there is always an equal and contrary Reaction; or the mutual actions of any two bodies are always equal and oppositely directed." The truth of this statement is based upon experimental evidence, but its universal applicability is, after consideration, seen to be reasonable enough; and in this sense Newton uses the word Axiom.

When a shot is fired from a gun, if the gun be free to move there is considerable recoil, the shot moving forward and the gun backwards. If the gun be fixed to the ground, the shot is apparently the only thing which moves. If the shot were held fast and the gun were free to move, the gun would move backwards. In this case we see, then, that to the action which impels the shot forward there is a contrary reaction which impels the gun backwards; and in the sequel we shall learn what evidence there is for the statement that that reaction is equal to the action.

When a man walks on firm ground, the action of his legs in locomotion tends to separate his body from the ground at each step. The action which tends to raise his body is contrary to the reaction tending to depress the earth, and at every step the earth is pushed down as a whole, or else if the soil be soft it yields locally and the foot sinks. Hence the difficulty experienced in getting out of boggy soil; the soft mud yields under the foot at each effort made by the individual, so that every step causes him to sink more deeply.

When a horse is loosely harnessed to a car, it may sometimes be observed that an inexperienced animal starts forward quickly; but suddenly the traces tighten, the car is jolted forward, and the horse is jolted backwards.

If a locomotive with a heavy train be suddenly started, it will be seen that its wheels may uselessly turn round; it has given a sudden pull to the carriages, and their reaction upon it is equivalent to a backward pull given to a moving engine.

The earth attracts the moon, and the moon equally attracts the earth. The former attraction mainly keeps the moon in her orbit, and the latter is one of the causes of tidal phenomena.

When a stone is thrown upwards from the earth, the earth is thrown back by recoil, and moves downwards to a very small extent as long as the stone continues to ascend: when the stone is at its highest point the earth is at its lowest, and as the stone falls the earth ascends to meet it. This is, of course, not the result of direct observation, but is deduced by way of inference from Newton's third law of motion, which is confirmed by all phenomena, terrestrial and astronomical, by which it can be put to the test.

The next statement generally applicable is that of the **Indestructibility of Matter.** This is, that Matter as we at present know it cannot be destroyed by any process with which we are acquainted. The limitations of this statement should be borne in mind, for there is no scientific warranty for saying that Matter is absolutely indestructible, and more than one consideration indicates that the structure of Matter may be such as to denote that in its present form it has had a beginning and may have an end. Within our experimental knowledge, however, Matter cannot be destroyed: and when it apparently disappears, as when a candle is burned in the air, Chemistry charges itself with the explanation of that disappearance, and shows what new forms the matter has assumed.

Another principle of the greatest possible use, and entirely the result of experiment, is that of the **Indestructibility** or the **Conservation of Energy.** Energy has been provisionally defined as the Power of doing Work; and this doctrine states that this power of doing work may alter its form but is never destroyed. A coiled watch-spring possesses power of doing work in virtue of its distortion; when it uncoils, it seems to lose this power of doing work, but the Energy thus lost is transferred to other bodies, while Heat, Light, or Sound produced, Work done, Electrical Condition set up, Friction overcome, etc., present the missing Energy in several apparently dissimilar forms, which all may be reduced, however, to two types: Energy due to Motion; Energy due to Displacement. The Energy of a body depends on the advantage which that body possesses either of motion or of position: the loss of that advantage can only occur through some other body or bodies simultaneously acquiring either motion or an advantage of position. If Energy disappear in one form, it will reappear in one or several others, and none of it is ever lost, though it may assume such a form that it is no longer a power of doing work available to man, namely, the form of uniformly diffused Heat. The principle of the Conservation of Energy, which is so important that the whole of Natural Philosophy may be said to be a commentary on it, will be better understood when the laws of Energy have been discussed, as they will be at greater length in Chapter IV.

A corollary to this principle takes the form of a statement of the belief that **The Perpetual Motion is impossible:** if the sum of the Energy in the Universe be constant, no machine in which this energy is employed in doing work, in which friction is overcome, in which sound is produced, and so on, can possibly go on for ever, for the reserve of energy at its disposal will ultimately be exhausted and become useless to that machine. Even the tides will ultimately cease, as the earth loses—we know it is losing—speed in its daily rotation round its own axis.

It cannot be too strongly insisted on that these general principles, the Constancy of Nature, the Law of Causality, Galileo's principle, the three laws of Motion, the Indestructibility of Matter and of Energy, are of no value for us except in so far as they are supported by experimental evidence. They are grouped together here, for the statement of them is necessary for comprehension of the results which have been obtained through their aid. We are not here called upon to go through the steps by which they have been arrived at, but we must bear in mind that no *a priori* deduction of them by any metaphysical reasoning is for a moment admissible. The doctrine of the Conservation of Energy is very simple when stated as the result of experiment, and its simplicity has led to statements that the contrary is unthinkable, and that a belief in this doctrine is deeply grounded in the constitution of the mind of man; but all conclusions derived from such reasoning must be regarded with suspicion, for we must take warning by the example of the ancients, who believed circular motion to be perfect and heavy bodies to fall faster than light ones, until experimental evidence was adduced to the contrary. The truth of these principles must be proved by their perfect accord with the phenomena which we may actually observe, and by their enabling us to predict results of hitherto untried experiments which agree with those actually obtained. Exact science depends directly for its facts and indirectly for its principles upon experimental evidence, and the true place of speculative imagination in scientific work is the conception of new combinations of circumstances, and hence of new fields of experimental Research, as also the construction of Hypotheses, which explain and co-ordinate observed facts, and which, when they are found to do this consistently, are raised to the rank of accepted Theories.

CHAPTER I.

TIME, SPACE, AND MASS.

So far as man's knowledge of phenomena occurring around him has become accurate, it has been obtained by means of precise Measurement; and the Fundamental Units in terms of which every measurement must be executed are those of **Time, Space,** and **Mass.**

The **unit of Time** is usually taken as one Second, and the time during which phenomena appear or are observed is reckoned in seconds, unless motives of obvious convenience cause it to be reckoned in minutes, hours, days, years, or centuries. The second is usually a second of mean solar time—that is to say, the $\frac{1}{86400}$th part of the average length of a solar day.

The solar day is the period which elapses between the sun's crossing the meridian, or being situated directly south (or in the southern hemisphere, directly north) of a place, and the next occasion on which it crosses that line. The sidereal day, in the same way, is the interval between two successive transits of any fixed star. The sidereal days are shorter than the solar; they are constant in length, because a sidereal day is the time of one complete rotation of the earth round its axis; but the solar day is not constant in length. A clock can keep time with the stars, and keep good "sidereal time;" but a clock of ordinary construction does not always indicate noon when the sun is highest in the heavens; it is sometimes apparently 14½ minutes fast, and sometimes appears to be 16¼ minutes slow. A good clock, however, is one which measures off and indicates as twenty-four hours a period of time equal to the average length of the solar day for a year or a century or an age, and such a clock is said to keep "mean solar time;" while the Second used in physical measurements is the second as indicated by a clock such as this; and it is the 31,558,150th part of a sidereal, the 31,556,929th part of a solar or tropical year.

Space.—When a single point moves it describes a Line: if it travel by the shortest distance between two points, its path is a straight line; and a line is an example of space of one dimension. Movement and measurement may be effected in a forward or a backward direction along it, but as a line has neither breadth nor thickness there can be no other.

Distance along a straight line may be measured in one direction arbitrarily chosen; let this be, for instance, the direction from left to right; if, then, a point travel towards the right its motion is positive, if to the left, negative. If it move a inches to the right and then b inches to the left, its distance from the starting point becomes $a-b$; while if it first go b inches to the left and then a to the right, its position will become $-b+a$ from that point; and these two positions are the same, for $a-b=-b+a$. Hence we learn that if a point move backwards and forwards by varying amounts along a line, it does not matter in what order it performs these operations: the spot ultimately arrived at will be the same in all cases.

In order to effect measurements along lines, we require a **standard of length.** This is taken as the Foot or the Mètre. The British standard yard, which is equal to three feet, is defined by law as "the distance between the centres of the transverse lines in the two gold plugs in the bronze bar deposited in the office of the Exchequer" at the temperature of 62° F. A number of authorised copies of this have been made and are deposited at the Royal Mint, the Royal Observatory at Greenwich, the New Palace at Westminster, and under the care of the Royal Society of London.

The Mètre is the distance, at the temperature of melting ice, between the ends of a platinum rod preserved in the Archives, and of which copies, to regulate French commerce, are preserved at the Ministère de l'Intérieure in Paris. It was originally intended to represent the ten-millionth part of the distance from the Equator to the Pole: the measurements of Delambre and Méchain, from which Borda made the standard metre according to a law of the French Republic passed in 1795, have been found not to be quite correct, for the earth's quadrant is now known to measure 10,000,880 mètres.

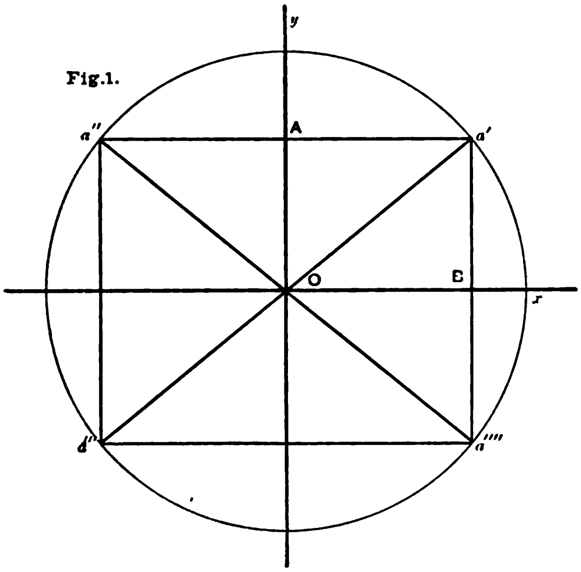
The metric system of measurement of length is decimal; each metre contains 10 decimetres, 100 centimetres, or 1000 millimetres: 1000 metres make a kilometre, which is, roughly speaking, about $\frac{5}{8}$ ($\frac{3281}{5280}$) of a mile; one metre is equal to 39·37043196 inches, or 3·28087 feet; a decimetre very nearly corresponds to 4 inches (really 3·937043196); a millimetre is very nearly equal to the twenty-fifth of an inch. For the purpose of physical measurement it is customary and convenient to make use of the

Centimetre* (·3937043196 inch) as a unit of length. One English foot is equal to 30·47972654 centimetres.

A plane surface has length and breadth but no thickness, and is therefore said to be space of two dimensions. Two terms are always necessary for the precise statement of the position of any point on a surface. The position of a ship at sea is determined when its latitude and its longitude are known.

The position of a point *a* on a plane surface is determined by choosing a fixed point O as the origin; then two axes, O*x* and O*y*, are chosen, generally at right angles to one another; *a*A is drawn parallel to O*x*, and *a*B parallel to O*y*, and the point *a* is said to be situated at a distance OA along the axis of *y*, and OB along the axis of *x*. If a point lie at the same time three miles to the north and four miles to the west of a given place, its true position (at the distance of five miles) can be easily indicated on a chart. The symbols + and − are also used here to denote that the measurement is to one side or the other of the point assumed as the origin. Points to the right of O have a positive, points to the left a negative, value of O*x*; points above O have a positive, points below a negative, value of O*y*. Thus (Fig. 1) the point a' has abscissa (or line cut off along the axis of *x*) OB, and ordinate (cut off along the axis of *y*) OA; the point a'' has abscissa − OB and ordinate + OA; the point a''' has abscissa − OB and ordinate − OA; that at a'''' has abscissa + OB and ordinate − OA.

Fig.1.



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* It is worth remarking that a French ten-centime piece measures 3 centimetres across, while a five-centime piece has a diameter of 2½ centimetres. Similarly an English halfpenny measures an inch, while a penny measures an inch and a fifth.

The Area of a Surface may be measured if we fix upon a **standard unit of area.** The unit of length may be made use of in order to obtain this. If a square be constructed, one of whose sides is one foot or one centimetre, we shall have a unit-surface whose area is known as one square foot or one square centimetre; and the areas of other surfaces may be measured by comparison with these standards.

A Solid has length, breadth, and thickness, and is said to occupy space of three dimensions. The position of any point in tridimensional space requires three numerical terms for its exact statement. The position of a balloon, for instance, will be definitely known if the latitude and longitude of the spot over which it stands and its height above that spot be ascertained.

Three terms are also required to define the position of a star: the telescope has to move so much "in azimuth" round a vertical axis; then so much in "altitude" round a horizontal axis; and thirdly, the distance of the star in a straight line must be known.

A cube whose side is one foot or one centimetre—that is, a cubic foot or a cubic centimetre—serves as the **unit of volume.** For convenience sake other units of volume are often chosen, such as the cubic inch, the cubic decimetre (otherwise known as the liquid measure, one Litre), the cubic metre, and so forth.

The remaining fundamental idea involving measurement is that of **Mass,** or **quantity of Matter.** The notion implied in this term is quite distinct from that of Weight. The weight of a certain quantity of matter depends upon the presence and nearness of other matter, which attracts it according to the well-known law of Gravitation. This may and, even within our terrestrial observation, does vary; the effect of gravity on a given mass—that is to say, its Weight—is greater as we near the Poles than it is at the Equator; and the weight of a substance varies, therefore, according to local causes, while the mass or quantity of matter in it remains the same. *Cæteris paribus,* however, equal masses will everywhere counterpoise one another in a balance, and we may define the **unit of mass** as that quantity of matter which will counterpoise in a balance a certain standard mass known as a standard Pound or Gramme.

The British standard Pound is a piece of platinum preserved in the same place as the standard yard, while authorised copies of it are preserved at the same institutions. The French standard is the Kilogramme (=1000 grammes), made of platinum, and preserved at the Archives in Paris. This is intended to have the same weight as a cubic decimetre of water at its temperature of maximum density—that is, 3·9° C. Since a kilogramme contains a thousand grammes, and a cubic decimetre a thousand cubic centimetres, it follows that the **gramme** is intended to be equal to the mass of one cubic centimetre of water at 3·9° C. Comparison of the actual standards shows, however, that a litre of water weighs, at 3·9° C., 1·000013 kilogrammes, and a cubic centimetre of water at 3·9° C. weighs therefore not 1 gramme, but 1·000013 grm. For most practical purposes the intended value may, however, be taken as correct. The British pound avoirdupois weighs 7000 grains, while the standard kilogramme weighs, according to Prof. W. H. Miller, 15432·34874 grains, and the gramme 15·43234874 grains.

It may be noticed that the British fluid ounce of water at 62° F. weighs one ounce avoirdupois; that a pint of water (20 fluid ounces) weighs therefore a pound and a quarter, and a gallon of water ten pounds. A French franc-piece weighs, when new, five grammes.

In British measurements the Foot, the Pound and the Second may be used as the fundamental units. In British Magnetic Observatories the units employed till lately were the Foot, the Grain and the Second.

The C. G. S. System.—For the international convenience of scientific men the C. G. S. or Centimetre-Gramme-Second system of units and measurements is in current use.

The gramme is chosen as a unit rather than the kilogramme, the centimetre rather than the metre; firstly, because the use of smaller units diminishes the need for working with decimal fractions; and, secondly, because on the C. G. S. system the density of water (p. 205) is equal to unity, which is a distinct advantage. If the kilogramme and the metre had been employed as units, the density of water—the number of kilogrammes in a cubic metre—would have been 1000.

The introduction of coherent systems of units for the measurement of all physical quantities has been an enormous stride in advance. When we have a problem to solve numerically, if we take care to put in all the terms in C. G. S. measurement, the answer comes out in C. G. S. units, ready for use without further reduction.

CHAPTER II.

NOTIONS DERIVED FROM THE PRECEDING.

WHEN a physical particle changes its position, it effects **Motion.** This Motion or Change of Position must be performed by passing along a definite continuous path—continuous because it is not possible for any physical particle to occupy two consecutive positions without traversing the intermediate space.

In this respect the path of a physical particle differs from many mathematical curves which abruptly end at one point and recommence their course at another. Obviously the path described by the moving particle may have any form, straight or curved; and the shortest possible path between the initial and final positions is a straight line.

We may remind the reader of Newton's use of the word Motion in the sense of Momentum (pp. 6, 19).

A moving body may travel rapidly or slowly: the rate at which it travels along its path is called its rate of motion, its rate of change of position, its **Velocity.** The Velocity of a moving body may be stated in units of length per unit of time, *e.g.* feet per second; and a body is moving with Unit velocity when it moves one foot in a second, or one centimetre in a second. It will be observed that it is necessary for us to make consistent use of the British or of the C. G. S. units of measurement, and not to use them confusedly within the limits of the same problem.

A body which moves sixty feet in five seconds has a velocity, evidently, of twelve feet per second. The velocity is equal to sixty divided by five—that is, to the whole space traversed divided by the time occupied in the movement. In algebraical language this is expressed thus: $v = \frac{s}{t}$, where the velocity, space, and time are denoted by their initial letters. Multiplying both sides of this equation by t, we get $vt = s$; the space traversed in a given time is equal to the velocity per second multiplied by the number of seconds.

Digression as to mathematical formulæ and the theory of Dimensions.—Each such formula is a kind of generalised shorthand blank form, waiting to be applied to particular cases by being consistently filled in with appropriate numbers. In words at full length we may affirm that the Number expressing a **velocity** is equal to the Number expressing the **space** traversed divided by the Number expressing the corresponding **time** taken; all these being of course systematically measured in consistent units. The numbers themselves in any particular case we may not know at present, and in the meantime we may not even care to know; for such a verbal formula is of a higher order of generality, of wider value than a mere statement of the particular numbers in any particular case. By way of rough jotting we may shorten the phrase "Number expressing a Velocity" down to the simple word "Velocity," and so on. Then we have the condensed note "Velocity = Space ÷ Time." This may be still further shortened by using initial letters only, in which case the symbols "$v = s \div t$," or "$v = s/t$," or "$v = \frac{s}{t}$" suffice to express the law; or we may agree that these unknown numbers shall for the time being be represented by letters arbitrarily chosen. Thus if we agree that the letter a shall stand for "number expressing velocity," or, as it is more usually phrased, that a shall represent velocity; and similarly that b shall represent space traversed, and c the corresponding time, the condensed expression of our law becomes $a = b \div c$. To apply this to any particular case we must know what the numerical values of two of the terms actually are; this much being determined, it is only an arithmetical matter to find the numerical value of the third term. For example, let v (the number expressing a velocity) be 12 (ft. or cm. per sec.), and let $s = 60$ (ft. or cm.), then replacing v in the equation by 12 and s by 60 we get $12 = 60 \div t$, and t cannot have any other value than 5 (sec.); all in units of the same system.

One great advantage attending the use of mathematical formulæ is their susceptibility to algebraic transformation. The above equation may be written $s = vt$ or $t = s/v$, either of which modes of expression, when translated into words at full length, is found to present the subject from so fresh a point of view as practically to amount in each case to the enunciation of an independent truth.

When a is stated by a formula to depend upon or to be "a function of" b, c, d, and of these only, it seems, when put into words, a truism to affirm that a is independent of variations in the values of e, f, g, etc.; yet this often leads to the enunciation of valuable principles, *e.g.* p. 197, line 35.

The Theory of Dimensions.—The number expressing a Velocity is the number expressing a Space divided by the number expressing a Time; $v = s/t$, as we have seen before. But there underlies this mode of expression a tacit understanding that we adhere consistently to some known system of units. The numbers must vary with the units conventionally employed, even when the same facts have to be expressed. Consequently we may, if we have in our minds a possible change of units, write such an equation as $v\,[\mathrm{V}] = s\,[\mathrm{S}] \div t\,[\mathrm{T}]$, where the italic initials represent numbers and the corresponding bracketed letters the respective conventional units. If v, s, and t in the above equation become all = 1, that equation becomes $[\mathrm{V}] = [\mathrm{S}/\mathrm{T}]$, an equation which refers to the conventional units only. Such an equation is technically known as an equation of Dimensions. Then if we change our conventional units from [V], [S], and [T] to others, say, $[l\mathrm{V}]$,

$[mS]$, $[nT]$, the last written equation must still hold good, and the new unit $[lV]$ is equal to $[mS/nT]$, or to $m/n\,.\,[S/T]$; that is, the new unit of velocity is equal to m/n times the old unit. The numerical value of any given velocity is accordingly n/m times as great when expressed in terms of the new units as it was when expressed in terms of the old units; that is, it varies inversely as the unit employed; just as a sum of £40,000 seems greater (one million) when expressed in the smaller French unit, the franc. Let us now set ourselves a problem: What is the ratio between the British and the C. G. S. unit of velocity? The former is 1 ft. per sec., the latter is 1 cm. per sec. Here $[V] = [S/T] = [\text{Foot/Second}] = [30{\cdot}478 \text{ cm./second}] = 30{\cdot}478[\text{cm./second}]$; the British unit is 30·478 times the C. G. S. unit. Consequently a velocity of 3047·8 cm.-secs. would be a velocity of only 100 if measured in ft.-seconds.

The equations of Dimensions thus explained were an invention of Fourier's, and were brought into prominence by Clerk Maxwell. Their use is twofold: (1) as a means of converting physical quantities expressed in one set of units into the same quantities expressed in other units; and (2) as a means of checking our equations, for the dimensions must agree on both sides, as will be seen in very simple examples on p. 59.

Velocity (resumed).—If a body move through equal spaces in equal times, its velocity is said to be **uniform.**

We are familiar with instances in which a body such as a railway train is said to be running at a certain time with a velocity of (say) thirty miles an hour. This indicates that if the train ran for a whole hour at the rate at which it was travelling at the instant of observation, it would at the end of an hour be thirty miles away from the point which it occupied at the beginning of it. But the train may possibly not have run more than a mile on the whole. The statement means, then, that during (say) a minute it ran half a mile, and that therefore during sixty minutes it might, at the same speed, have run thirty miles. But even during a minute it may have gained or lost speed, so as to render its motion not uniform but **variable**: the statement would be still more exact if we knew that in six seconds it ran the twentieth of a mile, or in one second the hundred-and-twentieth; for when the interval of time is very short, there is less possibility of variation during that interval, and the speed approximates more nearly to uniformity. Hence the variable velocity of any moving body at a particular instant is found by observing the amount of motion effected in a certain very short interval of time, and finding what movement would be effected in one unit of time if the velocity were to remain uniform during that period.

If a body move over a certain space, s (say thirty feet), in time t (say ten seconds), the equation $v = \frac{s}{t} = \frac{30}{10} = 3$ feet per second, shows what the mean or average velocity is during the motion. The **mean velocity** of a train which travels fifteen miles in one hour is a quarter of a mile a minute, or $\frac{1}{240}$th part of a mile in a second, although during some seconds or minutes it may be travelling at the rate of sixty miles an hour, at others may be standing still, and at others may be actually going backwards.

All velocities, mean and constant, uniform and variable, may be expressed in feet or in centimetres per second, and can, when so expressed, be compared with one another.

All measurable velocities are Relative; we know nothing about Absolute velocities in space, for we have no standard of comparison.

Problems.

1. If a body move 144 feet in 3 seconds, what will be its mean velocity? —*Ans.* 48 feet per second.

2. In the previous question: What will be its velocity during the second second if it travel 16 feet in the first second and 80 feet in the third?—*Ans.* 48 feet per second.

3. A body moves with a uniform velocity of 40 miles 1600 yards per hour: what is its velocity in feet per second; and how many feet will it traverse in 10 seconds?—*Ans.* 60 feet per second; 600 feet.

4. A railway train explodes two detonating signals placed on the rails at a distance from one another of 176 feet; an interval of exactly 2 seconds elapses between the explosions. Compare the velocity during that interval with the mean velocity, which is indicated by the statement that the train takes an hour and a half to perform the journey between two stations 45 miles distant from one another.—*Ans.* It is twice the mean velocity.

5. Which is the greater velocity, 40 miles an hour or 12 metres per second?

6. A train travels 10 miles at a velocity of 20 miles per hour; then 4 miles at an average rate of 30 miles per hour; then 6 at a uniform rate of 40 miles per hour; it takes 1 mile to come to rest, running at an average speed of 20 miles an hour; it stands for 7 minutes: it starts and runs for 20 minutes at the average speed of 21 miles an hour. What has been its mean velocity?—*Ans.* 32 feet per second.

Acceleration.—When the velocity of a moving body varies, the Rate of Change of Velocity is called its Acceleration. In popular language this word indicates increase of speed, but it is in this connection used to signify the rate of change, whether that change be an increase or a diminution of the velocity. If a body be moving at the rate of ten feet a second at the beginning of a certain second of time, and at the end of that second be found to be moving at the rate of eleven or of nine feet, it is said to have received during that second an acceleration, positive in the former case, negative in the latter, of one foot per second. Acceleration is usually indicated by the symbol a, and the Unit of Acceleration is the acceleration observed when a body alters its speed by one unit of velocity *every second*. A body, then, undergoing a unit acceleration (in British units) has its speed increased or diminished by one foot in one second, two feet in two seconds, and so forth; in C. G. S. units, by one cm. per sec., n cm. in n seconds.

The initial velocity may be zero, the body being originally at rest; in such a case the body will undergo unit acceleration in a given direction, if in that direction it acquire unit velocity in one second, or a velocity of n units in n seconds.

There are some cases in which the apparent effect of acceleration is only to change the direction of motion; but there is no essential difference between such cases and those upon which the definition here given is based; and such a result will be readily understood after we have discussed the composition of velocities and of accelerations.

If a body move with velocity V, and at the end of t seconds with velocity $V_{,}$, the total change of velocity during t seconds is $V_{,}-V$, the time during which this change is effected is t, and the acceleration per second is $\frac{V_{,}-V}{t}$. This is the mean acceleration during the time t: and the acceleration may during that period t be uniform or variable, but an approximation to its value at any instant may be found by making the interval t as short as possible.

Problems.

1. A body starts from rest under the influence of a force which produces acceleration $a = 2$ ft.: when will it have a velocity of 1000 feet per second?—*Ans.* At the end of the 500th second.

2. A body travels at 12 feet per second; in 10 seconds it is moving 7 feet per second: what is the mean acceleration?—*Ans.* $-\frac{1}{2}$ foot per second.

3. If in the last question the acceleration had been $+\frac{1}{2}$ foot per second, what would have been the rate of movement at the end of 10 seconds?—*Ans.* 17 feet per second.

4. A body moves in the first second during which it is under observation through a space of 16 feet; in the fourth second through 112 feet: what is the acceleration per second?—*Ans.* 32 feet, so that during consecutive seconds it moves 16, 48, 80, 112 feet. At the end of each successive second it moves with a velocity of 32, 64, 96, 128 feet per second respectively.

5. A body as it moves is made to record its own speed: it is found that at a certain instant it is moving at the rate of 112 feet a second; after an interval of $\frac{1}{20}$ second its velocity is 114 feet per second: what is its acceleration?—*Ans.* $\frac{V_{,}-V}{t} = \frac{114-112}{\frac{1}{20}} = 40$ feet per second.

6. A particle moves, during the first second, with diminishing velocity, at the *mean* rate of 10 centimetres per second; the next second it moves at the mean rate of 8 centimetres per second; the acceleration is constant: how

far will it travel, and what will it do when it has come to rest?—*Ans.* It will go on for $5\frac{1}{2}$ seconds, will traverse 30·25 centimetres, and will return, arriving at every point on its previous path with the same speed as that with which it left it, and will retrace the 30·25 centimetres in another $5\frac{1}{2}$ seconds, passing the starting point with a reversed velocity of 11 centimetres per second.

Momentum.—When a body whose mass is m moves with velocity v, it is said that the total *Momentum* or Quantity of Motion is the product of these two terms, namely, mv. The greater the velocity or the greater the mass moved, the greater the quantity of motion.

Force.—When a body which is at rest is set in motion, or one which is in motion is accelerated (positively or negatively) or deflected from its straight course, we attribute these effects to impressed force, or simply to Force. This is sometimes defined as any Cause which tends to alter a body's state of rest, or of uniform motion in a straight line. It is better defined as it is by Newton, not as a cause, an existing reality of any kind, but simply as an observed Phenomenon, a measurable Action upon a body, under which the state of rest of that body, or its state of uniform motion in a straight line, suffers change.

The presence and the mutual influence of at least two bodies are always essential to the production, in any one of them, of those effects of displacement which we commonly attribute to Force; and such displacement is always associated with a transformation or a redistribution of Energy.

Forces considered as measurable Actions are measured by the Masses set in motion, and by the Velocities imparted to them in unit of time—that is to say, by their Accelerations; and the equation $F = ma$ enables us to measure any Force F, as the product of a Mass m into the Acceleration a imparted to it, as found by observation.

Force may also be measured as an Observed Rate of Change of Momentum. Acceleration a = rate of change of velocity v; $F = m \times a = m \times$ rate of change of v = rate of change of mv = rate of change of momentum. A uniform Force is therefore numerically equal to the Amount of Momentum gained or lost by a body during each Second. (See further p. 40 and p. 46).

The product of a force acting into the time during which it acts measures the momentum imparted during that time; and this product is known as Impulse—a term of frequent use in the study of the working of machinery.

By a convenient form of speech a given Force is said to act upon a given body and to impart to it a given acceleration. It must be constantly borne in mind, however, that a Force is not a physical entity. It can never be measured until we already know, absolutely, or by comparison, the mass acted upon and the acceleration actually imparted to it; and force may be increased or diminished by varying the arrangement of the bodies to whose mutual actions it corresponds; as in the case of the Hydraulic Press, where the ordinary action presents an apparent increase of Force, while if the action be reversed, Force seems to be destroyed.

If a body weighing three pounds be set in motion so as at the end of one second to have had a velocity of four feet per second imparted to it, then $F = ma = 3 \times 4 = 12$ Poundals or British units of force. The same force would have imparted to a two-pound mass an acceleration of six feet, to a one-pound mass an acceleration of twelve feet, and to a twelve-pound mass it would have imparted in one second a speed of one foot per second.

If the mass moved be a unit, and the acceleration imparted be unity, the product $F = ma$ is also unity; and hence the **Unit of Force** is to be defined as that under the action of which a unit of mass will come to move with unit velocity when it has been acted upon for one second.

It will be observed that this definition of the Unit of Force is absolute, is not affected by local variations in the intensity of gravity, and is hence everywhere the same.

If the unit of length chosen be the centimetre, and the unit of mass the gramme, the Unit of Force will in one second cause a gramme-mass to acquire a velocity of one centimetre per second; and the unit of Force so defined is called a **Dyne.** Any force may be stated to be equal to so many dynes.

One million dynes make one Megadyne.

Problems.

1. A certain force acts upon m units of mass of matter, and at the end of a second that mass is found to be moving with a velocity of 32 feet per second: what velocity will be produced if the same force act upon $32m$ units of mass for the same period?—*Ans.* One foot per second.

2. How many dynes of force are required to set a mass weighing 50 kilogrammes in motion with a velocity of 12 metres per second, the force being supposed to act for precisely one second?—*Ans.* 60,000,000.

3. How many dynes are required to make a gramme-mass move with a velocity of 9·81 metres per second, the force measured in dynes being supposed to act for precisely one second? what, if it act for two seconds?—*Ans.* 981 dynes; 490·5 dynes.

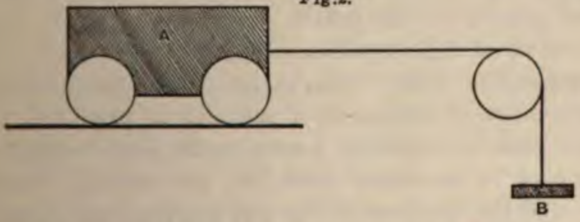
4. Compare the velocities produced by the action on masses of 2 kilogrammes, 750 grammes, and one gramme respectively, of forces measuring respectively 300,000, 112,500, and 150 dynes.—*Ans.* All equal; 150 centimetres per second if the action endure for one second.

5. Equal forces act upon the masses specified in the last question: what will be the relative accelerations produced?—*Ans.* 3 : 8 : 6000.

Weight.—We have here to anticipate what will be afterwards more fully discussed, by defining the Weight of any given mass as the actual measurable Action of Gravity upon it at a given spot, whatever may be the hitherto unknown cause of that action. This Action is (p. 19) identical with a Force acting upon that mass; and as every force may be represented as equal to ma, the weight of any given mass is the product of that mass into the acceleration produced when the body moves without restraint under the unhindered influence of gravitational attraction. Experiments made near the earth's surface show that every mass of matter acquires, if gravity act freely upon it for one second, a velocity of nearly 32·2 feet (981 centimetres) per second. This acceleration is found to be independent of the nature and of the size of the falling body. The force acting on a falling gramme-mass within small distances from the earth's surface is therefore $ma = 1$ grm. $\times$ 981 cm. $= 981$ dynes: the weight of a gramme-mass, that is to say, is equal to 981 dynes: and conversely, a C. G. S. unit of force, one Dyne, is equal to the weight of $\frac{1}{981}$ gramme.

The British unit of force (that force which, acting on a pound-mass for one second, produces an acceleration of one foot per second) is one poundal; the weight of a pound-mass (since it produces in that mass an acceleration of 32·2 feet per second) is equal to 32·2 poundals; or the poundal, as a unit of force, is equal to the weight of $\frac{1}{32{\cdot}2}$ of a pound-mass. Hence it is said that the British unit of force is nearly equal to the weight of half an ounce.

Fig.2.



If a pound-mass be divided into two parts, one of which, A, weighs $\frac{31{\cdot}2}{32{\cdot}2}$ lbs., and the other, B, $\frac{1}{32{\cdot}2}$ lbs., and if the weight of the smaller be employed to set both in motion, then the whole mass set in motion is 1 lb., and the force acting is $\frac{1}{32{\cdot}2}$ of the weight of 1 lb., *i.e.* $\frac{1}{32{\cdot}2}$ of 32·2 poundals —that is, one poundal. Hence $F = 1$, $m = 1$; and since $F = ma$, a, the acceleration, must be unity, and at the end of a second the whole mass will be found to be moving at the rate of one foot per second.

We can state, then, that any given *force* is *equal to the weight* of so many units of mass at a certain definite place; and hence the use of loose expressions such as "a force of 8 lbs." for "a force equal to the local weight (the place being mentioned or understood) of a mass of 8 lbs." The objection to such expressions is that, when we measure forces in pounds or in grammes, then, since gravity varies from place to place, we introduce the unnecessary complication of local variations in the value of our unit of force—that force, namely, which is equal to the weight of a pound-mass or of a gramme-mass, as the case may be. If properly understood, however, such expressions are compendious and not wanting in convenience, for the error introduced by local variations in gravity is, practically, within one-half per cent.

The reader will now be able clearly to distinguish between a Mass of 1 lb., a Force of 1 lb., and the Weight of 1 lb.

The observed acceleration (about 981 cm., or 32·2 ft. per sec.) is not constant from place to place on the earth's surface, and so weights vary while masses do not. This variable number is, however, usually represented by the letter g, which usually denotes, in cm. or in ft. according to the system employed, the local acceleration produced by gravity, and then the equation $F = ma$ is transformed for convenience sake into w (the weight of a mass m, or the action of gravity upon it) $= mg$.

The symbol g may, indeed, be said to have two meanings: (1) the local acceleration of a falling body $= 981$ cm. per sec., more or less; and (2) the "intensity" of gravity at a place, *i.e.* the weight of a unit-mass there, for when $m = 1$, $w = g$; and in this sense $g = 981$ dynes nearly.

For local values of g, see p. 191.

Stress.—The word Force is limited to the case in which some movement of masses or of particles is produced, varied, or checked: what is popularly known as the force tending to bring a spring back to its original form, but not actually doing so, is a Stress; and the condition of the spring under such circumstances is a Condition of Stress. A spring, when its form is altered, tends to resume its original form, and it exerts a pressure or a pull upon any object so placed as to prevent its doing so; but this object also exerts continuously an equal but opposed pressure or pull upon the spring. This mutual pressure or pull will cause motion if the bodies pressed upon or pulled become free to move; if not, the pressure or pull is continuously applied without producing movement, and such an inactive mutual pressure or pull is called a **stress.**

In popular language a Stress is called a Strain, as where it is said that a bridge or wire being exposed to too great a strain gives way and breaks or snaps. Properly the word Strain means Deformation of a body.

Every such stress implies at least two fixed points; these are either pressed together, or else the material stretched between them is in a condition of tension. In the former case, when the condition of stress ceases, the body previously compressed expands; in the latter case, when set free it contracts. If both ends of a stretched body be simultaneously liberated, the resultant movement is towards the centre; if one end only be set free, the movement is towards the end which remains fixed; and conversely for a body exposed to compression.

Stress therefore always implies mutual Action and Reaction; and we might, with Tait, paraphrase Newton's third Law thus: "Every action between two bodies is a Stress." A stress is always numerically equal to either the Action or the Reaction, as also to the force which is necessary to produce it, or to that which is developed when the condition of stress comes to an end. Stress cannot be said to have a determinate direction, positive or negative, in the line of its application, for it depends on extraneous circumstances which point or part of the stressed body shall be set free, and therefore what shall be the direction of the resultant movement; but it has a numerical magnitude, for it can be measured in dynes; and it may be numerically specified either (1) as Total Stress or (2) as Stress per Unit of Area of the common bounding surface between two bodies under mutual Action and Reaction.

In the first sense, stresses or total stresses may be measured as equal to the forces which produce them. A spring is pressed upon by a certain known weight; it yields to a certain extent; it is then caught by a ratchet, and the weight is removed. The force necessary to cause the given yielding of the spring is known, for it is the numerical value, in dynes, of the weight of the mass employed; the Total Stress established in the spring is numerically equal in dynes to the force used. The whole upward pressure of the spring on the ratchet must be numerically equal to the weight removed; so must the downward pressure of the ratchet on the spring. The opposite extremity of the spring imposes an equal and downward pressure on its support, opposed to which is an equal upward pressure of the support upon the spring.

In the second sense, the numerical value of the stress (*i.e.* Stress per Unit of Area) is obtained by dividing the total stress (in dynes) by the area (in sq. cm.) over which it is distributed; and this is otherwise known as the Intensity of the Stress.

In the sequel we shall, except where the context makes it plain, avoid the use of the unqualified word Stress, and shall endeavour to make it clear whether in any particular instance we refer (1) to a Condition of Stress, (2) to a Total Stress of so many dynes, or (3) to a Stress of so many dynes per square centimetre.

Pressure.—Suppose a heavy slab of iron weighing 100 kilogrammes to be laid upon a flat slab of indiarubber of sufficient size; and let its under surface be flat and have an area of 1000 sq. cm. Its total weight is 100,000 grms. × 981 = 98,100,000 dynes; and this is distributed over the underlying surface of the indiarubber as a Total Pressure of 98,100,000 dynes. When the arrangement is that specified, the indiarubber suffers (over 1000 sq. cm.) a downward pressure of 98,100 dynes on each sq. cm. acted upon; but it exerts on the iron an upward pressure of equal amount, for the pressure is mutual.

Now let the metal slab be mounted on four legs, whose joint cross-area is, say, 20 sq. cm.; and let the whole again stand upon the indiarubber. The total pressure is the same as at first; but it is now distributed over an area of only 20 sq. cm., for which reason the indiarubber and the metal are subject to a mutual pressure of 4,905,000 dynes per sq. cm. across the area of contact.

Here, therefore, we have again a number of different meanings. The word Pressure may mean:—

(1.) Between two objects having a common bounding surface, a Total Mutual Pressure of so many dynes over that whole surface.

(2.) From the point of view of one of the objects, the Total Pressure suffered by it or exerted by it across the whole area of mutual contact; the same number of dynes over the whole area as in the preceding case.

(3.) The Mutual Pressure per Unit of Area of the common surface.

(4.) The Pressure of the one body on the other across the common bounding surface, measured in dynes per sq. cm. (Pressure of A on B, or of B on A per Unit of Area).

(5.) Sometimes, in a quasi-popular sense, a general synonym for Force, as where the piston of a steam-engine is said to move under the influence of the Pressure of the steam in the cylinder.

As a rule the *fourth* of these is the proper meaning of the word Pressure when used alone; but we shall endeavour to make it consistently clear in which sense the word Pressure is supposed to be used in each particular case.

Tension.—If a mass of, say, 100 kilogrammes be hung upon a metallic rod of 1 sq. cm. cross-section, the metallic rod is under tension amounting to 98,100,000 dynes across that one sq. cm. of cross-section. If the same mass had been suspended on a metallic rod of, say, 10 sq. cm. cross-section, the tension would have amounted to 9,810,000 dynes per sq. cm. of cross-sectional area. In both these cases the Total Tension is equal to the weight of the mass suspended, viz. 98,100,000 dynes; and it is irrespective of the transverse-sectional area of the rod which is being acted upon. Here again we have thus to distinguish between a Total Tension (so many dynes) and a Tension per Unit of cross-sectional Area (so many dynes per sq. cm.); and here again we shall have, in the sequel, to make it plain to which of these reference is being made in any particular case.

As a rule the phrase The Tension of a Cord is supposed to mean the Total Tension acting across any transverse-section of a cord; this is the same at all parts of a cord stretched between two points, whatever may be the local variations of the thickness of that cord. Pretty obviously the thinnest part of a cord thus stretched between two supports is exposed to the greatest tension *per unit of cross-sectional area;* for, the Total Tension being uniform, it necessarily follows that where the cross-area is least, there the Tension per Unit of Area (*i.e.* the quotient $\frac{\text{Total Tension}}{\text{Cross-Area}}$, otherwise known as the Intensity of Tension) is the greatest; and, accordingly, a stretched string is most liable to snap where it is thinnest.

It is scarcely necessary here to point out for the sake of clearness that there are three other distinct meanings of the same word Tension, which will duly come up in their respective places. These are: (1) the Surface-tension of a Liquid, p. 252; (2) Electric Tension, a name given to the self-repulsion of electrified surfaces, p. 536; (3) "in Tension," an old-fashioned and obsolete phrase denoting a certain arrangement of cells in a galvanic battery, p. 586.

CHAPTER III.

MEASUREMENTS.

In the foregoing chapters we have become acquainted with the units of Space, Time, and Mass, and with those, derived from the preceding, of Velocity, Acceleration, Momentum, and Force; and it is for us now to ascertain what principles are made use of in the various measurements effected in terms of these units.

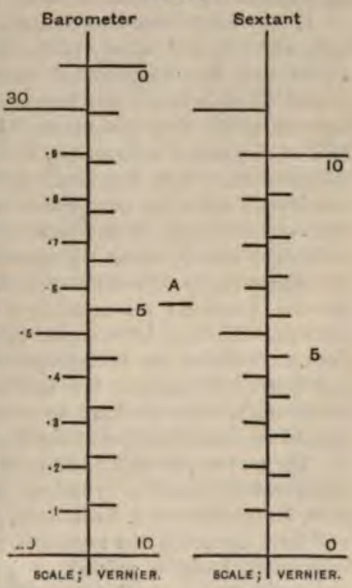
In the **Measurement of Lengths** two main methods are resorted to—Line measurement (*mesure à traits*) and End measurement (*mesure à bouts*). The former is the method in habitual use among carpenters, who lay off so many feet and inches by the aid of their pocket-rule; the latter is the method which they use when they measure the width of a cavity by means of a pair of callipers, which they open until it exactly fits the space.

Line Measurement.—The length of any line may be measured by a graduated scale, which may be, like the carpenter's pocket-rule, somewhat roughly graduated; or instruments on the same principle may be made use of which are finely and very accurately divided. The measurement is effected by observing the nearest coincidence of the marks on the scale with the length of the object, and by then reading off the value on the scale. This is a familiar operation, but it will be observed that it depends on the accuracy of the sight. The eye requires to be held directly first over one, then over the other end of the object to be measured, together with the corresponding part of the scale; for the true coincidences would be disturbed if the scale and object were looked at obliquely. It is found that in estimating measurements which are very small the eye gets confused; and besides this, the difficulty of making accurately and very finely graduated scales increases with the accuracy required. Thus it happens that, while the ordinary 24-inch rule divided into various fractions of an inch, or Whitworth's very convenient 20-inch rule, decimally divided, or a measure divided to half or quarter millimetres, may be used for measurements involving differences of the hundredth of an inch, it is only with difficulty that they can be so applied, and it is much more convenient in cases involving minute measurements, such as observations of the height of the barometer or thermometer, to use a contrivance called a Vernier. This is a subsidiary scale which slides up and down past the main scale, and is differently divided from it.

There are two kinds of Vernier in use, those known as the Barometer-Vernier and the Sextant-Vernier, which require separate description.

The Barometer-Vernier is thus graduated: a line is set off on the vernier equal to eleven divisions on the main scale. This line is divided into ten equal parts. Each of these parts therefore is equal to $1\frac{1}{10}$ division on the main scale. If the main scale be divided to tenths of an inch, the difference between a division on the main scale and one on the vernier is $\frac{1}{100}$ inch. Suppose the object measured to be more than 29·5 inches and less than 29·6 inches on the scale. The zero of the vernier is laid, as exactly as possible, opposite to the point whose position is to be found, the extremity of the object to be measured, or the height of the mercurial column in the barometer; then on looking *down* the vernier it will be found that at some point there is a coincidence between a graduation mark on the vernier and one on the main scale. The number of that mark on the vernier is noted, and that is the figure required in the second place of decimals. For example, let the point A be above 29·5, below 29·6 inches; the zero point 0 of the vernier is brought opposite to it: the point 6 of the vernier coincides with a division of the main scale; the length is 29·56.

Fig.3.



In the Sextant-Vernier, as it is called in this country, which is the form more usually found in instruments of foreign make, the divisions on the vernier run in the same direction as those on the main scale. A line is set off on the vernier equal to nine divisions of the main scale; this is divided into ten parts, each of which is equal to nine-tenths of a division of the main scale. In the same way the vernier is moved until its zero point is brought opposite the end of the object to be measured, and the mark *up* the vernier which first coincides with a division-mark on the main scale gives the figure as before.

Frequently, as in the sextant, a little magnifying glass is so placed that the zero point of the vernier may be by its aid brought more accurately opposite the object to be measured. When still greater accuracy is required, a microscope is so placed as to ensure the greatest possible completeness of coincidence between the zero point of the vernier and the end of the object to be measured, both of which are simultaneously brought into the centre of the field.

The Cathetometer is an instrument whereby vertical heights are measured. It consists of a vertical rod on which a finely-graduated scale is engraved. This carries a sliding piece to which is attached a telescope. In this telescope is fixed a pair of spider threads or fine platinum wires arranged at right angles to one another, and so placed (in the focus of the eyepiece) as to be visible simultaneously with the object looked at through the lenses. The telescope-carrier is placed in such a position on the vertical rod that the lower end of the object to be measured is seen, when looked at through the telescope, to coincide exactly with the point of crossing of the spider-threads in the field of view; then it is slid up until the upper end of the object to be measured appears to coincide with the same point; the distance along which the carrier has been slid along the vertical rod indicates the height of the object to be measured. Provision must be made in the construction of the apparatus for ensuring that the vertical rod is quite perpendicular to the horizon; this is effected by making it stand upon three screws whose heights can be adjusted until a spirit-level indicates the base of the apparatus to be quite horizontal.

It may be necessary to compare a standard measure with the length of a body which is very nearly of the same length as the standard. In this case, a microscope may be placed at each end. Coincidence as perfect as possible is established between the images of the object and the standard measure in the field of the first microscope. If, then, the coincidence be perfect in the field of the second microscope, the object is of precisely the same length as the standard. This but rarely occurs, and the object in view frequently is to ascertain what the error amounts to. The second microscope is provided with spider threads in the focus of the eyepiece, and the end of the object is brought exactly under the apparent crossing-point of these threads; then the microscope is moved along until the end of the standard appears to be in the same position; the extent to which the microscope has been moved indicates the difference between the two lengths compared. Since, however, the amount to which the microscope has been moved may be exceedingly small and difficult to measure, the methods hitherto described may be insufficient in accuracy, and we have to resort to those more delicate devices which depend on the properties of the Screw.

The Screw, as will be seen on examination of any specimen, presents a spiral coiled round a cylinder. If a screw, having twenty threads to the inch, be inserted in a fixed body, and turned round exactly once, its point will have advanced the twentieth part of an inch. If the head of the screw be connected with a pointer fixed on it at right angles, which can indicate on a graduated circle the amount of rotation of the screw, there will be no difficulty, even with roughly made apparatus, in causing the screw to execute a rotation of half a circle, a quadrant, 45°, 5°, or even 1°. If a screw, then, which has twenty turns to the inch be turned through one degree ($\frac{1}{360}$ of a complete turn), its point will have advanced or been retracted by $\frac{1}{360} \times \frac{1}{20} = \frac{1}{7200}$ inch. But this is rough measurement. By making the head of the screw part of a large wheel with graduated circumference and using a fixed vernier, rotation of the screw to the extent of half-a-minute of arc can be easily observed, and this would correspond to onward motion on the part of the point of the screw of $\frac{1}{768000}$ inch. The principle of the screw thus enables us to detect and to measure very small quantities of motion. If the second microscope in the last paragraph be connected with a graduated screw of this kind, the amount of its motion, indicating the difference of length of the two objects measured, can be very exactly determined.

In Sir Joseph Whitworth's measuring machines advantage is taken of another principle for producing and measuring very slight motion. The screw (twenty threads to the inch) is driven by a "worm-wheel," a wheel bearing 200 teeth on its circumference: this is propelled by a tangent-screw, a screw whose threads fit between the teeth of the worm-wheel: each turn of the tangent-screw sends each tooth of the worm-wheel forward into the position previously occupied by the tooth immediately before it—that is to say, causes the worm-wheel itself to revolve through the two-hundredth part of 360°, and to press the point of the screw forward by the four-thousandth of an inch. But the tangent-screw is itself driven by a wheel divided into 250 parts, so that if this wheel be turned round only one division, the tangent-screw is rotated $\frac{1}{250}$ of a turn, and the point of the "worm-wheel" screw is thus pressed forward the 250th part of $\frac{1}{4000}$, *i.e.* the millionth part of an inch.

If a screw be fixed at each end so that it can rotate but not progress, the "thread" of the screw will appear to travel when the screw itself is turned. If any object (the slide-rest of a lathe, or the like), have a female screw* cut in it, and be by means of that screw fitted upon a rotatory but otherwise fixed male screw; and if it be then placed between guides so as to be free to move backwards and forwards along the fixed screw but in no other direction: if then the fixed screw be rotated, the object borne by it will travel along it in one direction or the other, according to the sense of the rotation. This mechanism will be thoroughly understood on looking at the traversing-screw and slide-rest of a lathe. If the travelling carrier bear a pencil or a diamond, and mark paper or glass at equal intervals, as indicated by equal rotations of the driving wheel, we shall have a contrivance illustrating the main principle of the Dividing Engine which is used for graduating thermometer-tubes, etc.

End Measurement.—If a couple of rods, exactly ten feet in length, be placed on the ground end to end; if then the first rod be taken up and carefully laid down endways at the other end of the second; and if the second be taken up and placed in the same way beyond the first and just in contact with it, and so on: then a very accurate setting off of any multiple of ten feet can be easily effected, provided that the rods themselves are exactly ten feet long. Measurement of a given length can also be thus effected: if there be an odd number of feet and inches, they can be measured by a set of smaller rods, or by an ordinary tape measure.

In measuring or setting-off in this way, it is plain that we depend upon the sense of touch for the perception of the contacts set up between the ends of the rods. The sense of touch is found to give more satisfactory results in many ways than the sense of sight; for if one object be intended to fit into another, and have a diameter $\frac{1}{5000}$ inch less than what is exactly necessary, its fit will be perfectly loose. The eye could not perceive this directly without the intervention of lenses.

The Callipers used by carpenters can be opened out so as exactly to fit into a cavity, or exactly to grasp an object. They are usually made so that the one end serves for inside, the other for outside measurement. They are useful in comparing the dimensions of objects which should be of the same size; but it is difficult to take very accurate measurements off a scale with them.

Gauges are made of known sizes, and the size of the object to be measured is compared with that of the gauge by trying the fit. If the gauge be made conical, then from the extent to which it penetrates a given aperture can the width of that aperture be determined.

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* A screw cut out of a solid mass, through which another screw, the "male" passes. In the ordinary nut and bolt, the bolt bears the male screw, the nut the female.

The Spherometer consists of a disc of metal with graduated circumference. This is supported on three equal legs, which are furnished with hard steel points, equidistant, and rounded off so as not to pierce any object on which the instrument is set. In the axis of it is a screw, the steel point of which is also rounded, and forms a fourth foot. Any instrument which stands on three feet is certain to be steady, because three points are always in some one plane: one which stands on four feet will only be steady if the point of the fourth foot be exactly in the same plane as the other three. If it be above this plane the instrument does not rest on it at all; if it be below this plane the instrument can never stand on more than three feet at a time, and may be rocked from one set of three to another. If the spherometer be set upon a piece of glass, it will stand steadily; if the central screw be turned so as to bring down the fourth foot, the instrument will be easily rocked if it be brought down too far. The hand in perceiving and the ear in hearing this rocking, just at its commencement, concur in detecting very small motions of the screw just at that part of its movement. By means of a pointer attached to the head of the screw the exact position of the screw which corresponds to the commencement of rocking can be observed on the graduated scale. Suppose the thickness of a piece of microscopic cover glass is to be determined. It is placed under the fourth foot. This central foot of the spherometer is brought down upon it until the whole rocks; the central screw is then raised until the rocking ceases; it is turned back again till it just commences, and, as before, the position of the screw corresponding to the commencement of rocking can be observed by means of the pointer and the graduated scale. If the pointer had stood at 75° when the instrument stood on the plain glass, and at 3° when the central point was on the piece of thin glass, the difference of position of the pointer corresponds to 72°, or $\frac{72}{360}$ of the circumference; and if the screw itself have twenty turns to the inch, the thickness of the glass is $\frac{72}{360} \times \frac{1}{20} = \frac{1}{100}$ inch.

The curvature of a lens may be determined by this instrument, for if the lens ABD be placed under a spherometer, Fig. 4 shows that the amount of curvature determines the length of the line DE, and the radius R of the sphere of which the lens may be considered a part is related to the line DE (represented by a) and the distance l between the equidistant tripod feet, by the formula $2R = \overline{a^2/3l} + l$.

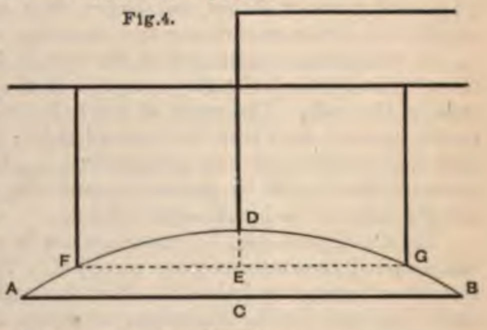


Fig. 4.

In Whitworth's Measuring Engine a bar representing the unit of length is placed between two jaws, which are made to move towards one another so as, without pressure, just to grasp it: they are then separated from one another, and the standard unit removed: the bar to be measured is placed instead of it, and the jaws are again brought together so as to grasp it in the same way. The jaws are brought together by fine screw adjustments, such as those previously described, so that the difference of the millionth part of an inch in two bars of metal can be detected. The precise position at which the jaws grasp objects without pressure is determined by a plane piece of metal, which is included along with them between the jaws, with its edges in a vertical plane. If the grasp be too loose, this piece of metal can be moved freely, and will fall back when lifted and let go; if the grasp be too tight, this metal plane cannot be moved; if it be exact, the metal plane can be raised, and will remain in any position in which it may be placed.

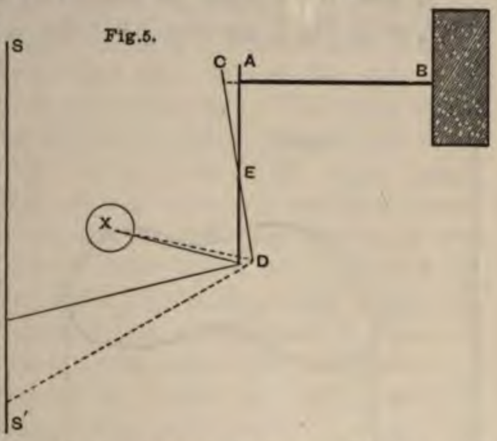


Fig.5.

Another plan by which an alteration in the length of a bar may be determined is the Optical. The end A of a bar AB rests against a strong framework at B, so that any alteration in its length may only affect the position of the point A. At A the bar is in contact with a lever CD, jointed at E, and bearing a mirror at D. A lamp at X casts a ray of light on the mirror; this is reflected to a screen SS′. If BA alter in length, or if another bar of slightly different length be substituted for it, the bar CD assumes another position, and the spot of light on the screen SS′ is deflected. From the amount of deflection may be calculated the alteration in length of the bar BA.

Good linear measurement, in whatever way effected, ought to present an error less than $\frac{1}{10000}$ %, or one-millionth of the whole.

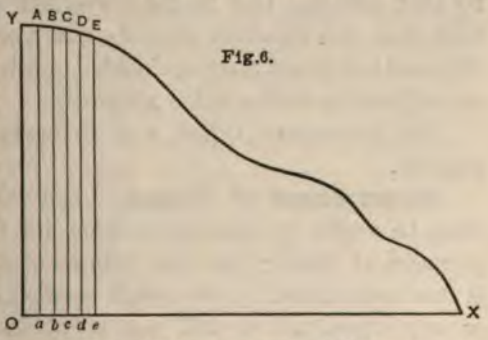


Fig.6.

Measurement of Surface.—If a surface be bounded by straight lines at right angles to one another, the parallelogram may be measured by the product of two adjacent sides: if it be of any other form bounded by straight lines, it can be broken up into triangles, and its area be found by the rules of trigonometry: if its boundary be a regular curve, its area can generally be found: but if the surface be bounded by an irregular curve, the determination of the area involves the following principle.

Let the figure YXO be bounded by the two rectangular straight lines OY and OX, and the curve ABCDEX. Find its area. Draw a series of lines parallel to OY; these will cut the curve in the points A, B, C, D, E, and so forth. Then the area

YXO is divided into a number of narrow parallelograms, OYA*a*, A*ab*B, B*bc*C, etc. Each of these is equal to the product OY × O*a*, *a*A × *ab*, etc.: these being all found and added together give the area of the surface.

If now the surface be completely bounded by an irregular curve, as in Fig. 7, the area ABCDEA is first found by the above method, then the area A*b*CDEA. The difference between these represents the area of the curved surface ABC*b*. This method is very difficult in actual practice, but all the mathematical methods of integration are based upon this principle. For actual work a convenient means of measurement of surface, which gives very fair results, and which is specially useful in those cases in which mechanical contrivances have registered their own performances on paper, is the following:—

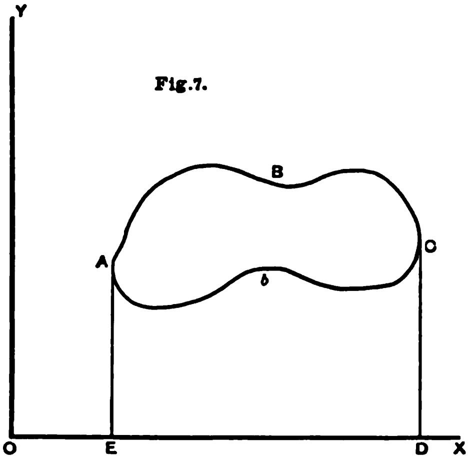


Fig. 7.

The paper on which the curve is drawn is laid on a flat board, and the outline of the surface very carefully traced by a sharp-pointed penknife, so as to cut out the part of the paper bounded by that outline: this is then weighed and its weight compared with that of a standard area, say a square inch of the same paper. This method is not unexceptionable, but it often gives a very useful approximation to the value required.

An instrument called a planimeter is also used for this purpose.

Measurement of Volume.—The volume of a substance may often be found by calculation from its form if that be a known geometrical figure; but the volume of a mass of irregular figure is best ascertained by the rough method of immersing it in water or any liquid which will not affect it, and by observing how much more bulk the whole now occupies than the water alone had done.

If, for instance, a piece of metal be placed with three fluid ounces of water in a measure, and if the whole measure exactly four fluid ounces, the piece of metal must occupy exactly the same bulk as one fluid ounce or $\frac{1}{16}$ gallon of water; that is, since a gallon of water occupies 277·274 cubic inches, (277·274 ÷ 80) or 3·466 cubic inches; and so for fractional parts of the units of liquid measure.

Measurement of Time.—It is not possible or necessary to do more in treating of this than to suggest one or two leading principles. A simple water-dropper, consisting of a vessel of water in the bottom of which there is a minute hole, through which the water falls, drop after drop, into a dish, was used anciently under the name of the Clepsydra. The water which fell through was kept in the lower vessel: the amount there accumulated, or equally the loss of level in the upper vessel, indicated approximately the lapse of time. It was found, however, that the flow of water from a vessel of this description was far from uniform. The use of Wheelwork set in motion by some constantly acting force was a fruitful suggestion: setting the wheels to indicate the amount of their own rotation by means of pointers connected with their axles was a plan early adopted; the train of wheelwork was set in motion by a falling weight; but these wanted some regulating contrivance by which the motion might be rendered uniform. A heavy flywheel was adapted to the mechanism, but without the desired result being fully attained; and it was only after Galileo's observation of the fact that the Pendulum oscillates from side to side in almost exactly equal periods of time, whether its arc of oscillation be great or small, that it was suggested that this property of the pendulum might be rendered available for regulating clockwork. This was effected by Huyghens; and the action of all pendulum clocks, however various the trains of wheelwork, depends on their regulation by an isochronously—*i.e.*, in equal times—oscillating pendulum. The simplest mode in which this regulation may be effected is the following:—One of the wheels of the train of mechanism bears on its circumference an appropriate number of teeth. The descent of the weight would, if there were no pendulum attached, cause the mechanism to run on continuously until the weight had run down to its lowest possible point; but at every stroke of the pendulum one of the teeth of the wheel is caught and the further progress of the wheelwork arrested.

The isochronism of the oscillations of the pendulum is not sustained; variations in the external temperature cause changes in the length of the pendulum, and hence in its rate of motion. The contrivances by which compensation is made for this cause of error, so that the rate of oscillation is maintained practically uniform, will be explained under Heat.

The measurement of small intervals of time is of great importance. A tuning-fork, if a writing-point be attached to it, will, when vibrating, describe wavy lines on a piece of smoked glass or paper drawn under the writing-point. If the tuning-fork vibrate 400 times per second, the time taken to draw each wave on the paper must be the four-hundredth part of a second; and if any other phenomenon be so produced and arranged as to record its own performance by a line on the paper or the glass, parallel to the wavy line of the tuning-fork, its duration may be estimated by counting the number of recorded vibrations of the tuning-fork to which that duration corresponds.

Measurement of Mass.—Masses are compared with one another by means of the Balance. The accurate and expeditious use of a delicate balance involves attention to certain practical rules, which will be found set forth in Kohlrausch's *Physical Measurements.*

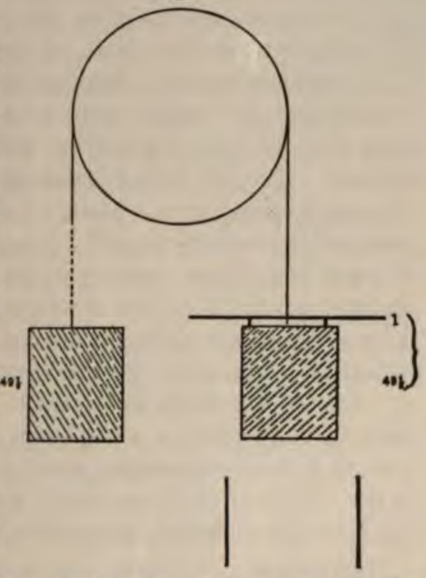
Measurement of Force.—There are four main methods of measuring any force. These may be stated as—

1. Direct Observation of Mass and Acceleration.
2. Direct Counterpoising.
3. Indirect Counterpoising.
4. The Method of Oscillations.

The first, the method of **direct observation** of the mass moved and the acceleration imparted to it by the force to be measured, is based on the equation $F = ma$; and if m the mass and a the acceleration be known, F, the Force acting, can easily be found. This method presents, however, serious practical difficulties in the observation of the acceleration produced.

One important problem to be solved by this method is the determination of the force with which gravity acts upon a unit mass of matter at any place. The equation $F = ma$ shows that if we use a unit mass, $F = a$; thus we need only find the acceleration produced. This is effected roughly by Attwood's Machine. In this the weight of one gramme is used as the force which sets a larger mass in motion. If it set only its own mass in motion, a velocity is acquired so great as not to be easily observed: if this limited force set a larger mass in motion, the speed acquired is less, varying inversely as the mass, for $a = F/m$. If a gramme in falling set a mass of 100 grammes (including its own substance) in motion, it can only acquire a velocity one-hundredth that which it would have acquired if it had fallen alone. The essential part of Attwood's machine consists of a wheel over which two masses are suspended. Let these masses be $49\frac{1}{2}$ and $50\frac{1}{2}$ grammes. The total mass set in motion is 100 grammes, and the force acting is the excess in weight of the heavier mass over the lighter—that is, $50\frac{1}{2} - 49\frac{1}{2} =$ the weight of one gramme. Let this gramme not be a fixed part of the heavier mass, but merely a piece of wire which can be removed by making the weighted mass fall through a metal ring. A pendulum which beats seconds regulates a timepiece; attached to the wheelwork of the timepiece is an "excentric," which works a lever; this lever, at a pre-arranged instant, pushes or pulls away a little plate which supports the heavier mass; this mass finds itself suddenly exposed to the action of gravity; the weight of the little gramme-load imparts to the whole mass a certain velocity; the ring is placed at such a position as to catch the wire exactly at the end of one second, this being indicated by the sound of the timepiece and pendulum coinciding with the click of the wire upon the ring which catches it as it falls. Thereafter there is no force acting, and the mass of 99 grammes continues to move uniformly according to the first law of motion. Its speed can be observed by comparing the distance it travels with the ticking of the timepiece. This is done by placing a little plate to receive the falling body. A slight sound will be made by the falling body touching this plate. If this sound and that of the pendulum coincide the plate is in the right position; if not, that position must be found by a process of trial and error. It is found that if a pair of masses of 49½ grammes each be suspended over the pulley, and one of them be loaded with one gramme so that the whole mass to be moved weighs 100 grammes; if the overweight be taken off at the end of one second by a ring; if the balanced masses be allowed to move onward with their then acquired velocity for one second; if a plate be so adjusted under the ring as to check this motion precisely at the end of a second—it is found that that plate must be 9·81 centimetres below the ring. This shows that the force acting (the weight of one gramme), acting for one second, is able to impart to a mass of 100 grammes a velocity of 9·81 centimetres per second. Hence by the equation $F = ma$, F, the weight of one gramme, is equal to 100 grammes × 9·81 centimetres = 981 dynes. This method can give no more than an approximation to the value required.

Fig. 8.



Much greater accuracy is attained by the use of the Pendulum. The time of oscillation of a pendulum, as we shall afterwards learn, varies inversely as the square root of the force of gravity at the place where the observation is made. The time of the oscillation of any pendulum can be very accurately known by observing the time taken to perform a certain sufficiently large number of oscillations, and dividing that time by the whole number of oscillations. From this observation can be deduced the local acceleration of gravity.

Measurement of Force by Direct Counterpoising.—In an ordinary balance whose arms are perfectly equal, the force with which gravity acts on the mass in one pan is equal to that with which it acts on the mass in the other. For one of these we may substitute another force of any kind but of equal amount. If, for instance, we use a balance with glass pans, we may lay one of the glass pans on the surface of mercury and determine what mass must be put in the other pan to pull the first from the mercury. Let this be 47 grammes, and the area of the glass pan 25 square centimetres. Then a force equal to the weight of 47 grammes is necessary to pull 25 square centimetres of the surface of glass away from mercury—that is, 1·88 gramme per square centimetre; and the force of adhesion between mercury and glass is for every square centimetre equal to the weight of 1·88 grammes—that is, a force of $1{\cdot}88 \times 981 = 1844{\cdot}28$ dynes.

A soap film tends to contract. If we find how much mass must be suspended on a soap film of a certain size in order to prevent it from contracting, the force of contraction will be equal to the weight of the mass which the film supports, and that force can hence be measured in absolute units.

This method, as well as the next, lends itself readily, so readily that no special explanation is necessary, to the measurement of stresses, pressures, tensions.

Measurement of Force by Indirect Counterpoising.—Let us suppose that we have access to a standard unit of mass. This is hung upon a spiral or spring of steel wire. It is observed to lengthen the spring by a certain measured amount. If another mass be hung upon the same spiral, and if the lengthening produced be the same, the inference is that the action of gravity upon the second mass is equal to that on the first, and hence, if the two observations be made at the same place, that the second mass is itself equal in quantity to the first. This is the principle of the Spring Balance. Different known weights may be suspended on such a spiral, and the elongations produced may be recorded on a scale attached to the instrument. If a mass of unknown amount be attached to the spiral, its weight may be found by reading on the scale the number of standard pounds and ounces, etc., requisite to produce the same distortion as the unknown mass causes when hung upon the spring.

The instrument known in one form as a spring balance is known in another as a Dynamometer. The form of the steel spring used is quite independent of the principle involved, which is that if two forces produce equal distortions in a body, these forces must be equal to one another. If a man can pull a spring out two inches, and if 200 lbs. must be hung on the spring to produce the some distortion, the man's pull is equal to the weight of 200 lbs.; similarly the force required to pull the spring out two inches is equal to that which must be exerted to raise a weight of 200 lbs.; and these can be translated, when we know the local acceleration of gravity, into forces measured in absolute units. If he can give it a blow which will compress it for a moment to the same extent as a weight of 140 kilogrammes placed on it would do, the force of his blow is equal to the weight of 140 kilogrammes—that is, $140{,}000 \times 981 = 137{,}340{,}000$ dynes. If he can, by closing his hand firmly, distort a spring to a certain extent, it can easily be ascertained what amount of weight acting on the spring is capable of producing the same distortion. This is usually done beforehand, and the instrument is provided with a graduated scale which indicates what amounts of weight—at the place where the instrument is made, be it remembered—correspond to the various readings of the pointer. When his flexor muscles contract so as to force the pointer of the dynamometer to indicate 84 kilogrammes, the distortion produced by them is equal to that produced by the weight of 84 kilos at Paris if the instrument have been made there—that is, since $W = mg$, 84,000 grammes × 981 cm. = 82,404,000 dynes.

Illustrations of this principle abound. The attraction of magnetism may be measured in a similar way. Let a magnet attract a piece of iron, which is attached to a spiral, to such an extent that the spiral is lengthened, say one inch, when the magnet is at a distance of a tenth of an inch from the iron. It is found that, say, 2 lbs. 3 oz. must be hung on that spiral to produce the same distortion; the magnetic attraction is equal to the local weight of a mass of 2 lbs. 3 oz. This is an undesirable method from the practical point of view, but it shows how magnetic and other attracting forces can be compared with forces whose absolute amounts we know.

If an electromagnet can hold ten pounds of iron, but cannot support ten pounds and a grain, the force of attraction is equal to the weight of ten pounds; for instead of the magnetic attraction, we might have used, in order to prevent the ten-pound-mass of iron from falling, another ten-pound-mass connected with it by a cord passed over a pulley.

If we take a bar of metal, suspend it on centres at each end, fix it firmly at one end so as to prevent that end from rotating, and hang a known weight over the side of that end which is free to rotate, we find that the bar is twisted; this effect is measurable. Whatever other force will produce the same effect must be equal to the known weight which caused it. If the body to be twisted be a glass or silk fibre, the amount of force required to twist it is small. To twist such a fibre through a certain number of degrees, a certain fractional number of grammes' weight must be applied at unit-distance from the centre. If an electric attraction be applied to a body suspended by such a silk fibre, the suspended body is attracted, the suspending fibre may be twisted; to produce the observed torsion or twist, a certain number of grammes' weight must be applied; the electric attraction can be stated to be equal to the weight of so many grammes, and therefore to so many absolute units of force.

Ruhelage: Equilibrium-position.—It is often advantageous to measure the force acting on a displaceable object by balancing that displacing force against another force, so adjusted as to bring the displacement back to zero value. A magnetic needle deflected by a current is twisted back into its original position by a twisted suspending fibre; the torsion imparted to the suspending thread is measurable and represents a known number of dynes. The force acting on the needle is thus measured. The advantage of this method is that we obtain precisely what we wish, the full force exerted by the current on the needle when in its original position, not the force acting on it in any other position; and we thus eliminate any disturbance produced by such variations in that force as may be due to variations in the position of the suspended needle itself.

The fourth method is that of **oscillations.** If a magnet be brought near another magnet it oscillates from side to side. If it be brought near a stronger magnet it oscillates more frequently. It can be proved that the velocities produced vary as the square root of the forces causing the oscillations. Hence we count the number of oscillations in a given period in two cases, and the ratio of their squares is the ratio of the two forces. If, for instance, a magnetic needle oscillate fifteen times a minute in presence of a magnet A, and sixty times in presence of a magnet B; the forces acting in the two cases are as the square of 15 is to the square of 60, or as 1 to 16. In this way we are able to compare the forces acting under the given conditions, but we do not learn the absolute amount of either. That must be ascertained by one of the other methods previously discussed.

CHAPTER IV.

WORK AND ENERGY.

Work.—When a force "acts upon" a body, and that body moves in the direction of the force, that force is said to Do Work, and the work said to be done *by* it is measured by the product of F, the force acting, into s, the space through which the body has moved—

$$\text{Work} = Fs = mas.$$

For example: Steam exerts on the piston of a cylinder a force or pressure of, say, 30 lbs. per square inch; the area of the piston is, say, 30 square inches; the whole pressure exerted is thus equal to the weight of 900 lbs. The piston is thrust through, say, 16 inches. The work done is 900 lbs. × $1\frac{1}{3}$ ft. = 1200 foot-pounds at each stroke.

Conversely, when a force acts upon a body and that body moves or is moved in a direction *opposed* to that of the force, that force is said to be Resisted, and work is said to be done *against* it; and Fs, the product of the force resisted into the space traversed, represents the work said to be done *against* the force so resisted.

When a ten-pound mass is raised ten feet against gravity, the work done against gravity is equal to the product of the space traversed into the force resisted—*i.e.*, 10 ft. × wt. of 10 lbs. = 100 foot-pounds. In this case Work = Fs as usual; but F, the force resisted, is the weight of a mass m, and therefore $F = mg$: consequently the work done $Fs = mgs$.

Suppose a man to walk against a heavy gale of wind, the mean pressure of which is 40 lbs. per square foot. If the surface presented to the wind-pressure be virtually 5 sq. ft., the total pressure of the wind will be 200 lbs., and the effort of walking against it will be the same as if the man pulled a weight of 200 lbs. out of a pit by means of a cord thrown over a pulley. If the man make his way for a mile he will have resisted a mean pressure of 200 lbs. through a space of 5280 ft. He will, therefore, have done 1,056,000 foot-pounds of work; an amount of work which, otherwise directed, would have sufficed to lift him up (his total weight being supposed to be 150 lbs.), to twice the height of Snowdon.

There is no Work done against or by the force acting unless there be actual Motion. We might imagine machinery to be driven by an avalanche during its fall; but not before, and not after. Gravity does no Work upon a resting stone: it does work upon a falling stone.

If $Fs = 1$, we have the Unit of Work. This is the case when $F = 1$ and $s = 1$; that is, a unit of work is done when a body acted on by unit force moves through a unit distance. In centigramme-second-gramme measures the unit of work is done by raising $\frac{1}{981}$ gramme (mass whose weight at Paris = 1 Dyne) to the height of one centimetre. This is the **Erg.** The erg is, however, a very small unit of work, and for many purposes it is convenient to use the Megalerg, which is equal to 1,000,000 Ergs, and would therefore be the amount of work done in raising $\frac{10000}{981} = 10{\cdot}19$ grammes through one metre; or the Ergten, 10^{10} or 10,000,000,000 Ergs; or the Joule, 10^7 or 10,000,000 Ergs.

In British measures 32·2 units of work are done in raising a pound-mass through one foot. Such units of work are called foot-poundals. British engineers are in the habit of using the foot-*pound* (the work spent in raising one pound one foot) as a unit of work. This would be satisfactory if foot-pounds were equal over the whole earth, but g, the acceleration of gravity, varies from place to place. Hence the foot-pound is from place to place a variable measure, varying between the Equator and the Poles by about one-half (0·512) per cent.; and it has to be reduced for each place to absolute units of work by the equation—$\text{Work} = Fs = \text{Weight} \times s = mgs$, and the foot-pound is equal to g foot-poundals, where g is measured in feet (32·2 nearly). The foot-pound is equal to 13,562,691 Ergs, when $g = 981$ cm. The kilogramme-metre, or French engineers' unit of work, is 1000 grms. × 100 cm. $\times g = 98{,}100{,}000$ Ergs.

Any amount of work may be specified as the product of two numbers, which represent respectively a force and a space. These may vary, but if they have the same product the amount of work done is the same. A pound raised 100 feet, 100 pounds raised one foot, fifty pounds raised two feet, four pounds raised twenty-five feet, all represent the same amount of work, namely, 100 foot-pounds, it being here assumed that the force of gravity is uniform within heights of 100 feet.

Since $\text{Work} = Fs$; it follows that $F = \text{Work} \div s$; whence Force is the number of units of work done upon or by a moving body divided by the number of units of space traversed by that body. Force is therefore a **rate** at which **work** is observed to be done, **per unit** not of time but of **space traversed.**

This looks like a definition obtained by reasoning in a circle; but if it be presented in the equivalent form—Force is the Rate at which a moving body gains or loses either potential or kinetic Energy per Unit of Space traversed—we shall presently understand that it is not a truism, for Energy is a physical entity.

The Mean Rate of Doing Work is the whole work done in a given time divided by the time. If an engine can raise 1,980,000 pounds one foot in an hour, its mean rate of doing work, or, as Sir William Thomson phrases it, its Activity, is 33,000 foot-pounds per minute, or 550 foot-pounds per second. This particular mean rate is known by Engineers as a Horse-power; and an engine of one-horse power can do this amount of work. A horse can, according to General Morin, do 26,150 foot-pounds per minute, and a labourer from 470 (lifting earth with a spade) to 4230 (raising his own weight, treadmill exercise) per minute. The French horse-power (cheval-vapeur) is 75 kilogrammeters, or 7,500,000 $g = 7{,}357{,}500{,}000$ Ergs per second; whilst the British horse-power is equal to 7,459,480,050 Ergs per second, when $g = 981$ cm.

The Unit of Activity is frequently taken as one Watt,* which represents 10 Megalergs per second. The British horse-power is thus equal to 746 Watts nearly, the French to 735$\frac{3}{4}$.

If a man weighing 14 stone run upstairs at such a rate as to gain 3 feet in vertical height every second, his muscular system is doing every second the work of carrying 196 lbs. up 3 feet, *i.e.*, 588 foot-pounds. If this could be kept up for a minute, $60 \times 588 = 35{,}280$ foot-pounds would be done, and the man would be, in the case supposed, undergoing an exertion which for the moment would be much greater than a horse can keep up, and seventy-five times that which a continuously-toiling labourer, lifting earth with a spade, can sustain; and in the most favourable circumstances, a labourer, raising his own weight merely, can only keep up one-eighth of this effort.

Activity is also measured as Fv, Force × Velocity; for $v = s/t$; and, therefore, Activity $= Fs/t = Fv$.

Energy.—When a body weighing ten pounds is raised ten feet, and prevented by a catch from falling, the work done upon it—100 foot-pounds—can be recovered by permitting it to fall upon a train of mechanism. If the mechanism were perfect, the work would be so restored that another ten-pound mass might be projected by it to a height of ten feet, a fifty-pound mass to a height of two feet, and so on. The fact that we cannot obtain perfect mechanism does not affect the principle. The body at a height has therefore a power of doing work equal to the work done upon it in lifting it. In this case the power of doing work has been conferred upon a body by the separation of it from the earth against the action of gravity: as it remains in its elevated position, there is a stress, or pull, or attraction, tending to draw it down, and it is only in virtue of this stress that it has any power of doing work. If the earth and the elevated body ceased to attract one another, the body would, if liberated, not fall down, and would not restore the 100 ft.-lbs. of work spent upon it. We know that the work done in raising a mass m through a height s against gravity is mgs: the energy stored up in the body is therefore equal to mgs, and is seen to depend on the mass of the body, the height at which it is placed, and the local accelerative effect of gravity. Energy, or power of doing work stored up in this way, is called **Potential Energy,** or Static Energy, or Energy of Position, or Energy of Stress. As an example of Potential Energy we may take that stored up in a mill-pond. The number of units of Energy in such a pond may be found by taking the product of the quantity of water in it and the average height at which it is placed, and multiplying that product by the local value of g. A small quantity of water at a great height may obviously have the same amount of energy stored up in it as a larger quantity at a lesser height. If the question be put, How much work could be got by appropriate mechanism from the rise and fall of the tide?—we consider (1) the total amount of water carried into the area which can be brought within the range of the mechanism, (2) the average height to which it rises, and (3) the local value of g.

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* There is some confusion as to this. Sir C. W. Siemens suggested (*Brit. Assoc., Presid. Address*, 1882) that 10^7 ergs be called a Joule, and 10^7 ergs-per-second a Watt. Practical men are, however, drifting into the habit of speaking of a Watt (10^7 ergs) or a Watt-per-second, as the case may be.

We have also energy stored up in such bodies as watch-springs. Work is done upon them in distorting them, and producing a movement, not of their mass as a whole, but a relative displacement of their parts. This work is restored and utilised in producing movement of the mechanism attached. When a watch-spring is distorted and held fast so that the distortion or strain persists, the whole mass remains in a condition of Stress, and tends at the first opportunity to restore the work done upon it.

If we look at our previous example of the earth and a stone lifted from its surface, we see that the phenomenon is on the large scale one of the same order. The earth and the stone together constitute a system: when this is deformed by pulling the stone away from the earth, the system tends to return to its original form, and there is a stress between the earth and the stone, which continues until the stone is allowed to fall back to the earth. If the stone had been connected with the earth by a band of indiarubber, we would have seen the indiarubber to be stretched or under stress, and would easily see that if the stone were liberated it would be pulled back towards the earth; but the question is, What is under stress in the actual case? for there is no visible connecting cord between the stone and the earth. If we could state what this was, we would be able to arrive at the cause of Gravitation. As it is our knowledge ceases. That there is some medium, and that it may be under stress, is a theory necessary for the exposition of Electricity, of Light, of Magnetism, and of Heat; but we are by no means, as yet, entitled to say that stress in this medium is the cause of gravitation.

Work may be done, then, in altering the relative configuration of a system, whether this consists of large masses or of smaller particles. If this system be what is known as a "Conservative System," in which a stress may be established depending upon the configuration, and only upon the configuration (not in any degree upon the history of any antecedent deformations through which the configuration in question may have been arrived at), the system will tend when work has been done upon it to return to its original form, and to restore the work done upon it. If its relation to surrounding objects be such that it cannot so return, it will be under stress, and will continue under stress until its relations to surrounding objects have become such as to permit it liberty of restitution; then, at the first opportunity, it will restore the work done upon it.

The change in its relations to surrounding objects necessary to render this restitution possible may be very small; for example, a heavy mass may be prevented from falling by a very small catch, but when the catch is removed the body falls. The cause of the body falling is not simply the release of the catch, but also the previously existing conformation of the distorted system.

Similarly, the ingredients of Gunpowder have a tendency to combine: its particles are chemically separate, but chemically attract one another, and therefore possess potential energy; the application of a very small amount of heat, as by a spark, liberates these particles, which can rush together and form new and stable compounds, which have no longer any tendency to alter their chemical constitution, being no longer under the same stress, having no longer the same potential energy. As it happens that in this special case the new and stable compounds formed are mainly gaseous at the ordinary temperature and pressure, the products of combination occupy a much larger bulk than the original gunpowder, and the result is an explosion. The spark only produces its own small effect; the previous arrangement of the particles of the powder is responsible for the rest.

poundals. The one kind of energy, the potential, is transformed into another, the kinetic, and there is in the system (earth and stone) neither gain nor loss of energy during the transformation. This is the simplest case of a widely applicable principle, that of the **Conservation** or **Indestructibility of Energy.**

This principle is, that if a system of bodies have a certain amount of energy in one form, it must retain that energy in one form or another unless it come into such relations with other bodies as, together with them, to form a larger system in which the energy becomes differently distributed; and if the system be so large that there is no other body with which it can enter into such relations—that is, if the system which possesses the energy be the whole Universe—that system cannot gain or lose energy by sharing with other bodies, and hence the total amount of Energy in the Universe is invariable and numerically constant.

If we take the instance just discussed, that of the earth, the bullet, and the gun pointed vertically upwards, these three bodies possessed before the explosion a certain amount of energy, potential in the gunpowder: just as the bullet left the gun, kinetic in the bullet: when the bullet was detained at the summit of its course, potential between the bullet and the earth, but always equal in amount—the same number of foot-poundals. While the kinetic energy was being transformed into potential, work was being done in establishing a state of stress. During this period the bullet and the earth were relatively moving, and the acceleration associated with the transformation of one kind of energy into another is attributed to a force acting during that period. Force is associated with a variation in the rate of change of the configuration of a system under which the energy in that system is altered in its distribution and form, and is said to act only as long as that variation continues.

Transformations of Energy.—Energy, however, assumes other forms than the two discussed. If the bullet in the case adduced be allowed to fall to the ground, it falls more and more rapidly until it regains its original velocity, and therefore its whole kinetic energy. But this bullet may suddenly strike the ground and lose all its kinetic energy: it has already lost all its potential energy; what has become of the energy of the system? We find that the bullet and the part of the earth on which it has fallen are warmed, and we learn from a wide induction of similar cases that Heat is one of the forms of Energy. It is proved to be so by the observation that the same amount of work, if entirely spent in producing heat, will always produce the same amount: 772·55 foot-pounds of work were found by Joule to correspond to an amount of heat capable of raising the temperature of a pound of water from 60° to 61° F. The Heat possessed by a body is explained as being the Energy possessed by it in virtue of the motion of its particles. Just as a swarm of insects may remain nearly at the same spot while each individual insect is energetically bustling about, so a warm body is conceived as an aggregation of particles which are in active motion while the mass as a whole has no motion. Heat is therefore a form of kinetic energy: and the more heat is imparted to a body the greater is the kinetic energy of each particle. If $\overline{\mathrm{M}}$ represent the average weight of the particles, and $\overline{\mathrm{V}}$ their average velocity, $\frac{1}{2}\overline{\mathrm{M}}\overline{\mathrm{V}}^2$ represents the average kinetic energy of each particle; and the sum of all the masses multiplied by half the square of the average velocity represents the intrinsic kinetic energy of the whole mass. The words "sum of" are expressed by the symbol Σ. Hence, Intrinsic Kinetic Energy $=\Sigma(\frac{1}{2}\overline{\mathrm{M}}\overline{\mathrm{V}}^2)=\frac{1}{2}\overline{\mathrm{V}}^2\Sigma(\overline{\mathrm{M}})=\frac{1}{2}m\overline{\mathrm{V}}^2$, where m is the whole mass.

When a bullet possessing actual energy of motion impinges on a target there is a certain amount of Heat obtained, and the bullet may be partly fused: there is also a flash of Light and a certain amount of Sound. Light seems to be a phenomenon of wave-motion in that Ether whose existence throughout space is apparently a necessary hypothesis; so also is Radiant Heat, such heat as streams to us from the sun, or from a fire across a room; and in that Ether, partly swinging, partly distorted by the passing waves, the energy is partly kinetic, partly potential: thus we say that the Energy of Light—or, briefly, Light itself—is a distinct form of Energy.

When a tuning-fork is made to vibrate, work is done upon it in giving it in the first place a distorted form. Its arms swing like pendulums, but their vibration gradually dies away and the energy of vibration of the fork becomes converted into the partly kinetic, partly potential energy of vibration of the air—that is, into the Energy of Sound; and ultimately it is converted into uniformly-diffused Heat.

Energy may appear, then, as Energy of Mechanical Position or Motion, as Heat, as the Energy of Light, of Sound, and again as that of Electrical or Magnetic condition; and a great part of our work is to study the modes in which the various forms of Energy are transformed and redistributed, and the forces and the phenomena attributed to forces which are associated with these transformations and redistributions.

A few other examples of Transformation of Energy may here be added. A man, ascending a stair, gains some potential energy : it is found (Hirn) that he is perceptibly cooler for a moment. The heat of his body has been partly transformed into potential energy. Of course the exertion of his muscles and the excitement of his circulation cause him to become warm immediately afterwards. When he comes downstairs he sacrifices the potential energy which he had possessed when upstairs in virtue of his elevated position, and which he might conceivably have utilised by dropping himself out of the window on an appropriate machine placed on the pavement. This energy is not lost, for he is (Hirn) perceptibly warmer at the bottom of the stair than he had been at the top. At every step downstairs he had arrested his own fall, and had consequently converted a part of his potential energy first into kinetic energy and then into heat.

When a quantity of water is decomposed by an electric current, the electric current is diminished and work is done in tearing asunder a certain number of particles of oxygen and hydrogen. These separated atoms tend to fall together again and form the stable compound, water. The mixture of oxygen and hydrogen thus formed by "electrolysis" possesses potential energy of chemical separation. When a flame is applied to the mixture, a process of recombination commences, and the whole of this potential energy is sacrificed as such, but appears in the form of heat, light, and sound, and may in an appropriate gas-engine be partly spent in doing mechanical work.

The heat and light produced by combustion and by chemical combinations in general are forms of energy obtained by transformation of the potential energy which the particles had previously possessed in virtue of their chemical separation and chemical affinity. Under certain circumstances this potential energy may not be transformed into heat or light, but, as in the galvanic battery, into the energy of a current of electricity, which may in its turn be made to do work, be transformed into heat, into light, into sound, or be spent in setting up magnetic condition, and so on.

When an engine goes round without doing work the steam remains hot. When the engine does work the steam is cooled, and the researches of Hirn have shown that the amount of work done is exactly equivalent to the heat which has disappeared.

The energy of an engine is derived from the heat evolved by the combustion of the coal. The coal of the furnace and the oxygen of the air rush together and sacrifice their energy of chemically-separate position, which was originally obtained by the action of the chlorophyll in the coal-producing plants.

When a plant is exposed to sunlight it has the power, by means of the chlorophyll or colouring matter of the leaves, of breaking up carbonic dioxide, CO_2, of evolving part of its oxygen in the free form, and of depositing the carbon in a less oxidised form in its own tissues. The work thus done by the plant in tearing asunder the constituents of CO_2 it is enabled to do by the energy supplied to it in the form of Light and Heat radiated from the Sun.

The Sun's radiant energy has next to be accounted for. This is not derived from combustion, for the sun would last but a comparatively short time if its energy were derived from any such source: its radiation of energy seems to correspond to 500 horse-power from every square yard, and such an enormous outflow would soon exhaust the store of energy if the sun were merely a huge fire: it is said that it could not last more than about 8000 years. It has been suggested that the meteorites which fall into the sun in great numbers are capable of accounting for the sun's energy; of the thickening of the sun due to this cause a very small amount corresponds to a very large amount of energy. Those meteorites which strike our own earth's atmosphere are retarded and greatly heated in their course through the upper regions of the air. If they be small enough they are entirely broken up, and their dust, characteristically ferruginous, settles down on the surface of the earth, and may be recognised in the dust collected from some specially favourable spots, such as glaciers, roofs, and snowy wastes, and the bottom of the sea. The kinetic energy lost by a meteorite falling upon the earth becomes distributed between it and the earth in the system of which the meteorite becomes a part, and this contributes to the total energy possessed by the earth; while its material goes to increase the earth's mass. In this way Nordenskjöld computes that the earth gains every year at least half-a-million tons. In the same way, the meteorites which fall on the sun must produce a flash of light, some heat, and a slight thickening of the sun. It has also been suggested that a very slight shrinking of the sun's mass would evolve a large amount of energy, its particles not being so far from one another after this contraction in bulk.

If the meteorite theory be accepted, the question arises, How did the meteorites get their energy of motion? This would relegate us to the consideration of the universe as a system of masses and particles containing as a whole a fixed quantity of energy: and this would bring us to the problem of the origin of this system.

Availability of Energy.—When a certain amount of energy has been spent in rubbing a button, the button is perceptibly warmed. The heat produced is exactly equal to the work done in rubbing. It is $\frac{1}{2}m\overline{V}^2$, where m is the hot mass and $\overline{V}$ the average velocity of its moving particles. All this we know. If a little time elapse, the button is no longer perceptibly warm: it has shared its heat with surrounding objects: their particles have been induced to oscillate more rapidly. Heat has thus a tendency to become uniformly diffused. It is then no longer available *to man* for doing work. It ceases to be power of doing work as far as he is concerned; but none the less do the particles of a hot body set in motion the particles of a cooler body, and the energy which has thus been imparted to these they can in their turn share with the particles of other cooler bodies. The temperature of a hot body tends uniformly to diffuse itself throughout the whole material universe.

In every Transformation of Energy we find that some energy is wasted through conversion into Heat, the result, direct or indirect, of friction, noise, flashes of light, and so on. This heat is presently distributed pretty uniformly among the surrounding objects, and can no more be made use of by us for the sake of producing work. A large quantity of the Energy of the Universe must have already assumed this relatively-useless condition, and in the course of time the whole of the Energy in the Universe will have assumed it. The Energy of the Universe is a constant amount: some of it is available, some is non-available: the former is in every phenomenon somewhat diminished but never increased: the non-available energy is constantly increasing: hence, as Tait put it, the "**Entropy**" (available energy) of the Universe tends to zero.

Clausius, the originator of the word Entropy, used it to signify a certain mathematical expression, the $\Sigma(H/\theta)$ of p. 366, which, as applied to the whole Universe, tends constantly to increase to a maximum. Thomson expresses this by saying that the **Motivity** (the proportion between the available energy and the whole energy) of the Universe tends to zero.

If with this clue we trace back the history of the Energy of the Universe we find, as we go back, less and less of the total Energy of the Universe to have become non-available. On going back far enough we arrive at a definite period when none of the total energy had become non-available. But in every actual phenomenon there is always dissipation in this way of some part of the total energy of a system. Hence we find that we are forced to realise a precise instant before which there were no phenomena such as those with which we are now acquainted, and since which such phenomena as are due to those relations of matter and energy which are within our knowledge have been occurring: while in the future we have to contemplate a moment at which the whole physical universe will have run itself down like the weights of a clock, and after which an inert, uniformly-warm mass will represent the whole material order of things.

The only way of escape from this conclusion is to lay emphasis on the fact that one part of the total Energy of the Universe is unavailable *to man*, and to suggest that at some time a state of things may supervene, as a result of which the molecular motion which is implied in a state of uniformly-diffused heat may be so arranged and directed as once more to produce a state of things such that particles may become aggregated into masses, in which all the particles may move on the whole in the same direction. This is what Clerk Maxwell's "Demon" is pleasantly imagined to do; he separates those particles which he prevents from going in one direction from those which he allows to go in another, so that ere long, without expending any work, he has the particles divided into two groups, moving in opposite directions. This is interesting, but it is not pretended that it is any other than a speculation.

"Conservation of Force" an erroneous phrase.—There is now no warranty for this expression. It was originally a translation of the German *Erhaltung der Kraft*, where *Kraft*, meaning strength or force, was used in 1847 by Professor Helmholtz, for want of a better term, to indicate what is now rigorously named *Energie* or Energy. Forces are of the same order as pressures exerted, pounds' or grammes' weight, resistance overcome; forces may be represented by *lines* which indicate their magnitude and direction. Energy is of the same order as work accomplished, as pounds or grammes lifted or resistance overcome through a certain number of feet or centimetres, and it may be represented by *areas* which are independent of direction.

The hydraulic press apparently creates force, and if its action be reversed, force disappears; but the work done upon it must be the same as the work done by it, and though there is no Conservation of Force, yet there is strict Conservation of Energy in this as in all those other mechanical contrivances in which force is altered in amount.

We have seen that Energy may be represented by Fs, the product of force acting or resisted through space s; by mgs where mass m is raised through space s against gravity whose local acceleration is g: by $\frac{1}{2}mv^2$ when a mass m has a velocity v imparted to it. We shall further see that Energy may be represented by the product CV of a charge C of Electricity into a numerical quantity V, called Electric Potential; by the product qp of a quantity q of fluid forced into a space against an average pressure p units of force per unit of area of the bounding surface of the fluid; by the product of a chemical affinity (which is equal to the work done in separating the atoms of an equivalent of a chemical compound) into the number of electro-chemical equivalents which enter into combination; and in other similar ways. These things, however, will find their explanation in due place.

Problems.

1. Energy is power of doing work: this depends on $\frac{1}{2}mv^2$; a body moving with a certain velocity $v_{,}$ can pierce a plank of thickness t; if it moves with velocity $v_{,,}$ what thickness can it pierce?—*Ans.* $t_{,,}=t\,(v_{,,}/v_{,})^2$.

2. A shot travelling at the rate of 700 feet a second is just able to pierce a 2-inch board. What velocity is required to pierce a 3-inch board?—*Ans.* $700\times(\sqrt{3}\div\sqrt{2})=857{\cdot}42$ feet.

3. A shot travelling at a certain rate can bury itself 10 feet in sand: how far could a shot travelling with double that speed bury itself?—*Ans.* 40 feet.

4. If a mass of 154·51 pounds be allowed to fall 10 feet, but in its fall be made to set a train of mechanism in action, and if that mechanism do no other work than to stir up a pound of water with a paddle, how much will the water thus stirred up be warmed?—*Ans.* 2° F.

5. If a locomotive weighing 5000 kilogrammes run at the uniform rate of 10 metres per second round a circular railway whose radius is 2 kilo-

metres, what will be its kinetic energy ?—*Ans.* $m = 5,000,000$ grms.; $v = 1000$ cm. per second; $\frac{1}{2}mv^2 = 2,500000,000000$ Ergs, or 250 Ergtens. The energy does not depend on the form of the path traversed, but only on the velocity at each instant along that path; for kinetic energy is independent of direction.

Graphic Representation of Energy.—The representation of work by the product F*s* (force acting into the space through which it acts or is resisted) finds its graphical equivalent in the representation of work done as a rectangular Area, the product of two lines, of which one represents the Force acting and the other the Space through which a body has been moved. If any instrument can be devised which will mechanically describe such an area, the amount of work done by a moving body can be recorded; such an instrument is a Dynamometer. This name is, as we have already seen, applied to the apparatus in which an elastic spring is deformed, the extent of its deformation showing, by comparison with that produced by a given weight, the amount of Force acting on the instrument. The same name has, however, been given to instruments designed to record not only the force acting on the spring at any given instant, but also the whole Energy spent in producing the deformation, and measured by a simultaneous record of the force acting and of the space through which it has acted.

If a distorted spring have a writing-point attached to it, as the distortion of the spring varies the pencil will move backwards and forwards in one line; if a piece of paper be held against the writing-point as it travels back and fore, the tracing produced is not instructive, for it is simply a line traced over and over. If the paper be drawn past the writing-point at a uniform rate, the line drawn is a curve, from which may easily be deduced the mean value of the deforming force during the whole time of observation. If, however, the paper be moved not uniformly but at a varying rate, proportioned at every instant to the space passed through by the moving body during given successive equal periods of time (that is, to the rate of change of deformation of the spring), then there are two factors recorded in the same tracing—first, the amount of Space passed through (this being indicated by the amount of paper unrolled under the writing-point) in a given period of time; and second, the Force which has acted in producing deformation (this being recorded by the oscillations of the writing-point attached to the deformed spring).

If the writing-point thus attached to the spring be supposed to draw the curve ABCDEF of Fig. 9, the various parts of the line give rise to the following discussion. The line O*abcd* shows the various spaces traversed by the body set in motion; the lines *a*A, *b*B, *c*C, etc., show the various pressures or forces in action at successive instants of time. The condition of affairs is more easily realised if we consider a cylinder the steam in which pushes a piston. Then the expansion of the steam is correlated to the movement of the piston, which may be represented by distances along the line O*x*. Then *ab* denotes the expansion of the steam as its volume increases from O*a* to O*b*, *a*A or *b*B its pressure, and hence the work done by the steam during the increase of volume *ab* is represented by the rectangle *a*AB*b*. When the steam expands still further, so that its increase in volume is *bc*, the pressure or force acting is again constant, and the work done is represented by the rectangle *b*C. Next, when the volume O*c* becomes O*d*, the pressure sinks from *c*C to *d*D; the average value of the pressure is $\frac{1}{2}(cC + dD)$, and the work done (or avetage pressure × space *cd*) $= \frac{1}{2}(cC + dD) \cdot cd$. This is the area of the figure *c*CD*d*, which accordingly represents the work done during the increase *cd* of volume. The area D*e* is made up of numerous rectangles, and if these be made sufficiently numerous the line DE is a curved line. The area comprised between the curved line DEF, the ordinates *d*D and *f*F, and the abscissa *df*, represents the work done by the steam during expansion of the steam from the volume O*d* to the volume O*f*. The area *a*AF*fa* represents the work done during expansion of the steam from volume O*a* to volume O*f*.

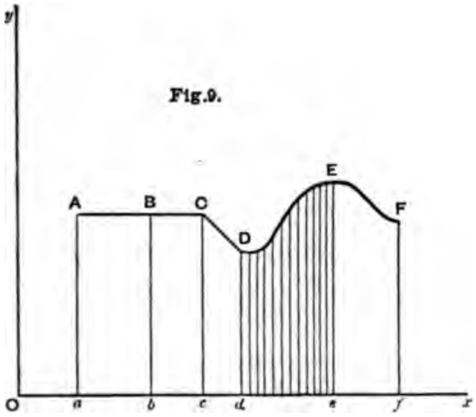


Fig.9.

The Indicator Diagram.—Since the time of James Watt engineers have been accustomed to make their engines record their own working by the method just discussed. In the Indicator-Diagram, as the curve traced out is called, the two factors which it is desired to record are the Space traversed, which is measured by the amount of movement of the piston, and the Force acting, which is the pressure of the steam in the cylinder. The former factor, the space moved through by the piston, is determined by making the piston or any part of the machinery upon which it directly acts set in operation the mechanism that unrolls the paper upon which the record is to be preserved. This paper is drawn over the writing-point at a rate depending on the velocity of the piston: hence the spaces traversed by the piston during successive intervals of time are proportional to the amount of paper which is drawn under the writing-point.

The pencil is borne by the piston of a small side cylinder attached to the main cylinder. The steam is let into this, and presses the little piston outwards; it presses it against a spring until the resistance offered by that spring prevents further propulsion. If the pressure were constant, the little piston would remain at the same level; but as the pressure of the steam varies, the position of the piston also varies, as it lies between the opposing spring and steam. Its displacement, then, is at each instant proportional to the pressure of steam in the cylinder, which is the second factor. If the paper be rolled over the pencil-point when no steam has access to the side cylinder, a smooth line is drawn, the Line of No Pressure; if the steam be allowed to enter the side cylinder, the divergences of the line then produced from the line of no pressure measure the variations of the pressure of the steam.

While the machine is working, steam is not allowed to enter the side cylinder until the apparatus is ready to record. The small piston is consequently at rest. While the piston of the main cylinder is moving in one direction, paper is rolled over the pencil-point; as the piston slackens speed the paper also slackens in speed; as the piston stops, the paper does the same; as the piston travels in the opposite direction, the paper travels in the opposite way at a proportionate rate. As long as the steam is not admitted into the side cylinder, the paper travels backwards and forwards over the pencil, and the same straight line is traced and retraced. The steam is admitted to the side cylinder for the space of one complete oscillation of the main piston, and the pencil-point itself travels in accordance with the varying pressure of steam during that period. The curve traced thereby is composed of two parts. The one, ACB, is produced during expansion. The work done by the steam in the cylinder during its expansion is the area ACB*ba* if O*ab* be the line of no pressure. When the piston has finished its stroke, it—and

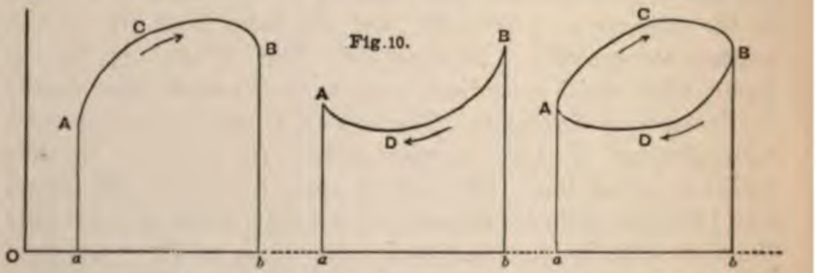


Fig.10.

therefore the paper—stands for an instant at rest. Then the piston is pressed against the steam either by other steam or by the atmosphere, and the paper is drawn backwards. Work is thus done against the steam during the backward stroke, and it is represented by the area BDA*ab*, where BDA is the line recorded.

The difference between the areas ACB*ba* and BDA*ab* represents the excess of work done by the steam over that done against it: hence the total work done by the engine is represented by the area of the surface ACBD traced out by the pencil in the formation of the so-called Indicator-Diagram.

The pencil of the Indicator in tracing out such a curve mechanically performs an operation equivalent to that which the mathematician effects when he sums up areas by means of the algebraic processes of the Integral Calculus.

In some cases it is sufficient to know the mean force acting. In such cases the space traversed being a known quantity, the energy can be determined if the mean force alone be recorded. For example, if the mean force required to pull a vehicle be found, it is a very simple matter to multiply the recorded mean force by the space traversed in order to find the total amount of energy expended. In Marey's investigations (*Trav. du Laborat.*, 1875) into the comparative total work expended on a vehicle, according as an elastic spring is or is not placed between it and the draught animal, a capsule was so arranged that the air in it suffered irregular compressions and rarefactions, corresponding to the irregular jolts between the animal and the car. The writing-point, set in movement by the correspondingly-irregular oscillations of one of the walls of the capsule, which was flexible, described irregular lines when there was no elastic intermediary, and more regular ones, nearer the line of no disturbance, when there was such an intermediary introduced; from these it was found that the mean force, and therefore the total energy expended in the two cases were in the ratio of about 4 to 5, showing that the use of an elastic spring between a draught animal and the vehicle which it draws results in an economy of labour amounting to about 25 per cent.

CHAPTER V.

KINEMATICS.

To the part of Science which deals with Motion, considered *per se* and without reference either to the force producing it or to the body moved, is given the name of **Kinematics.** The nature of the questions discussed under this title is essentially mathematical; and though no great acquaintance with mathematical methods is presumed in the reader of this volume, it will be necessary to assume in him a certain amount of knowledge of the most elementary geometry and algebra.

GENERAL PROPOSITIONS.

Direction.—There cannot be Motion without Direction; we cannot think of a body or a point as moving, and yet not moving in any direction. If it move at all, it must either move so as to travel constantly in the same direction, in which case it is moving in a straight line; or else the direction of its motion must change as it proceeds from point to point of the path traversed, so that the body travels in some kind of curved line.

In the great majority of those curves which possess physical interest as being those in which bodies actually do move, it is possible to draw at any point of the curve a straight line known as the Tangent to the curve at that point. The Tangent to the Circle at any point is familiar enough, and is easily understood to be a straight line at right angles to a radius connecting the centre of the circle with that point of the circumference at which the tangent is to be drawn; and the characteristic property of the line as a tangent is that it touches the circle without cutting it. Tangents may in a similar way be drawn to most curves, so as at any determined point to touch but not to cut the curve unless the curve changes its curvature beyond the point at which the tangent touches it.

If a circle be drawn on a very large scale, and a tangent be drawn to it at any point chosen, it will be found that the larger the scale the more nearly will the circle appear to coincide with the tangent at the point of contact. This can easily be seen by actually drawing such a figure. In fact, if a circle be drawn on a very great scale, any very little part of its circumference will appear to be practically straight. Of course it is not straight, but by drawing the circle sufficiently large, and by diminishing the size of the little part of the circumference considered, the approximation to perfect straightness in the little part or "element" considered may be rendered as close as may be desired. Such a circle may, then, be considered as a polygon, having an infinite—greater, that is, than any definite assignable—number of sides, the length of each of which is indefinitely small, and each of which coincides for an infinitesimal distance with the tangent which is drawn past it.

What is true of a large circle is true of a small one, and hence motion in a circle may be considered as motion round a polygon of an indefinite number of sides, whence the following proposition.

As a body or point moves round a circle, the direction of its motion is that of the tangent at each successive instant. Similarly, the direction of motion of a body which travels in any other curve is, at each successive instant of time, the same as the direction of the tangent to the curve at the point of the curve momentarily occupied by the moving body.

Velocity.—We have already anticipated some kinematical statements in discussing the velocity of a moving body. This was defined as the distance passed over in a unit of time by a body in motion; and if we consider, not the moving body, but the motion itself, we may say that one of the necessary properties of pure Motion is Velocity. It is not possible to think of Motion without thinking of a corresponding definite Rate of motion, which, if there be motion at all, cannot be zero, and on the other hand cannot be infinite, so long as Space and Time are related to Motion in the way in which experience shows them to be; and the idea of Rate of Movement is as necessary a constituent of the idea of Motion as is that of Direction.

Velocity may be **uniform** or **variable.** The measurement of uniform velocity is simple enough; and it has already been explained on what principle the measurement of variable velocity is based. Whether the direction of motion be constant or variable—whether the moving particle travel in a straight line or in a curve—the principle involved is always the same, namely, that the velocity of a moving particle is the length of path traversed by it in a unit of time, or the length of path which would have been traversed by it during a unit of time if the speed had remained uniform during that period. In the case of motion in curved paths, there arise subsidiary mathematical difficulties in the estimation of the precise length of the path traversed, but these only arise in the determination of the value of one of the terms of the formula $v = s/t$, and do not affect the validity of the formula itself.

Velocity may be otherwise defined as the relation of change of position to change of time. If at a certain instant of time the moving point be at a distance s from a fixed point chosen as a standard of reference, then, as time goes on, the position of the body, and therefore the value of s, changes. Let the time during which motion is going on be δt, a very small element of time, and the corresponding change of position be δs, then this relation of the change of position to the change of time may be expressed by the fraction $\delta s/\delta t$, which, when δt is chosen sufficiently small, becomes the function familiar to students of the Differential Calculus as ds/dt. This change of s in accord with the passage of time may be very advantageously represented as $\dot{s}$, where the dot above the letter indicates "the value of the change in unit of time of" the quantity expressed by the letter over which the dot is placed, when the unit of time chosen is very small. This is the notation employed by Newton in his *Fluxions*, $\dot{s}$ being the change or Fluxion of s. If the velocity ($\dot{s}$) itself change, the change of velocity in unit of time—which we otherwise know under the name of Acceleration—would be represented by the symbol $\ddot{s}$. So if the acceleration ($\ddot{s}$) itself varied, the change of acceleration per unit of time would be represented by the symbol $\dddot{s}$. Such a number of dots as this rarely occurs in physical problems.

We may here make use of previous discussions to bring together some of the symbols used to express frequently-recurring terms.

Distance of particle from point of origin $= s$.
Velocity, $v = \dot{s} = s/t$.
Acceleration, $a =$ change of $v = \dot{v} = \ddot{s} = v/t$.
Force, $f = ma = m\dot{v} = m\ddot{s}$.
Work $= fs = m\ddot{s}s$.
Rate of doing work (Thomson's *Activity*, Newton's *Actio Agentis*) $= fs/t = fv = m\ddot{s}\dot{s}$.
[Action (Maupertuis) $= vs$ or $\Sigma(vs)$; held by him to be always a minimum in unguided motion of a conservative system: shown by Hamilton to present, in unguided motion between fixed positions, either a minimum or a maximum value or else a value little affected by slight variations in the path traversed.]
Energy $= \frac{1}{2} mv^2 = \frac{1}{2} m (\dot{s})^2 =$ Kinetic energy.
Do. $= fs = mas = m\ddot{s}s =$ Potential energy.

Dimensions.—Distance of a particle from a point of reference may be represented by a straight line which is measurable in units of Length. Unit-distance would be represented by unit of length: distance, therefore, is said to be of one dimension in length, and a unit-distance may be represented by the symbol [L]. Velocity is a Length (space traversed) divided by a Time, and the unit of velocity may be represented by the symbol [L/T]. Momentum is Mass × Velocity; its unit is [ML/T]. Acceleration, the velocity acquired per unit of time, is a Velocity divided by a Time, and the unit of Acceleration may be represented as (*i.e.*, the "Dimensions" of

Acceleration are said to be) $[L/T] \div [T]$, or $[L/T^2]$. Force is a Mass × Acceleration, and its dimensions are accordingly $[ML/T^2]$. Weight has the same dimensions as Force. Energy, if we take the expression $\frac{1}{2}mv^2$, has the dimensions $[M]\,[L/T]^2$ or $[ML^2/T^2]$; while if we take the expression mas, it is found to have the dimensions $[M]\,[L/T^2]\,[L]$ or $[ML^2/T^2]$, the same result. The dimensions of Work are the same as those of Energy. Those of Activity are Work done ÷ Time $= [ML^2/T^3]$.

Examples.—1. How many British units of force is a Dyne equal to? The Dyne $= [ML/T^2] =$ [Gramme × Centimetre / Second²] $= \left[\frac{15{\cdot}432}{7000}\,\text{lb.} \times \frac{1}{30{\cdot}48}\,\text{ft./Second}^2\right] = \left(\frac{15{\cdot}432}{7000} \times \frac{1}{30{\cdot}48}\right)[\text{lb.-ft./second}^2] = \frac{15{\cdot}432}{7000 \times 30{\cdot}48}$ British units.

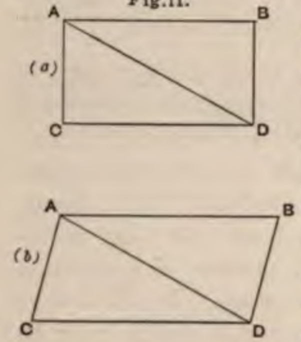
2. Suppose it were affirmed that Force is Rate of gain or loss of Energy as Time goes on; test the statement. We would have an equation of the following kind; $f\,[F] = e\,[E] \div t\,[T]$. Putting aside the numbers this would be $[F] = [E/T]$ or $[ML/T^2] = [ML^2/T^2] \div [T]$, which is obviously wrong.

3. Test the statement that Force is measured by Time-rate of Change of Momentum. Similarly $[F] = [\text{Mom.}] \div [T]$, or $[ML/T^2] = [ML/T] \div [T]$, which is consistent.

4. Test the assertion that Force is the Rate at which a body gains or loses Energy as it traverses Space. $[F] =$ [Energy/distance traversed] $= [E/L]$, or $[ML/T^2] = [ML^2/T^2] \div [L] = [ML/T^2]$, which is consistent.

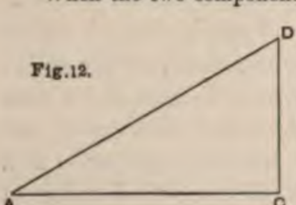
Simultaneous Motions.—If a particle have by any means two separate independent motions communicated to it simultaneously, each will produce its own effect, and the total movement of the particle can be found by any process of summation which may be found mathematically appropriate. It will always be found as the result of experiment on bodies which may be taken to represent particles, that if the motions imparted to the particle be themselves constant in velocity and direction, the result of their concurrence is a single motion in a straight line with a single velocity and direction. The single motion which is produced as the result of the concurrence of two motions is called their **Resultant**, and they with regard to the resultant are called its **Components.** If a ship travel from west to east and a man on board also walk from west to east, the speed of the ship and the speed of his walking will have to be added together to find the rate at which he is moving eastwards: if the ship travel from west to east and he walk along the ship from east to west, the difference between his own speed and that of the ship is the rate at which he is travelling eastwards. In the latter case the result may be positive—*i.e.* he is really going eastward; negative—*i.e.* he is really going westward; or zero, in which case he has no movement at all, the ship carrying him east just as much as he walks to the west, so that he is really beating time in the same place. If a steamer travel to the east and be at the same time carried to the north by a current, the path traversed by the steamer will be a line which is the diagonal of a **parallelogram** whose sides represent the eastward and northward velocities respectively. The steamer will describe this diagonal line in the same time as it would have taken to have steamed or to have drifted along one or the other side of the parallelogram if the steaming or the drifting respectively had been the only cause of its movement. Hence, to find the resultant of two simultaneous velocities, the rule is: Construct a parallelogram whose adjacent sides represent in magnitude and direction the velocities produced, and the diagonal which lies between these adjacent sides represents the resultant velocity. If the lines AB, AC (Fig. 11), represent in direction and, on any conventional scale, the magnitude of the velocities simultaneously imparted to the particle A, the particle A will move along the line AD to the point D in the same time that, under the influence of the velocity AB alone, it would have taken to reach B, or, under the influence of the velocity AC, to reach the point C. The actual construction of the diagram representing the resultant of any two velocities is an easy matter: the calculation of the value of the resultant—that is, of the length of the diagonal—involves a little geometrical working.

Fig.11.



When the two components are at right angles to one another, we resort to Eucl. I. 47, which shows that in a right-angled triangle ACD (Fig. 12), of which the right angle is at C, $AD^2 = AC^2 + CD^2$. In the parallelogram Fig. 11 (*a*) it is plain that $AD^2 = AC^2 + CD^2$; but $CD = AB$; hence $AD^2 = AC^2 + AB^2$; or in words, the square of the resultant is equal to the sum of the squares of the two components, if these be at right angles to one another.

Fig.12.



Again, if they be not at right angles to one another, they must make either an acute or an obtuse angle. In the former case we resort to Eucl. II. 12, which shows that if the parallelogram be drawn (Fig. 13) and the side AC be produced so far that a line DE can be drawn at right angles to it from the point D, the equation $AD^2 = AC^2 + CD^2 + 2AC{\cdot}CE$ is true; this enables us to find the value of AD if we know that of AC and AB (which is equal to CD), and if we can find that of CE. In the latter case, where the angle BAC between the components AB and AC is obtuse (Fig. 14), Eucl.

Fig. 13.

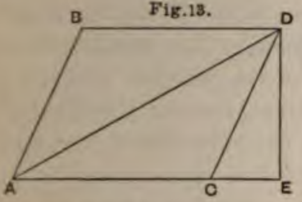
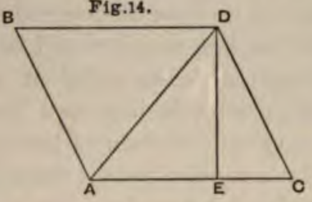


Fig. 14.



II. 13 shows us that if we drop a perpendicular DE from D upon the base AC, the equation $AD^2 = AB^2 + AC^2 - 2AC.EC$ holds good, and enables us to find the value of AD.

Problems.

1. A person on board a ship which is going eastwards walks back and fore at the rate of 4 miles an hour relative to the ship: the ship is travelling at the rate of 12 miles an hour. What is his absolute velocity when he is walking forward? what when he is going aft? what is his average absolute velocity?—*Ans.* 16 miles an hour; 8 miles an hour; 12 miles an hour.

2. A point moves with velocity a eastwards and velocity b westwards simultaneously. What is its eastward velocity?—*Ans.* $a - b$.

3. Interpret the result if b is greater than a.—*Ans.* The eastward velocity $= a - b$; this is negative: the velocity must therefore be westward and $= b - a$.

4. Interpret the result if $b = a$.—*Ans.* The eastward velocity $= a - b = 0$: or the body is absolutely at rest.

5. If in a railway carriage compartment a man walk across at the rate of 5 miles an hour while the train goes forward at the rate of 12 miles an hour, what will have been his real path and velocity relative to the railway line underneath?—*Ans.* In Fig. 11 (a), if $AB = 12$ and $AC = 5$: the real path is AD, which has a value of 13.

6. To the same particle are imparted a velocity of 12 and one of 6 feet per second in directions which stand to one another at an angle of 60°: what is the direction and the amount of the resultant velocity?—*Ans.* In Fig. 13, if AB represent the velocity of 6 feet per second, and AC on the same scale that of 12 feet, the angle BAC being one of 60°, then the line AD will indicate the direction of the resultant movement, and the equation $AD^2 = 12^2 + 6^2 + \overline{2 \times 12 \times CE}$ (CE being seen, since the triangle CDE is half an equilateral triangle, to be equal to half CD—that is, to 3), or $144 + 36 + 72 = 252$, shows that $AD = \sqrt{252} = 15{\cdot}822$.

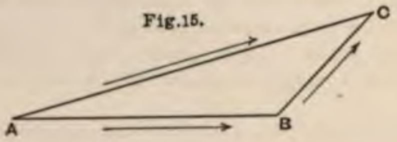
7. If in the last question the angle between the directions of the velocities had been 120°, what would the resultant velocity have been? —*Ans.* In Fig. 14, if AB be 6 and AC 12, the angle BAC being 120°, $AD^2 = 12^2 + 6^2 - \overline{2 \times 12 \times CE} = 144 + 36 - 72 = 108$; whence $AD = \sqrt{108} = 10{\cdot}392$ feet per second.

8. What would have been its direction?

In the same figure, 14, the triangle ADC has its sides $AC = 12$, $CD = 6$, and the angle $DCA = 60°$. By trigonometry we find that the angle DAC is 35° 16′, and hence the direction of the resultant motion is inclined to those of its components AC and AB at the angles of 35° 16′ and 84° 44′ respectively.

Triangle of Velocities.—If the figures just made use of be reduced to their simplest necessary elements, it will be seen that there is no need to describe a complete parallelogram in order to find the line which would be its diagonal. The three sides of a triangle are quite sufficient to express the relation between two component velocities and their resultant, and for the determination of the resultant of two velocities the rule may be thus stated: Take a starting point; from it draw a line representing in magnitude and direction one of the component velocities; from the point thus arrived at—that is to say, the end of the line thus drawn—draw another line similarly representing the second component velocity. The third side may now be laid down, and the problem is reduced to the form which in trigonometry is simple enough, namely—Given two sides of a triangle and the angle between them, to find the third side, and the angles which it makes with the two sides given. In the triangle ABC (Fig. 15), if the sides AB and BC represent the component velocities in amount and direction, AC in the same way represents their resultant; and it will be observed that if the sides of the triangle be taken consecutively in the "cyclical" order, AB, BC, CA, the direction of the resultant is in this diagram opposed to that of the components.

Fig. 15.



Resolution of a Velocity into Components.—The converse proposition is one of very general utility. In the former case, by Composition, that single resultant was found which was the effect of two simultaneously-imparted movements. A single movement may, conversely, be considered as the resultant of two component movements which we may wish to find. The process of finding them is known as the Resolution of a motion into its components. A given pair of velocities can only have one resultant, for if two sides of a triangle be fixed, there is no scope for variation in the position or length of the third side; but if the resultant be given, it can be resolved into components in an indefinite number of ways, for there can be an infinite number of triangles made by supplying two sides when only one side is determinately fixed. Hence the question how to resolve a velocity into its components, set in this vague way, never arises; but the question how to resolve a velocity into its components in certain fixed directions is of constant occurrence. Such a question is generally solved by construction in the following way:—Let AB (Fig. 16) be a line indicating the direction and rate of movement of a particle. It is required to know what are the corresponding components in two directions, LM, NO, arbitrarily chosen or determined by the conditions of the problem. Draw lines from the extremities of the line AB parallel to the directions assigned. In this way a parallelogram will be formed in which AD, CB will be parallel to LM, and DB, AC to NO. In this parallelogram AB represents the single motion whose components are to be found: the length of AD or CB represents the proportionate value of the component parallel to LM, and DB or AC the proportionate value of that parallel to NO. Hence the problem is solved. Very little practice enables one to dispense with drawing a complete parallelogram, and to find the components by constructing either the triangle ABC or ABD. If numerical values are required, we can find them by means of the known angles which the directions of LM and NO make with that of AB: the value of these two angles together with the numerical value of AB give by trigonometry the numerical values of AD and DB, which represent the components. If we wish to resolve a single velocity into components at right angles to one another, the process is precisely the same, LM and NO being drawn at right angles to one another. A modified form of the problem which we very often encounter is — Given a velocity in a certain direction, what is the value of its component in another assigned direction? This is solved by the following construction: — Let AB (Fig. 17) represent the given velocity and LM the direction of the required component of AB. Draw through A a straight

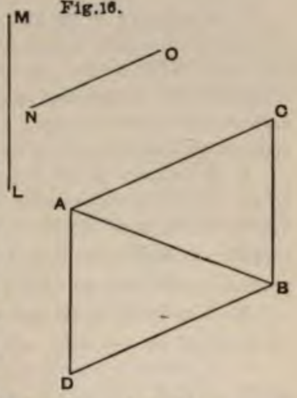


Fig.16.

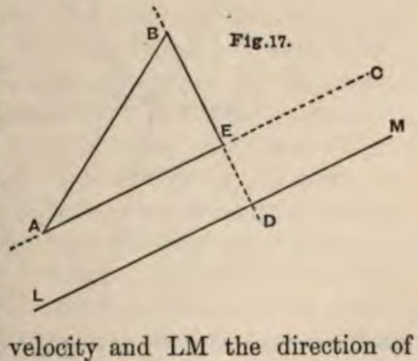


Fig.17.

line, AC, indefinite in length, but parallel to LM. From the other extremity, B, of the line AB, draw a line BD *at right angles* to AC, cutting it in the point E. AE is the component required in the direction parallel to LM.

Problems.

1. A velocity of 30 feet per second: what is the value of its component in a direction which makes with its own an angle of 60°?—*Ans.* In Fig. 17, if the angle ABD be 60°, the line AB may represent the velocity given; EB represents its component at an angle of 60° with it. The triangle BEA is half an equilateral triangle, and EB is half of AB; it represents therefore a component velocity of 15 feet per second.

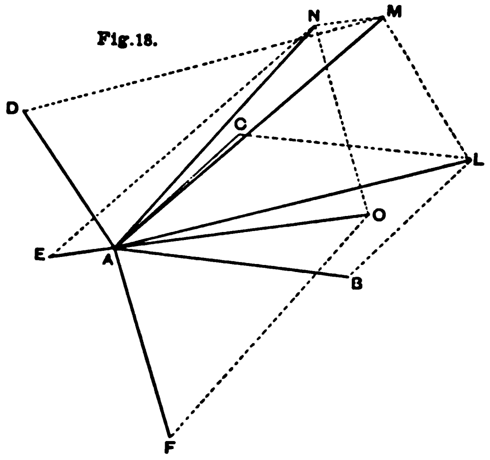
2. A velocity of 20 feet per second: what is the value of its component whose direction makes with its own an angle of 30°?—*Ans.* In the same figure, if the angle EAB be 30°, and AB represent the velocity 20 feet per second: in such a triangle EA : AB :: $\sqrt{3/4}$: 1, and the value of the component is $\sqrt{3/4} \times 20 = \sqrt{300} = 17{\cdot}32$ feet per second.

3. A velocity of 60 feet a second: what is the value of its component at an angle of 45°?—*Ans.* $\sqrt{1/2} \times 60 = 42{\cdot}42$ feet per second.

4. A velocity v in a certain direction: what is its component at right angles to that direction?—*Ans.* It has none.

Composition of more than Two Velocities.—If more than two velocities be imparted to a body the resultant is always, if they themselves be uniform in amount and direction, a single uniform motion in a straight line. If the several velocities imparted be all in the same plane, their resultant may easily be found by finding the resultant of any two of them, compounding the resultant thus obtained with any other of the velocities imparted, and so on, till all the velocities have been taken into consideration,

Fig. 18.



and the final resultant obtained. Let the several velocities which are imparted to a particle be represented by the lines AB, AC, AD, AE, AF (Fig. 18), all in one plane. It is required to find their resultant. The resultant of AB and AC is AL; the resultant of AL and AD is AM; the resultant of AM and AE is AN; that of AN and AF is AO, the final resultant of the five velocities AB, AC, AD, AE, AF. It does not matter in what order they are compounded; it may be left as an exercise for the reader to show that the same result is always obtained whatever be the order followed.

The Polygon of Velocities.—If, in the last diagram, the figure ABLMNOA be traced out, it will be seen that it is a polygon whose sides represent the various velocities and the resultant: for these sides are AB, BL (= AC), LM (= AD), MN (= AE), NO (= AF), and AO, which represents the Resultant. Hence the method of finding the resultant of any number of forces in the same plane may be exemplified as follows:—Take a starting-point K (Fig. 19); from K draw the line KP, representing AB in magnitude and direction; from P draw PQ, representing AC; from Q QR, representing AD; from R RS, representing AE; from S ST, representing AF; then join KT. KT represents the Resultant sought. It will be seen that the direction of the resultant is opposed to that of the other sides of the polygon taken in cyclical order. The rule, then, for the composition of a number of velocities in the same plane is—Construct a polygon with lines representing them (it being a matter of indifference in what order they are taken, or whether they cross one another or not), and if there be a side missing, complete it; it will represent the magnitude of the resultant, and its direction will be opposed to that of the other constituent sides taken in cyclical order. If the two points K

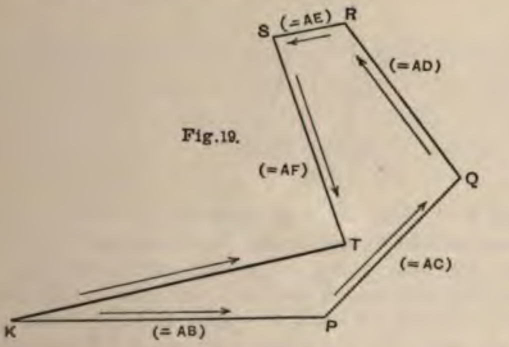


Fig. 19.

and T coincide, then the line KT has no value, there is no resultant motion, and the result of the simultaneous velocities is, in such a case, a state of rest.

Reference to Axes.—It is often as convenient, or more so, first to resolve each velocity into two components, which are made parallel to arbitrarily chosen axes. Let the same velocities, AB, AC, AD, AE, AF, be supposed as in the previous paragraphs. Through the point A (Fig. 20) draw

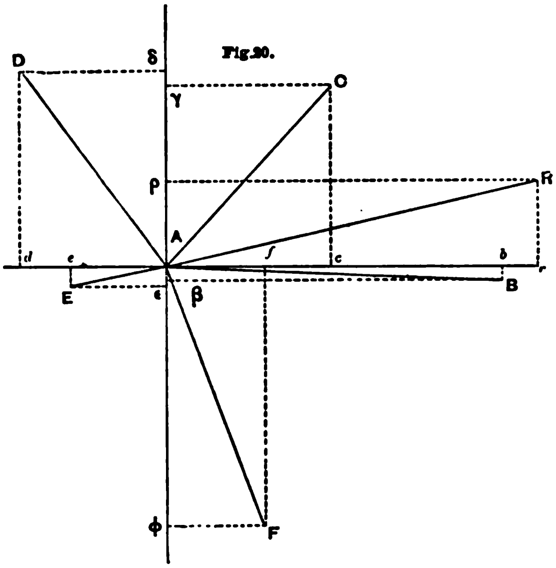
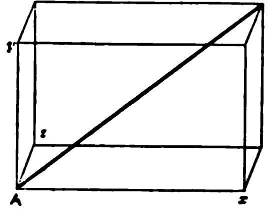


Fig. 20.

axes of x and y at right angles to each other. Resolve each velocity into its components parallel to these axes. AB is resolved into Ab and Aβ; AC into Ac and Aγ, and so on. The value of the resultant is found after summing up with reference to each axis separately. The total result with reference to the axis of x is (Ab + Ac − Ad − Ae + Af), which has a value, say, + Ar. In the axis of y the total result is ($-$Aβ + Aγ + Aδ − Aϵ − Aϕ), which has the aggregate value, say, + Aρ. The resultant therefore is to be drawn from A to a point R, which has co-ordinates, $x = +$Ar, $y = +$Aρ.

Velocities not in one Plane.—The same essential principles apply here as in the preceding paragraphs. In the case of a railway train travelling at the same time northwards, westwards, and upwards, the motion, while it may be represented by a straight line, is the resultant of three components at right angles to each other. The proposition in three dimensions, which corresponds to that known as the parallelogram of velocities in bidimensional space (in a plane), is called the **parallelepipedon of velocities.** If the three velocities, Ax, Ay, Az (Fig. 21), at right angles to each other, be compounded, the resultant is expressed by a line drawn from A to the opposite angle of that parallelepip-

Fig. 21.



edon of which Ax, Ay, Az measure the length, breadth, and thickness. If Ax, Ay, Az be at right angles to one another, $AR^2 = (Ax^2 + Ay^2 + Az^2)$, while if they be not at right angles to one another, AR is the diagonal of an oblique prism.

Any rectilineal velocity may be resolved into three components in an indefinite number of ways, for there may be an infinite number of prisms constructed on a given diagonal line; but there can only be one way of resolving such a movement into components if these must be at right angles to one another, or if the directions of two of them be assigned.

The **Polygon of Velocities** also applies when the component movements are not restricted to one plane, for a so-called "*gauche polygone*," or "skew-polygon," may be realised, no three of whose contiguous sides are in the same plane; the only essential criterion of such a polygon is that it shall be continuous and closed. If such a polygon whose sides represent velocities be realised, but be incomplete or "unclosed," the missing side represents the Resultant, and the direction of the resultant—opposed to that of the rest of the sides taken in cyclical order—and its magnitude are found in the same way as if the polygon had been restricted to a plane surface.

The method of **reference to axes**, illustrated by Fig. 20, is of special use when extended to tridimensional space. Of a number of velocities in different directions in space, each may be resolved into three components, parallel to the axes of x, of y, and of z, and the resultant is found after summing up the effects produced with reference to each of these axes respectively.

Change of Velocity.—This phrase is sometimes employed, as when the statement is made that a certain velocity has been changed into another, and the question is asked, What has been the "Change of Velocity?" Another way of stating the same is—A known component and an unknown one have produced a given resultant: *what was the value of the unknown component?* This is easily solved if the direction of motion have not changed; while if the direction have also changed, the question is answered by the aid of the triangle of velocities; the two sides being known, the third side is easily found.

Parallelograms, etc., of Accelerations.—What is true of simultaneous velocities imparted in general is true of velocities simultaneously imparted in unit of time—that is, of Accelerations, and hence, if a body receive two accelerations, these must be compounded in exactly the same way as two velocities. So every one of the geometric propositions just laid down with reference to velocities finds its exact counterpart in a proposition relating to accelerations, and we thus have such propositions as the Parallelogram, the Triangle, the Polygon, the Parallelepipedon of Accelerations.

Acceleration may therefore result in mere change of direction of motion: for the original velocity compounded with that produced in a given time by the acceleration may yield a resultant velocity which is the same in amount, but not in direction, as the original velocity: the triangle of velocities is then an isosceles triangle, the two equal sides in which represent the original and the resultant velocities respectively.

Problems.

1. If the same particle be simultaneously affected by a northward velocity of 10 feet per second, an eastward of 8, one towards the S.W. of 7, to the W. of 8, to the S.E. of 5, and to the N.E. of 7, find the resultant movement, and show that it does not matter in what order the components are resolved.

2. If the axes of x and y be drawn at right angles to one another through the common point A; if then the point A be supposed to be simultaneously affected by velocities represented by the following lines, viz. (a) one drawn making an angle of 15° with Ax, and of such a length as to represent a velocity of 10 metres per second; (b) one making an angle of 45° with Ax, and representing a speed of 15 metres per second; (c) one making an obtuse angle of 120° with Ax, and representing 8 metres per second; and (d) one at an angle of 195° with Ax, and representing a rate of 12 metres per second. Find the resultant velocity (1) by the polygon, and (2) by reference to axes.

3. If a body moving 10 miles an hour northward come to move at the same rate southward, what is the change of velocity?—*Ans.* 20 miles an hour.

4. If a body be moving with a velocity 4 miles an hour northward, and is after some time found to be moving at the same rate eastward, what is the change of velocity?—*Ans.* $4 \times \sqrt{2}$, acting towards the S.E.; the hypotenuse of a right-angled triangle.

5. If a body moving at the rate of 10 feet a second is found after some time to be travelling at the same rate, but in a direction inclined at an angle of 60° to its former one, what is the change of velocity?—*Ans.* 10 feet per second, making, with the original component and the resultant, an equilateral triangle.

Accelerated Motion.—If a body be moving at a rate which increases or decreases with the time, its velocity is said to be accelerated. The acceleration is said to be positive when the velocity of the motion is increased, negative when it is diminished. It is measured by the amount of increase or decrease of the velocity per unit of time. If a particle be at a certain initial instant moving at a rate V, and if its acceleration be $\pm a$, in the same straight line, then its various rates of motion are—

At the initial instant . .	V.
At the end of one second . .	$V \pm a$.
At the end of two seconds .	$V \pm 2a$.
At the end of t seconds . .	$V \pm at$.

Hence we arrive at a general equation expressing the relation between v the **velocity acquired** at the end of t seconds, V the original velocity, and $\pm a$ the acceleration, namely,—

$$v = V \pm at, \quad (1.)$$

in which the + or the − sign is used according to the positive or the negative character of the uniform acceleration a.

It is supposed that the acceleration is uniform, and hence the **average velocity** during any interval of time is the arithmetical mean between the velocity (V) at the commencement and the velocity $(V \pm at)$ at the end of the interval; that is to say, it is equal to half their sum or $\frac{1}{2}\{(V) + (V \pm at)\} = (V + \frac{1}{2}at)$. This being the average velocity during the interval, the **space traversed** will be found by multiplying the average velocity by the time, and hence we have (s being the space traversed)—

$$s = t(V \pm \tfrac{1}{2}at) = Vt \pm \tfrac{1}{2}at^2. \quad (2.)$$

From equations (1) and (2) we may eliminate t,* and thus obtain a third equation—

$$v^2 = V^2 \pm 2as, \quad (3.)$$

which expresses the relations between the original and acquired velocities, the space passed over, and the acceleration.

All elementary problems concerning accelerated movement in one direction, which give a sufficient number of terms to enable a conclusion to be arrived at, can be solved by the aid of these equations.

Problems.

1. If the velocity at the initial instant be 10 feet per second, and the acceleration be + 2 feet per second, what will be the speed at the end of 13 seconds?—*Ans.* Here, by equation (1) (v being the unknown term);

$$v = V + at = 10 + (2 \times 13) = 36.$$

2. If the acceleration be − 2 feet per second, what will be the velocity? —*Ans.* $v = 10 - (2 \times 13) = -16$; that is, 16 feet per second, in a direction opposed to the original velocity.

3. If the terminal velocity be 20 feet per second, the acceleration be 4 feet a second, and the initial velocity 4 feet per second, what was the time spent in acquiring the ultimate speed?—*Ans.* Here, by equation (1) (t being the unknown term),

$$20 = 4 + 4t, \text{ whence } t = 4.$$

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* From equation (1): $t = \pm \frac{v - V}{a}$.

Substitute this value for t in equation (2)—

$$s = \pm V\left(\frac{v-V}{a}\right) \pm \frac{a}{2}\left(\frac{v-V}{a}\right)^2, \text{ or,}$$

$$V^2 \pm 2as = v^2.$$

4. A body travels with accelerated velocity; its acquired velocity is 100 feet per second, its acceleration is 10 feet per second, and it has been gaining speed for 8 seconds. What was the initial velocity?—*Ans.* By equation (1), V being the unknown term, $100 = V + (10 \times 8)$, whence $v = 20$.

5. A body falls from rest: its velocity increases by 32·2 feet per second. What will be its speed at the end of 5 seconds?—*Ans.* By equation (1), v being the unknown term, and $V = 0$, $v = 0 + (32{\cdot}2 \times 5) = 161$ feet per second.

6. What space will have been traversed, the terms remaining as in the last question?—*Ans.* By equation (2), s being the unknown quantity, and $V = 0$, $s = 0 + \frac{1}{2}(32{\cdot}2 \times 25) = 402{\cdot}5$ feet.

7. What time will a body take to fall 502·5 feet if it be thrown down from a cliff at the initial rate of 20 feet per second, and if the acceleration of a falling body be 32·2 feet per second?—*Ans.* Here, by equation (2), t being the unknown term, $502{\cdot}5 = 20t + 16{\cdot}1t^2$, a quadratic: whence $t = 5$ seconds.

8. If its initial velocity had been 20 feet per second *upwards*, how long would it take to fall?—*Ans.* Here the acceleration is opposed to the original velocity, and equation (2) becomes $502{\cdot}5 = 20t - 16{\cdot}1t^2$, whence $t = 6{\cdot}24$ seconds.

9. What speed is acquired by a falling body if it start from rest and fall 1610 feet?

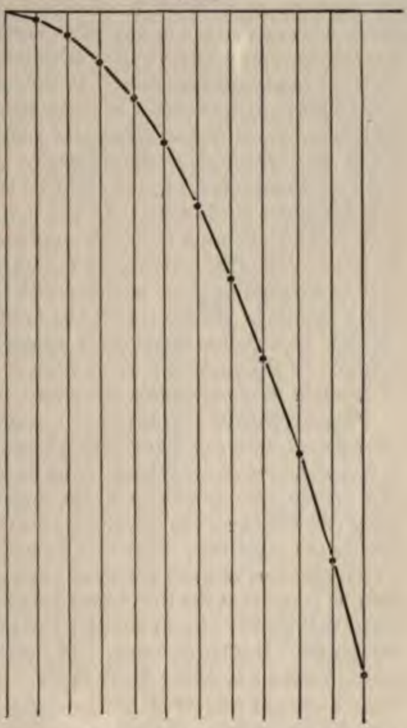
Here v is unknown, $V = 0$ and $a = 32{\cdot}2$. By equation (2),

$$v^2 = 0 + (2 \times 32{\cdot}2 \times 1610),$$
$$v = 322 \text{ feet per second.}$$

10. If a body start with initial velocity V, and the acceleration be a, what will be the space traversed in the first, in the second, in the third, in the fourth seconds respectively; and what will be the space traversed in 4 seconds?—*Ans.* $V + a/2$; $V + 3 . a/2$; $V + 5 . a/2$; $V + 7 . a/2$; $4V + 16 . a/2$.

Composition of uniform with accelerated motion.—If a particle be affected with both a uniform and an accelerated motion, and if these be in the same straight line, we have simply the problem dealt with by the aid of the last three equations. When, however, the uniform and the accelerated velocities are not along the same line, but are in directions inclined to one another, the resultant must be found by a geometrical or an algebraical process of composition of velocities. If, for the sake of fixing our ideas, we consider such a motion as that of a projectile fired horizontally from a gun placed on a height, we see that the ball is affected with two simultaneous motions, the one horizontal and uniform, the other vertically downwards and accelerated. If we consider the positions reached by such a body in successive equal intervals of time, we find that while it passes forward, by reason of its horizontal component, over spaces varying directly as the time, the amount of its vertical drop due to the downward accelerated component is proportional (Equation 2, where $V = 0$) to the square of the time during which it has been in motion; so that if we separately find its various positions at the end of successive small intervals of time, we can draw a line joining these positions, which line we find to be a curve known as a Parabola (Fig. 22).

Fig.22.



If, again, we consider that the one movement is in the axis of x, and is uniform, so that at the end of time t the horizontal component motion has carried the body along the axis of x a distance $x = vt$; while the vertical fall represents the distance y, along the axis of y, at which the body is situated at the end of the same time t, so that according to equation (2) above, $y = \frac{1}{2}at^2$; then we find* that in time t (whatever this time be) the body moves to a position such that its vertical distance y from the starting point bears to x^2, the square of its horizontal distance from the starting point, a constant ratio, or in symbols $kx^2 = y$, which is recognised as "the Equation to" a Parabola. This indicates that in order to preserve the given relation between the values of x and y, the path of the body as it moves from point to point must be in a curve known as a parabola.

It is stated that the body will move in *a* parabolic not in *the* parabolic path; this is because there is an indefinite number of parabolic paths possible, there being an infinite number of parabolic curves (just as there may be an infinite variety in the forms of a jet of water expelled from a fire-engine), which resemble each other in having some constant proportion between the values of the one co-ordinate and of the square of the other, but which differ in the numerical value of that ratio.

Degrees of Freedom of a Particle.—If a particle be free to move in any direction in space, it is said to have three "degrees of freedom," because it may move in tridimensional space; it may move, *e.g.* (1) up or down, (2) forwards or backwards, or (3) to the right or left; or more generally, it may move in the direction of any of the three axes arbitrarily chosen at right angles to one another, by reference to which we agree to specify any given direction in space, or it may move in any other direction, motion in which may be considered as the resultant of simultaneous motions in the three directions assumed as axes of reference. If the particle be restricted to a surface, it cannot move in a direction at right angles to that surface, and is accordingly said to have one degree of freedom less; it has now two degrees of freedom, for it may travel along the surface in two main axial directions (*e.g.* (1) forward or backward, (2) to the right or left), or in any direction derived from the combination of these. If the particle be restricted to two surfaces, on both of which it must lie at the same time, it can lie nowhere but on the line in which these two surfaces cut one another, and it has now only the one degree of freedom implied in the possibility of moving (backward or forward) along this line. If the particle be restricted to three surfaces which cut one another in a point, the particle cannot leave that point without leaving one or other of the surfaces; its position is definitely fixed, and it has no degree of freedom to move in any direction.

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* This is an instructive example of a method frequently in use. One consideration leads us to the equation $x = vt$: another to the equation $y = \frac{1}{2}at^2$: the question is, what law governs the relations of x and y? The two equations are combined in any way so as to represent x as some multiple (or other "function") of y, and also, if possible, so as to eliminate a letter common to both equations. Here the value of $t\,(=x/v)$ derived from the first equation may be substituted for t in the second, thus making it $y = \frac{1}{2}a\,(x/v)^2$, whence $x^2 \div y = const.$

Translation.—If there be a system of separate particles, all of which are affected with equal and parallel velocities, each particle will move in such a way as to retain its relative position with regard to its fellow-particles, and the system will move as a whole, undergoing no deformation, just as a company of soldiers, all the constituent units of which march in the same direction and at the same rate, retains its formation. If a straight line be drawn between any two of these particles when the system is in its initial position, it will be found that the line drawn between the same particles after such movement will always remain parallel to its former position, and will be unaltered in length. Motion in which every such line remains parallel to all its previous positions is called Simple Translation. If we study the motion of such a straight line or of the particles between which it lies, we shall have complete knowledge of the positions of the various particles of the system, if that system be restricted to a plane surface. If the system be not restricted to a plane surface, then it is possible that though one line and all lines parallel to it may continue to be parallel to their former positions, the whole system may have rotated round one of these lines as round an axis; and hence in this case it is necessary, before the motion of the system can be said to be a motion of simple translation, that not a line only, but any plane through the system—or, which amounts to the same thing, every line in any such plane—should retain parallelism to its initial position.

Rigid Body.—This is an ideal, not physically realisable. A rigid body may be regarded as a system of particles which may move as a whole with reference to surrounding objects, but in which there can be no displacement of its particles with reference to one another.

Centre of Figure.—There is in the case of every body of any shape whatsoever some one point occupying a definite position, which position may be described as the average of all the respective positions of the several particles of the body. A body suspended in the air somewhere towards the N.W. will have (generally within it) a point which is not only situated at an average distance to the north of the point of reference, but is also an average distance to the west and at an average height; and this point is the **Centre of Figure.** Not only with respect to the planes chosen as those of reference is this point the centre of figure, and its distance from each of these planes the average of the several distances of all the particles, but it has this property with reference to any plane whatsoever.

The centre of figure of a straight line or linear body is its middle point; the centre of figure of a circle is its centre; the centre of figure of a sphere, of an ellipsoid, of a spheroid, is equally obvious; that of a hollow spherical shell is the centre of the corresponding solid sphere, and is therefore not within the substance of the shell; that of a parallelogram is the point at which the diagonals cross one another. That of any regular plane figure is obtained by dividing it into numerous thin strips and bisecting these; by joining the points of bisection a line is drawn in which the centre of figure must lie. By repeating the process another such line may be obtained. These two lines will cross one another in some point, and the point where they do so is the only point which lies in both the lines, and is the centre of figure. This holds good only when the lines thus containing the centre of figure are straight; if they be not so the construction fails, and we may modify one of the experimental methods described under the Centre of Gravity, further on.

The importance of the Centre of Figure lies in this: that if a rigid body be subject to translation without rotation, the motion of the body may be quite effectively studied by considering the movement of the centre of figure, and, on the other hand, if a rigid body be subject to accelerations whose resultant passes through the centre of figure, the whole rigid body will participate in the movement of its centre, and there will be *translation:* while if the resultant of parallel accelerations do not pass through the centre, there will be *rotation.*

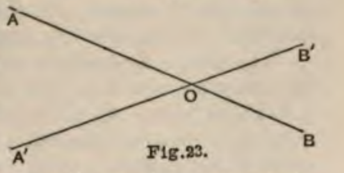

Fig. 23.

Rotation takes place when a straight line drawn through a moving body or system of particles does not continue to be parallel to its previous directions in space. Let us suppose the moving system to be restricted to a plane surface. Then a determinate line AB, arbitrarily chosen in the body, may move so that its ultimate position is A'B' (Fig. 23). Obviously the line AB, and with it the system, has rotated round the point O. Again, the relative positions of the same line may be AB and A'B' in Fig. 24. In this case a point O may be found,* round which the line AB has rotated so as to acquire its new position A'B'. If the lines AB and A'B' be very nearly parallel to one another, construction will show that the point O is at a great distance. When the lines AB and A'B' are perfectly parallel, the

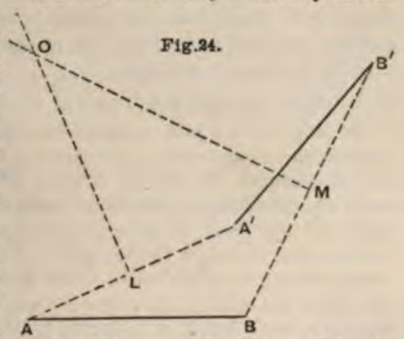

Fig. 24.

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[*] Join AA'; bisect AA' in L; draw LO at right angles to AA'. Join BB'; bisect BB' in M; draw MO at right angles to BB'. LO and MO will intersect in O; O is the point required. Join OA, OA', OB, OB'; the triangles OAB and OA'B' are equal in every respect.

point O is at an indefinitely great, an infinite distance. Thus we see that though it is convenient to regard translation and rotation as distinct forms of motion, yet translation may be considered as a limiting case of rotation effected round an infinitely distant centre.

Any translation of AB into the parallel position A′B′ may be resolved into a succession of rotations first round one extremity as A, then to an equal extent but in an opposite direction round B: this is easily verified by construction. If the line AB cannot be brought into coincidence with the line A′B′ by a single pair of such rotations, a sufficient number of pairs of rotations will certainly effect this.

Composition of Rotations.—If by reason of a rotation round the point O, the line AB be brought into the position A′B′; if it be then rotated round the point O′ so as to assume the position A″B″: the two rotations can be compounded into one round a point O″, which is found directly by comparing the initial and final positions AB and A″B″, without reference to the intermediate position A′B′.

If a solid body, of which one point is fixed, move in any way whatsoever, the result is the same as if it had revolved round some definite axis passing through that point. Any movement under this condition is equivalent to a single rotation.

If a body be subject to two or three simultaneous rotations round axes which meet in a fixed point, the resultant movement is rotation round a single axis, which is found by a construction precisely the same as that of the parallelogram or the parallelepipedon of velocities: the sides of the figure represent in direction the axes round which the rotations occur, and in length the amount of angular velocity;* the diagonal obtained represents in the same way the axis and the angular velocity of the resultant rotation.

The most indeterminate motion of a rigid body may always be resolved into the same motion as that of a screw in its nut, namely, a Rotation and a Translation. As the body continues to move, the axis of the imaginary screw may change its direction in space; but when considered at and for the space of any particular very small instant, it may be regarded as fixed, and is then called the Instantaneous Axis. As limiting cases, the translation may = 0, when there is Simple Rotation; or the angular velocity of rotation may = 0, in which case we would have Simple Translation. The ultimate position attained may always be reached by means of a single translation and a single rotation round some axis.

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* When a body moves in a circular path, its velocity in that path may be measured by the length of the path traversed divided by the time, as usual; or it may, in many respects, more conveniently be expressed in terms of **angular velocity.** Here the path is measured, not directly in terms of its own length, but with reference to the angle which it subtends and to the length of the radius. The **unit angle** is that angle (57°·29578 = 57°·17′·44″·8 nearly) which is subtended by a part of the circumference equal in length to the radius. Hence the circumference = 360° = 2π × **unit angle**, π being equal to 3·1416. Unit angular velocity is that under which a particle travelling in a circle whose radius = 1 would itself describe a path = 1—that is, unit angular velocity is that of a rotating body which traverses the unit angle—in unit of time. If the radius be r and the angle traversed be θ, the part of the circumference passed over is $r\theta$, and if this be accomplished in time t, the linear velocity of a particle on the circumference is $r\theta/t$; that of a particle nearer or farther from the centre is proportionately less or greater; while the Angular Velocity of all the particles of a rotating wheel is the same, namely, θ/t. The Dimensions of angular velocity are an Angle (= Arc ÷ Radius) ÷ a Time = $[(L \div L)] \div [T] = [T^{-1}]$.

Degrees of Freedom of a Rigid Body.—When a rigid body is absolutely free to move in any direction in space, it is said to have six degrees of freedom. These are (1) three degrees of freedom of translation, like those of a simple particle ; and (2) three degrees of freedom of rotation round three axes arbitrarily chosen at right angles to one another. Any such body may move, for example, (1) upwards or downwards, (2) to the N. or S., (3) to the E. or W., or it may rotate round (4) a vertical axis, (5) an axis lying N. and S., or (6) an axis lying E. and W. Any rotation not round these axes, or any translation not in the direction of these axes, may be resolved into its components, round or parallel to them ; and as any change of position whatsoever may be produced by a single translation and a single rotation, any motion whatsoever may be effected by a body which has these six degrees of freedom.

If one point in a rigid body be fixed, there can be no translation, and three degrees of freedom are thus lost ; the body has, however, unlimited freedom of rotation round any axis passing through the fixed point, and thus retains three degrees of freedom. If a line in the body be fixed in position, there can be no translation, and there can be no rotation except round this fixed line, and so there can be only one degree of freedom, which corresponds to that rotation. If a surface (or, which amounts to the same thing, if three points) in the body be fixed in position, there can be neither translation nor rotation, and the rigid body has no freedom.

If a point in the body be restricted to motion along a given line, there can only be one translation, but there may be any rotation, and so the rigid body has four degrees of freedom. When a given line in the body must coincide with some part of a line assigned in space, there can be only one translation—that along the line assigned, and one rotation—that round the line ; and here we find the rigid body to have two degrees of freedom. If a point in the body be restricted to a given surface, the only motion which is impossible is translation in a direction at right angles to the surface, and hence the body has in this case five degrees of freedom. If a line in the body be restricted to a given surface, one translation is impossible, as in the previous instance, and there are two rotations possible, the one round the line which is restricted to the surface, and the other round an axis at right angles to the surface : in this case there are accordingly four degrees of freedom. If three points in a body be restricted to a surface, there can be rotation round an axis at right angles to the surface, and there can be translation in any direction along the surface but not away from it, so that in this case we have three degrees of freedom.

Strain.—When a body is not rigid, its particles may so move with reference to one another that their displacement produces deformation, and such relative motion of the particles of which a body is made up is called a Strain of the body.

Suppose a circular plate to be expanded uniformly, as a disc of iron is when heated ; the radius will enlarge in the ratio of (say) 1 to α ; the area of the plate increases in the ratio $1 : \alpha^2$. The linear expansion is the difference between the initial and the final length of the radius, *i.e.*, $r(\alpha - 1)$ where r is the original radius, and is hence proportional to $(\alpha - 1)$. If the body have contracted, α is less than 1, and $\alpha - 1$ is negative ; hence the linear expansion is negative. The superficial expansion is the difference between the areas before and after the strain, viz. $\{\pi(\alpha r)^2 - \pi r^2\} = \pi r^2\ (\alpha^2 - 1)$;[*] hence the superficial expansion is proportional to $(\alpha^2 - 1)$. A square similarly affected has its sides and area increased or diminished in the same ratios: so would a parallelogram or any other plane figure, if the linear expansion were the same in all directions. Again, suppose a globular body to be thus uniformly expanded; it increases in size and becomes a larger globe: if its radius increase in the ratio $1 : \alpha$, its bulk will increase in the ratio $1 : \alpha^3$, and its cubical expansion will be proportional to $\alpha^3 - 1$. Cubes, parallelepipeda, and all other solid figures, would under the same circumstances become larger or smaller cubes, parallelepipeda, etc., whose sides and bulk would bear similar ratios to their original dimensions.

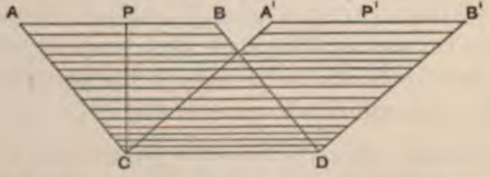
Suppose a square to be unequally dilated or contracted along axes parallel to its sides, the square will become a parallelogram. A circle will thus become an ellipse; an ellipse will become an ellipse of another form. As a circle is an ellipse of a particular form whose length (its major axis) is equal to its breadth (its minor axis), any ellipse may be converted by a strain into a circle, if its axes be in due proportion lengthened or shortened. If the expansions along the two rectangular axes be in the ratios $1 : \alpha$ and $1 : \beta$, the area of the resultant parallelogram, ellipse, or circle, will be to that of the original figure in the ratio $\alpha\beta : 1$.

If the body be a cube and be unequally expanded in directions parallel to its sides, it becomes an unequal-sided parallelepipedon. If the several sides expand in the respective ratios $1 : \alpha$, $1 : \beta$, and $1 : \gamma$, the bulk of the parallelepipedon will bear to that of the cube the ratio $\alpha\beta\gamma : 1$. A sphere strained in the same way will become an ellipsoid, a so-called "strain-ellipsoid": any ellipsoid will become an ellipsoid of a different form, and may become that particular kind of ellipsoid known as a sphere, an ellipsoid whose three axes are equal to one another.

It will be seen on drawing any of the figures just described that any two parallel lines drawn through the body in its original form will be parallel after the strain. In this kind of strain, called **Homogeneous Strain**, there are always three axes, which were at right angles to one another in the original position of the body, and continue to be so after the strain. These are the axes of the strain-ellipsoid into which an imaginary sphere existing in the body would be transformed by the strain.

Shear.—If a body be so distorted that one plane passing through it is fixed while others move past this at rates proportional to their distances from it;—if, for example, the body ABCD (Fig. 25) be so distorted that CD remains in its original position,

Fig.25.



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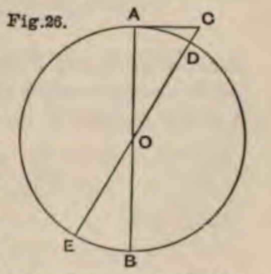
[*] $\pi = 3{\cdot}14159\ldots$, the ratio of the circumference of a circle to its diameter. The area of a circle $= {\cdot}7854 \times \text{diar.}^2 = 3{\cdot}1416 \times \text{rad.}^2 = \pi r^2$.

while the line AB travels into the position AB′, the body is being sheared, or is undergoing the kind of deformation or strain known as a **Shear.**

The greater the distance of any plane in the body from the plane passing through CD, the greater will be its displacement from its original position; and a body so sheared we may conceive as made up of an indefinite number of indefinitely thin layers which relatively move by slipping over one another. A shear is measured by the amount of relative motion between two non-distorted planes which are situated at a unit distance from one another, and which remain parallel. In Fig. 25 the amount of the shear of ABCD is the ratio of the displacement AA′ or PP′ to PC, the shortest possible line drawn between the two parallel planes AB and CD, and vertical to them both; that is, it is equal to $\frac{\mathrm{PP'}}{\mathrm{PC}} = \tan \mathrm{PCP'}$ if $\mathrm{PC} = 1$, or to $\mathrm{PC} \times \tan \mathrm{PCP'}$ if PC have any other value.

Circular Motion.—If a body move in a circular path, as, for example, a stone whirled in a sling, its motion at every instant is compounded of a tangential motion and a motion towards the centre. If it pass the point A in Fig. 26 with such a velocity that it would in a unit of time, if not drawn towards the centre, have reached the point C, and if it be at the end of that interval found at the point D, it is evident that the acceleration towards the centre must have produced in unit of time the change of position represented by the line CD. In other words the whirling sling-stone is constantly being drawn in from the tangential path, which, in virtue of its inertia, it would at every instant naturally take; and a planet in its orbit is constantly falling towards the sun, but does not proceed straight towards it, for the resultant of its tangential and its centripetal motions is an elliptical path which is approximately circular.

Fig. 26.

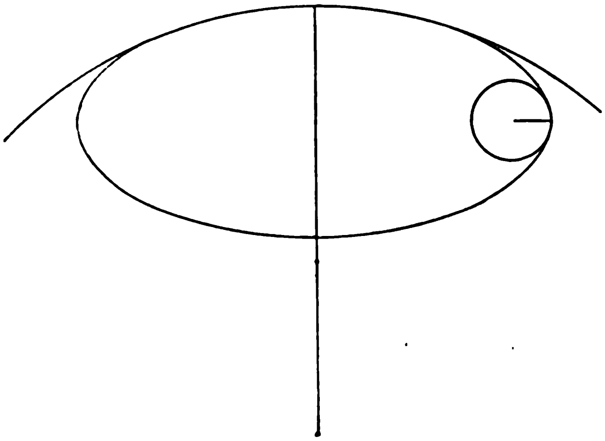


If the line CD be prolonged through the centre to the point E, Eucl. III. 36 shows that $\mathrm{CD} . \mathrm{CE} = \mathrm{AC}^2$. If v represent the velocity of the body in the direction AC, $v_{,}$ the average velocity in the direction CD, and r the radius of the circle, we thus find $v_{,}(2r + v_{,}) = v^2$; or $2rv_{,} + v_{,}^2 = v^2$. (i).

If the unit of time taken be sufficiently small, the square of the small quantity $v_{,}$ will be so small as to be negligible, and the above equation will become $2v_{,}r = v^2$. (ii). But $v_{,}$ is the average velocity of fall from the direction AC during the instant in question, and hence the velocity at the end of the interval is $2v_{,}$; this is the velocity acquired in unit of time, and hence the acceleration towards the centre is $a = 2v_{,}$. Hence the equation (ii) may be written $ar = v^2$ or $a = v^2/r$; the Acceleration towards the centre of the circular path in which a body is moving is numerically equal to the Square of its Velocity at any instant divided by the Radius of curvature. If a body be travelling in any other curve, there can at every instant be found a circle, a part of the circumference of which coincides, to an indefinite approximation, with the curve at the instant.

Curvature.—Any curve may be considered as made up of successive elements, each of which approximately coincides with a part of the circumference of a particular "**osculating**" **circle,** which may always be found. For each element of the curve, the radius of the corresponding osculating circle—whose circumference coincides with that element, and which would have the same tangent—is called the instantaneous **radius of curvature**; and as the curve passes from point to point the osculating circle may be

Fig.27.



changed in respect of its radius or its centre. Thus in an ellipse, near the extremity of the major axis, the osculating circle of curvature is smaller than it is near the end of the minor axis, as is shown in Fig. 27. Accordingly,

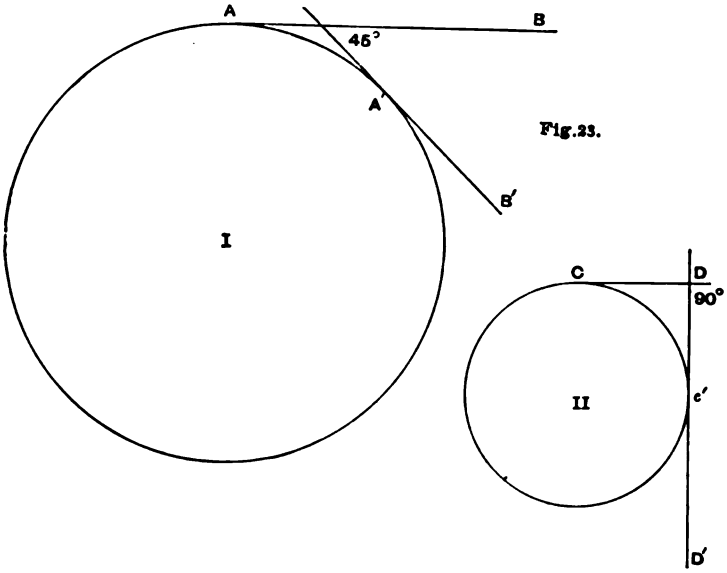


Fig.23.

if a body move in a curved path, its acceleration at every instant towards the instantaneous centre of curvature is numerically equal to the square of the instantaneous tangential velocity divided by the instantaneous radius of curvature, ρ.

But in a curve, the Curvature is the angle through which the tangent sweeps round per unit of length of the curve, and this varies inversely as the radius, as may be seen on comparing the circles in Fig. 28. The radius of I is twice that of II: the length AA′ = Cc′ is supposed to be a unit of length. In I the tangent AB has swept round into the position A′B′ through an angle θ: the tangent CD has swept through twice as great an angle, the length of circular path traversed being the same: wherefore the curvature (as above defined) of the circle II is twice that of the circle I, and that of any circle is inversely as the radius: and since curvature and acceleration towards the instantaneous centre both vary inversely as the radius, they are proportional to one another, and therefore the acceleration of a body moving in a curved path is directed towards the instantaneous centre of curvature, and is equal to the product of the square of the instantaneous velocity into the curvature. $a = \frac{v^2}{r}$; $\frac{1}{r} = c$; $\therefore a = v^2 c$, where c is the curvature. Hence a comet turning sharply round the sun, the curvature of its path being very great, has a very great acceleration inwards.

SIMPLE HARMONIC MOTION AND WAVE-MOTION.

Motion in a circle may be practically effected by a heavy ball suspended by a string, and set to swing in a circular path. A pendulum set to swing in this way goes by the name of the "Conical Pendulum." If the path of the bob of the so-called conical pendulum be looked at from above, it appears circular: if looked at from a point somewhat to one side, it appears elliptical: as the eye approaches the level of the plane in which the bob travels, its path appears to be an ellipse comparatively long and narrow; and as the eye is placed exactly on a level with that plane, the bob appears to travel backwards and forwards in a straight line. In a similar way, the satellites of Jupiter, which travel round that planet pretty nearly in the plane of the Ecliptic,* and therefore astronomically on a level with ourselves, seem to travel backwards and forwards in lines nearly straight. The bob of the conical pendulum and the satellites of Jupiter appear to move very slowly at the end of their apparently linear courses. This is because the moving body is really travelling either towards the eye of the observer or away from it at the time when it appears to be at the end of its swing. When it is travelling right across the field of view, when it is in the middle of its apparently linear path, it seems to travel rapidly. Just in the same way a railway train seems to be moving very much faster when it runs right across the field of view than when it is coming or going round a curve, and is seen not broadside but end-on.

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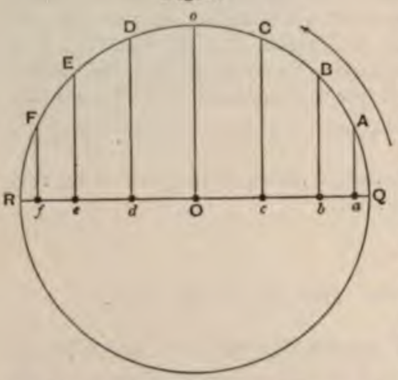
* The bodies which make up the solar system may be said in a rough way to be situated in a plane fixed in space, and called the Ecliptic, from which they do not very widely depart. Objects moving in their respective orbits in this plane may appear to pass and obscure—*i.e.* eclipse—one another, like so many ships at sea.

If we represent the circle in which the body is moving by the circle QAR, and its apparent linear path by the line QR, and if we represent a certain number of positions of the body in the circle by the points A, B, C, D, etc., we may define the apparent motion of the body in the straight line QR by finding the points *a*, *b*, *c*, etc., to which the respective positions of the body in the Circle of Reference correspond. This is done by drawing lines A*a*, B*b*, etc., at right angles to the line QR. It will be easily seen that if QA, AB, BC, C*o*, etc., be equal to one another, the corresponding lines drawn along QR are longer near the centre of that line than near its ends; and these represent the spaces apparently traversed in equal intervals of time.

Fig. 29.



A representation of the relative values of these lines Q*a*, *ab*, etc., is obtained as follows. If the line QR be taken as the axis of X, a line OA, joining the centre O and the corresponding position A of the particle, may be supposed to sweep round into the successive positions OB, O*o*, OC, etc. As it does so it forms an increasing angle with the line OQ. Then the lengths of the lines O*a*, O*b*, O*c*, etc., bear to one another the ratios of the *cosines* of the angles QOA, QOB, etc.

These angles will, if the corresponding motion in the circle of reference be uniform, depend directly on the time. If the angle swept through in unit of time be θ, that swept through in time t is $t\theta$: hence, if the starting-point in time be that instant at which the body is at the point Q, the apparent distance of the body from the point O will be proportional to $\cos t\theta$, and, if a be the radius, will be equal to $a \cos t\theta$. Hence $x = a \cos t\theta$. When $t\theta = 0$, since $\cos 0° = 1$, $x = a$: when $t\theta = 90°$, since $\cos 90° = 0$, $x = 0$: when $t\theta = 180°$, since $\cos 180° = -1$, $x = -a$: when $t\theta = 270°$, $x = 0$: when $t\theta = 360° = 2\pi$, $x = a$. As the radius continues to sweep round, the values of x repeat themselves.

Such a motion as that apparently executed backwards and forwards along the line QR is called **Simple Harmonic Motion** or S.H.M. Such motion requires to be studied with great care, for actual instances of it occur throughout the phenomena of Optics and Acoustics, and of many other parts of physics.

The length OQ or OR of the swing from the median position O is called the **Amplitude,** a, of the S.H.M. Simple Harmonic motion, then, is motion which is a periodic[*] function of the time (*i.e.*, repeats itself at regular intervals), which is effected backwards and forwards along a line, and which may be studied by comparison with uniform motion round a **circle of reference,** of which the line of S.H.M. is the diameter, and of which accordingly the amplitude of S.H.M. measures the radius.

The **Period** of a S.H.M. is the *interval of time* which elapses between the passage of the moving particle over a certain point and the next passage of the same particle over the same point in the same direction. This corresponds to the time during which one complete revolution would be effected round the circle of reference.

It is understood that when the moving body appears to travel from left to right, its motion is positive; when it moves from right to left its motion is in the negative direction. When the particle is at Q in Fig. 29, it is said to be in its position of greatest positive elongation: when at R it is in its position of greatest negative elongation.

At any instant the position of the particle executing the S.H. Motion may be stated in terms of the **Phase** of the S.H.M. at that instant,—the Phase being the interval of time, the fraction of a period, which has elapsed since the particle last passed through O, the middle point of its course, in the direction reckoned as positive.

If the starting-point in time be not the instant at which the particle was at the point Q in the circle of reference (Fig. 29), but so many units of time after that instant that the angle traversed is not $t\theta$ but $(t\theta + \epsilon)$, then the motion along the axis of X is $x = a \cos (t\theta + \epsilon)$. This term ϵ is called the **Epoch.**

Acceleration in S.H.M. Proportional to Displacement.—In Fig. 29 the moving particle as it describes its circular path does so under the influence of an acceleration V^2/r towards the centre. This may be resolved, when the particle is at any point A, into $(\cos AOQ.V^2/r)$ parallel to QR, and $(\sin AOQ.V^2/r)$ at right angles to it. The former component is alone effective in reference to a body moving in S.H.M. in the line QR, and, being always towards the centre, it is alternately in the same and in the opposite direction to the movement of the body. But this acceleration towards the centre $= (\cos \text{AOQ} \,.\, \text{V}^2/r) = (\text{O}a/r) \,.\, (\text{V}^2/r) = \text{O}a \,.\, (\text{V}^2/r^2) = \text{O}a \times$ a constant number, for V and r are constant. The Acceleration when the particle is at a in its S.H.M. is thus proportional to Oa, the Displacement from O.

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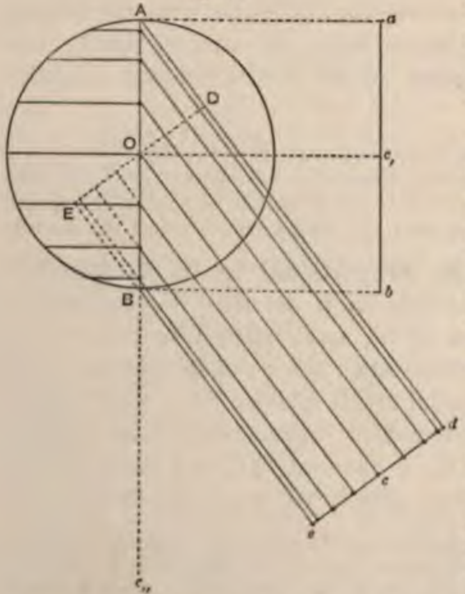
[*] If x vary when y varies, as, for instance, if $x=ay$, or if $x^2=y^3+ay^2+by+c$, or $x=\log y$, or if in any other way whatsoever the value of x depend on that of y, x is said to be a *function* of y; and if x recur to the same value while y goes on uniformly increasing or diminishing, x is said to be a *periodic* function of y: if $x = \cos y$, or $= \tan y$, or $= \sin y$, etc., as y goes on increasing, x recurs to the same values, for $\cos y = \cos (2\pi + y) = \cos (4\pi + y) = \cos (6\pi + y) = \cos (8\pi + y)$, etc.

Again, the constant number (V^2/r^2) is the square of $\text{V}/r = \omega$, the angular velocity in the circle of reference. Therefore

$$\omega = \sqrt{\frac{\text{acceleration at any point.}}{\text{displacement at that point.}}}$$

Isochronous S.H.M.'S.—Since ω is the angular velocity (page 74, *note*), and since the time taken to execute one complete revolution round the circle of reference is $\tau = 2\pi/\omega$, then *if* ω, the angular velocity in the circle of reference, be constant, the time τ—that is, the period of the S.H.M.—is independent of the amplitude; for the amplitude does not enter into that formula which expresses the value of τ, namely, $\tau = 2\pi/\omega$. This criterion, the constancy of ω, is satisfied if the quotient $\frac{\text{acceleration}}{\text{displacement}}$ be a constant number. In other words, if the acceleration with which a particle tends to return to its median position bear a fixed proportion to the displacement, the particle will execute a S.H.M. whose period is independent of the amplitude of oscillation. This proposition is one of high importance in the theory of the Pendulum, of Elastic bodies, of Sound, of Heat, and of Light.

Fig. 30.



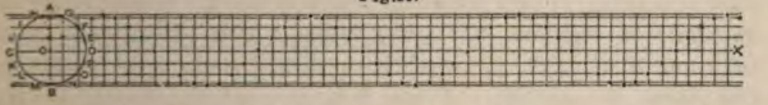
Projection of a S.H.M. always an Apparent S.H.M.—It is understood that when a line AB is looked at from the position c in Fig. 30, that line appears to be shortened, and to assume the length DE, and the line DE, or de, at right angles to cO, is called a Projection of AB. There may be as many projections as there are possible directions of the line O*c*. When the eye is placed somewhere in the line $Oc_{\prime}$, the line AB does not appear to be shortened, and the projection *ab* of the line AB is equal to that line itself; when the direction of sight has become $ABc_{\prime\prime}$, the projection of the line AB, thus seen end-on, is a point merely.

If the position of the point of view be intermediate between these extremes, as, for example, at *c*, the projection *ed* is to the original AB as the cosine of the angle AOD in the figure is to unity.

If now a body execute S.H.M. on the line AB, the diagram will show that, if regarded from *c*, it will appear to perform a S.H.M. corresponding in period and phase, though not in amplitude, in the line DE; or, in other words, the projection of a S.H.M. is itself an apparent S.H.M.

Harmonic Curve.—If a S.H.M. in one line be compounded with a uniform motion in a direction at right angles to that line, the resultant path may be found by the following construction. Let A and B (Fig. 31) be the points of greatest elongation of the particle. Let the particle be also made at the same time to travel uniformly from left to right. Draw ACB, the circle of reference. On it lay off (say) sixteen equidistant points, D, E, F, G, etc.; lines drawn through these at right angles to the line AB determine the points on that line which define the positions of the particle, so far as these positions are determined by the S.H.M., at equal intervals of $\frac{1}{16}$ of the period. These lines being drawn as in the figure, other lines may be drawn parallel to AB and cutting the axis OX at equal intervals, each of which represents the amount of motion from left to right during $\frac{1}{16}$ of the period of the S.H.M. The previous examples of composition of simultaneous motions will show us that the successive positions after successive intervals ($\frac{1}{16}$ of a period in this case) will be found by marking off points, such as those indicated in the diagram, the distances of which along the axis of X represent the displacements due to the uniform motion, and whose distances along the axis of Y represent the displacements due to the S.H.M. If these points be joined they give rise to a very characteristic curve, the Curve of Sines, or the Harmonic Curve. The geometrical property

Fig. 31.



of this curve evidently is that, of any point in it the Abscissa (along X) is proportional to the time, while the Ordinates (or distances from the axis of X) are proportional to the sines of angles, which are themselves proportional to the time. The ordinates, therefore, pass through positive and negative values alternately, while the abscissæ uniformly alter in value. There may be several curves of sines, differing from one another in the amplitude of the S.H.M., or in the rapidity of the uniform motion in the direction of the axis of X. Evidently, if the amplitude of the S.H.M. be greater or less, the undulations of the Harmonic Curve will be deeper or shallower; while, if the motion along the axis of X be slower or quicker, the undulations of the resultant curve will be closer together or farther apart.

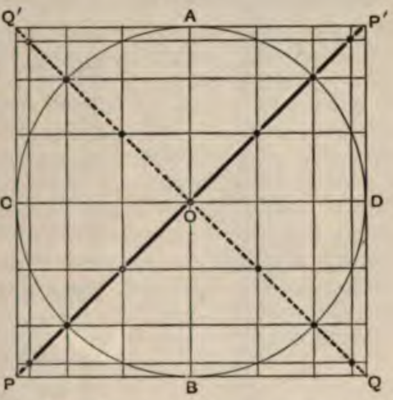
When a pendulum is set to swing, its oscillatory motion is visibly quickest at the middle of its course and slackens towards each end of it; so that the motion of a pendulum is very much like S.H.M., and hence, if a pendulum be made to carry sand and to drop it as it travels, it will deposit a trace which is much thicker at each end of its course, where its bob is moving slowly, than it is at the middle where its course is rapid. If the pendulum be made to oscillate, while the frame which supports it is at the same time made to travel in a direction at right angles to the plane of oscillation of the bob, or—what amounts to the same thing—if the surface on which the sand is received be made to travel under the oscillating pendulum, which is suspended from a fixed support, the sand is deposited in a curve which can hardly be distinguished from the Harmonic Curve. But any motion which, when compounded with a uniform rectilinear motion, produces the characteristic Curve of Sines must itself be a S.H.M., and hence the motion of the pendulum, projected on the plane which receives the sand, is approximately a S.H.M.

That the trace left by the falling sand does not with perfect exactitude coincide with any harmonic curve is due to the fact that though the motion of the bob in its curved path is nearly S.H.M., the necessary divergence between that curved path and the flat plane on which the sand is deposited expresses itself as a slight distortion of the resultant trace. If the arc of oscillation be small, this distortion is so very small that most of the properties of Simple Harmonic Motion can be practically demonstrated by the use of pendulums which record their own movement in some such way as that mentioned.

The trace left by such a moving body does not present, in the parts corresponding to the greatest positive or negative elongation, so steep an ascent or descent as it does when it crosses the axis CX (Fig. 31); this indicates that the body moving in S.H.M. is moving more rapidly at the centre of its course than at its ends.

Composition of Simple Harmonic Motions.—If the same body be subjected to two different S.H.M.'s, the problem of their composition may in general be solved with great ease by the use of the respective circles of reference. (1.) Let the two motions be equal and in the same direction: the resultant will be a S.H.M. of double amplitude. (2.) Let the two motions be equal and in the same line, but differing from one another in phase by half a period, the resultant will be Rest. (3.) Let the two S.H.M.'s be equal, at right angles to one another (AB and CD, Fig. 32), and in the same phase, so that when the moving particle is at O it is moving in a positive direction with reference to both axes: its real course will be in a line PP′, making an angle of 45° with both AB and CD. (4.) If it be half a period behind in one of the S.H.M.'s, so as to be moving in the + direction (from O to D) with reference to one axis, and negatively (from O to B) with reference to the other, the resultant will be S.H.M. in the line Q′Q. (5.) If the one S.H.M. be a quarter period behind the other, so that while the moving particle is at the middle of (one, say) its vertical oscillation, it is only just leaving the point of greatest negative elongation, in respect to the other—its horizontal oscillation—its motion will be compounded of one forwards, from C towards O, and one upwards, past O towards A; the result will be that the motion of the body will be restricted to the circumference of the circle DBC, and the body will move round that circle in the direction CA. Similarly (6), if the horizontal movement be in advance of the vertical by a quarter period, so as to be already bringing the body back from its position of greatest positive elongation while it is still moving vertically upwards past O towards A, the body will travel in the circle DAC in the direction DA. Hence we have the very important proposition that motion in a circular path may be considered to be made up of two S.H.M.'s, the one a quarter

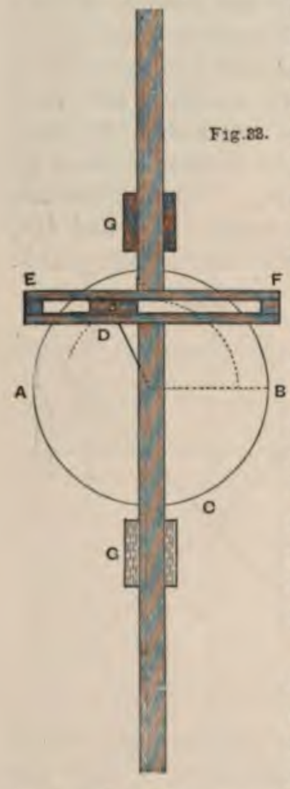
Fig. 32.



of a period in advance of or behind the other, according to the direction in which the body travels in the circle.

Though the conical pendulum shows this when its motion is watched, perhaps the simple piece of mechanism drawn in Fig. 33 may make it even plainer. The circular plate ACB has a pin D set in it. This pin works in a sliding piece within a slot in the frame EF. The frame EF is connected with two sliding bars, which run between the guides G, so that lateral motion is impossible. Let the circular plate ACB be rotated uniformly: the frame EF will be moved upwards and downwards alternately, while the pin D will move in the slot from right to left and from left to right alternately. It will easily be seen on making a model, or on imagining the diagram to act, that the oscillations of D in its slot, and those by which it produces alternating ("reciprocal") motion of EF, do not agree in phase, but differ by a quarter of a period, the one being at the middle when the other is at the end of its course.

Fig. 33.

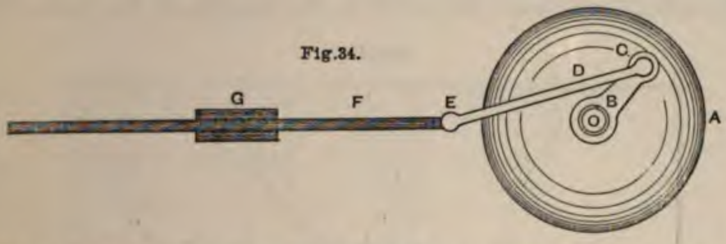


The circular motion of the pin D is, therefore, compounded of two S.H.M.'s, of which one is easily conveyed to the frame EF, while the other cannot, because of the arrangement of the guides G, be so conveyed. The apparent conversion of the circular motion of the disc ACB into the reciprocal motion of the sliding-bars is in reality, then, due to the suppression of one of its simple harmonic components; and the motion of the sliding-bars is exactly S.H.M. if D rotate uniformly.

Circular transformed into Reciprocating Motion.—In accordance with this principle, mechanism intended to transform rotatory into reciprocating motion is in reality mechanism which, with more or less completeness, suppresses one of the S.H.M.'s of each particle of a rotating body. The most usual device is that of a **crank**; this may be seen in one form or another in almost every piece of machinery worked by steam-power. In Fig. 34 the wheel A is rotated almost uniformly, and the crank B is turned round along with it; attached to the crank B by a joint at C is the rod D, which is, in its turn, attached by the joint E to the rod F; this runs between the guides

G. Here the motion of the bar F between the guides is only approximately Simple Harmonic, but approximates more and more nearly to that condition the longer the bar D and the shorter the crank B, or, in other words, the less F and D together diverge from a straight line. The rod F may be made to work a pump-handle, a saw, or any such contrivance whose use requires reciprocating motion.

Fig. 34.



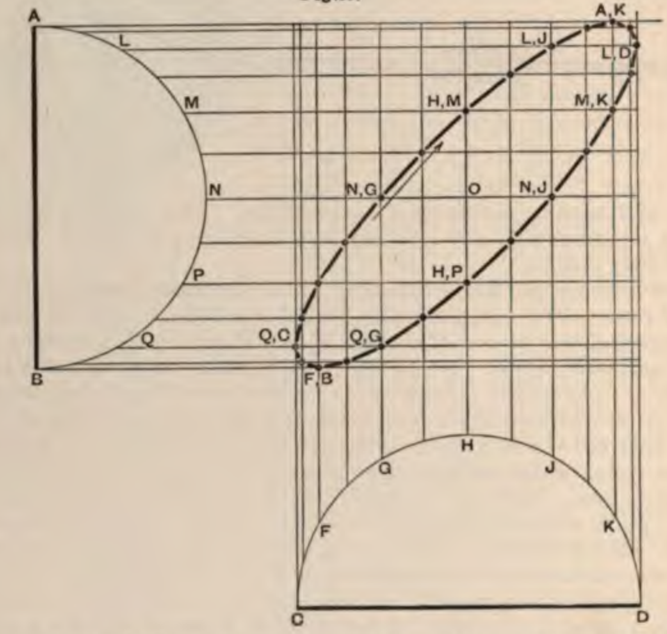
Conversion of Reciprocating into Circular Motion.—If in figure 34 the rod F be supposed to be pushed towards the crank B, then D will be pushed over towards C, and the wheel will turn until E, O, and C are in the same straight line. No further pushing will make the wheel A turn any farther; neither will pulling, when the crank is in this position, have any effect. If in the same figure the rod F be pulled instead of pushed, the points E, C, and O will come to be in the same straight line; and in this position, again, neither pulling nor pushing will have any effect in making the wheel turn. There are therefore two positions or **dead points** in which a piston cannot by means of a crank set a wheel in motion. If, however, the wheel A be heavy, or, better still, if it be connected with a heavy flywheel, when it arrives at the dead points its Inertia, or that of the flywheel, manifests itself by the wheel A rotating past these unfavourable positions into others in which the reciprocal movement of F can act effectively; the wheel A is thus set in continuous motion. An example of this is furnished by the treadle and flywheel of a lathe or of a sewing-machine.

In the marine-engine, since there can be no flywheel on board ship, some other contrivance is necessary. Two cylinders, and therefore two reciprocating pistons, both acting on the same wheelwork, are so arranged that when the one crank is at its dead points the other is at the middle of its course, and therefore at its position of greatest advantage.

Composition of Simple Harmonic Motions (resumed).—Let the two S.H.M.'s differ by some other fraction of a period than the half or the quarter, as, for instance, the twelfth: if they be equal in amplitude they both have the same or equal circles of reference. Let the circumference of these be divided into twenty-four equal parts, as in Fig. 35, in which only a part of each circle of reference is shown. Let chords be drawn dividing the arcs AB and CD in the points F, G, etc.

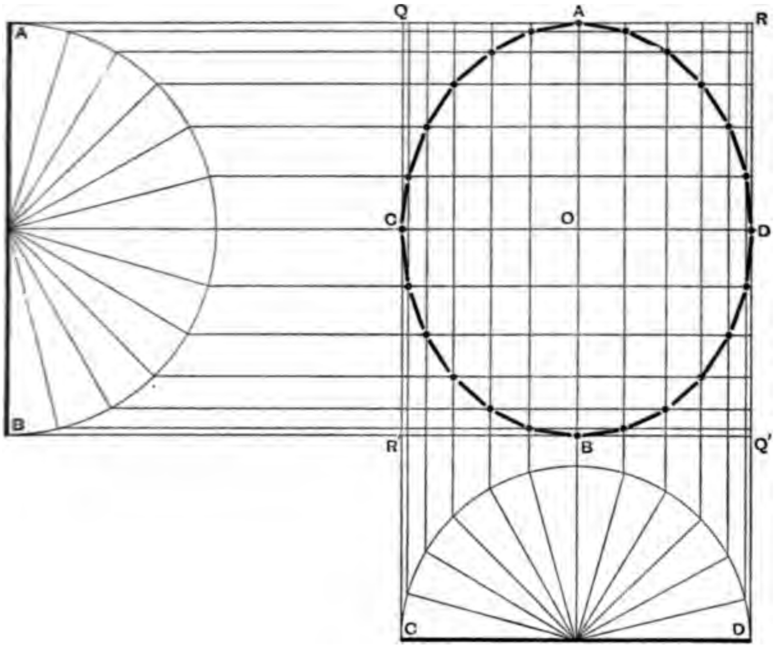
If, now, the particle be at the middle of its course with reference to the S.H.M. along the axis BA, and if it be at the same time $\frac{1}{12}$ of a period behind (so as not yet to have arrived at the central point) in its execution of the S.H.M. referred to the axis CD, the point at which it must be situated in order to satisfy both these conditions must be N, G. When $\frac{1}{12}$ period has elapsed, it will have advanced to the middle of its horizontal course, but will have moved vertically as far as the point M, H; at the end of another $\frac{1}{12}$ it will be at the point L, J, then at A, K; then it still advances horizontally to the limit of its course, but returns along AB to L, thus reaching the point L, D; then it returns to M, K, and so on, and in this way it describes an **ellipse**. When the difference of phase is less, the point G is nearer to O, and the ellipse is narrower; when there is no difference of phase, the points G and O coincide, and this ellipse is a straight line, as has been already learned (Fig. 32). When, on the other hand, the difference of phase is greater, the point G is farther from O, and the ellipse widens out until, when the difference of phase is $\frac{1}{4}$ period, the point G coincides with C, and the ellipse is a perfect circle.

Fig. 35.



When the amplitudes are not equal, the circles of reference will not be equal. If the two S.H.M.'s be AB and CD, the corresponding construction is shown in Fig. 36. If they be in the same phase, the resultant is S.H.M. in the line R′OR; if they differ in phase by half a period, the resultant is S.H.M. in the line QOQ′; if the difference of phase be $\frac{1}{4}$ period, the path is the ellipse ABCD, traversed in the direction BC if the S.H.M. in AB be $\frac{1}{4}$ period in advance, and in the direction BD if it be $\frac{1}{4}$ period in arrear. If the difference of phase be any other fraction of a period, the resultant will be motion in some other ellipse contained within the same bounding rectangle QRQ′R′. The construction is the same as that of Fig. 35.

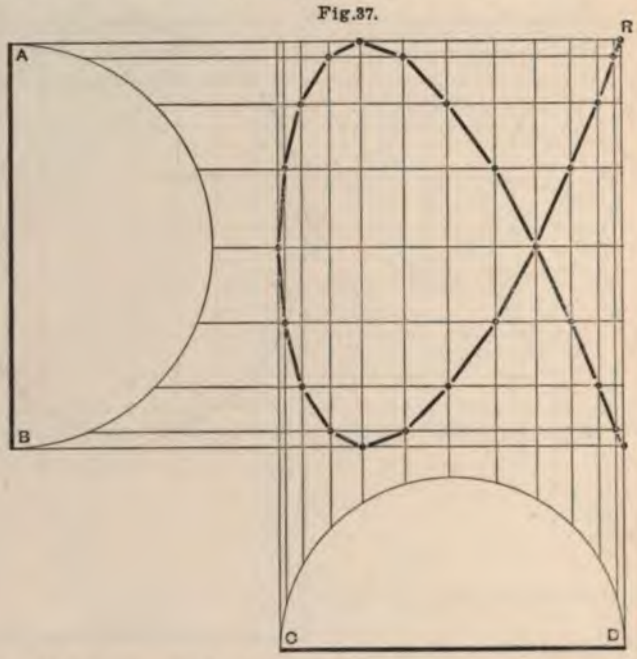
Fig. 36.



Composition of S.H.M.'s of different period.—The same method with little modification may be here employed. The respective circles of reference are drawn and are divided into arcs corresponding to equal intervals of time. The lines representing the S.H.M.'s are divided in accordance with the now well-known construction and the positions of the body traced out accordingly.

In Fig. 37 the periods are as two to three, the period of the vertical S.H.M. being the shorter: the ranges of oscillation are represented by the lengths of AB, CD respectively. The respective circles of reference are drawn: they are equably divided into arcs corresponding to intervals of time arbitrarily chosen, say the sixteenth part of the period of the more rapid oscillation in AB, this being the $\frac{1}{24}$ of the period of the slower oscillation in CD. The arcs AB and CD having been thus divided into segments corresponding to equal intervals of time, the usual construction enables us to trace out, point by point, the path of the body, which—if we assume the body to be in the position of greatest positive elongation with respect to both S.H.M.'s simultaneously, and therefore to touch the point R, there being there no difference of phase—we find to be a looped curve over which the body travels backwards and forwards without quitting it.

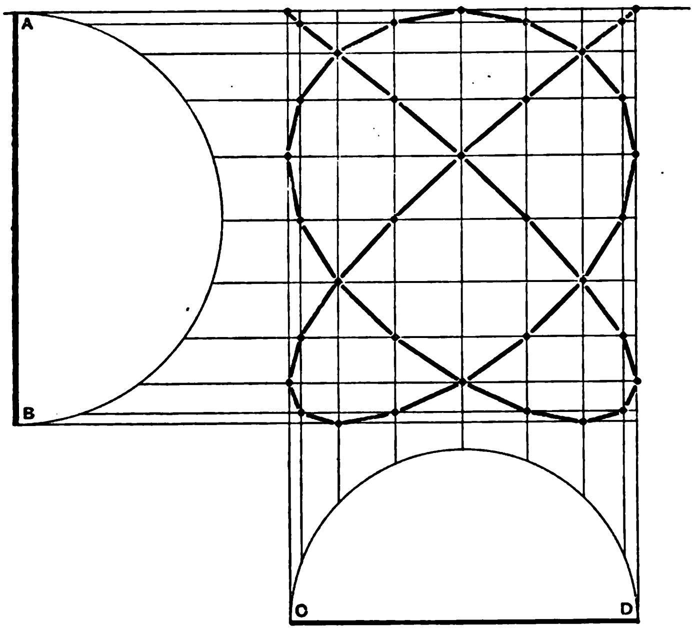
Fig. 37.



In Fig. 38 the period of the S.H.M. in CD is $\frac{4}{5}$ of that in AB. The construction is essentially the same. The arcs cut off on the circle AB must subtend angles at their centre $\frac{4}{5}$ of those similarly subtended by the arcs on CD. Hence the circle AB has been divided into arcs, each of which represents one-twentieth of the circumference, while the circle CD has been divided into 16. These curves may assume a variety of forms depending on variations in the relative lengths of AB and of CD.

Composition of S.H.M.'s differing in Phase.—If the two S.H.M.'s differ in phase, and if those chosen as illustrations be, for convenience of reference, the same as those of figures 37 and 38, the same circles may be drawn and divided in the same way. If the difference of phase between the two S.H.M.'s correspond to such a fraction of the period of the more rapid oscillation as may be represented by the arc CE (whether this be an aliquot part of the circumference or not), the body cannot be at the extremity of both its S.H.M.'s at one time, and when it is opposite the point B it will simultaneously be opposite not the point C but the point E. Figs. 39 and 40 indicate the modifications undergone by the resultant curve of Figs. 37 and 38 in consequence of this difference of phase.

Fig. 38.



These resultant curves vary considerably in form, according to the amount of difference of phase of the component S.H.M.'s. Fig. 41 shows, for example, a series of modifications of the curve of the ratio 1, 2, in which the more rapid oscillation is in advance by periods which successively differ from one another by one-quarter of the period of the more rapid oscillation.

A S.H.M. in a third dimension may be compounded with two in a plane.

Composition of non-commensurable S.H.M.'s.—The periods in all the cases already considered have been commensurable, *i.e.*, they have borne to one another ratios expressible in whole numbers, and consequently, after a certain number of oscillations, the moving body has returned to the starting-point, and the path has been a closed curve which the body has traversed repeatedly.

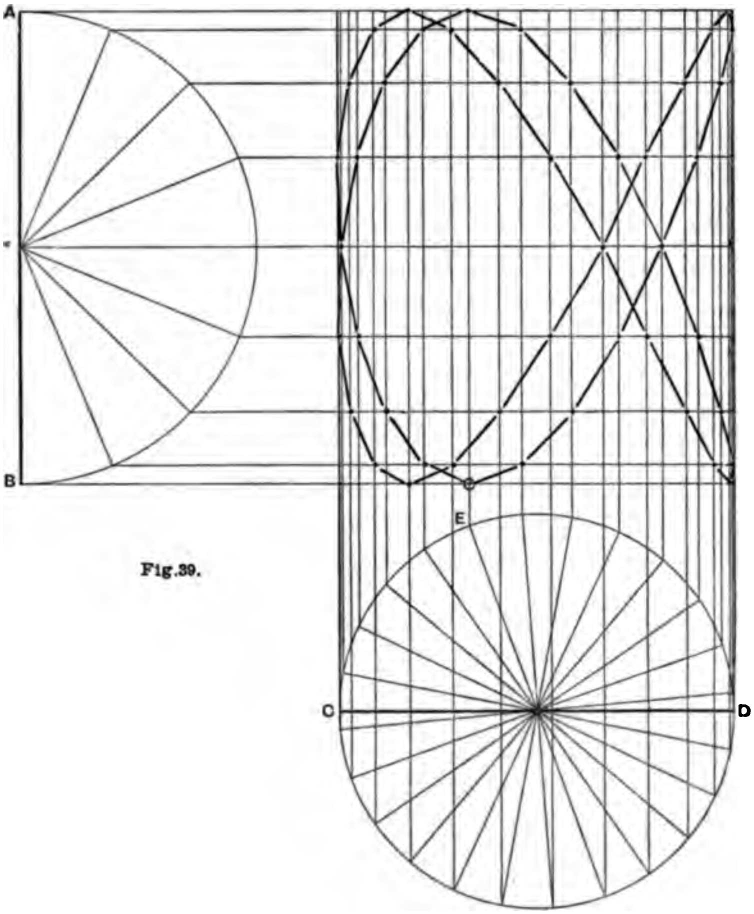
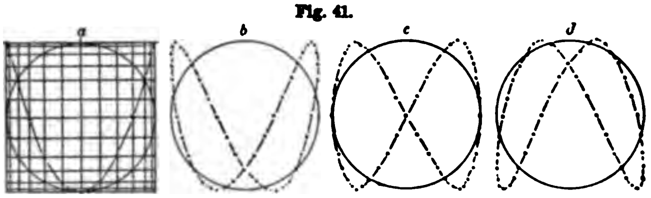


Fig. 39.

Fig. 41.



If, however, the periods be not commensurable, the body cannot return to the starting-point after any definite number of oscillations, and the path never becomes a closed curve.

Composition of S.H.M.'s whose periods approximate to an aliquot ratio.—If the periods of the two component S.H.M.'s be, for example, very nearly as one to two, but not exactly so, the

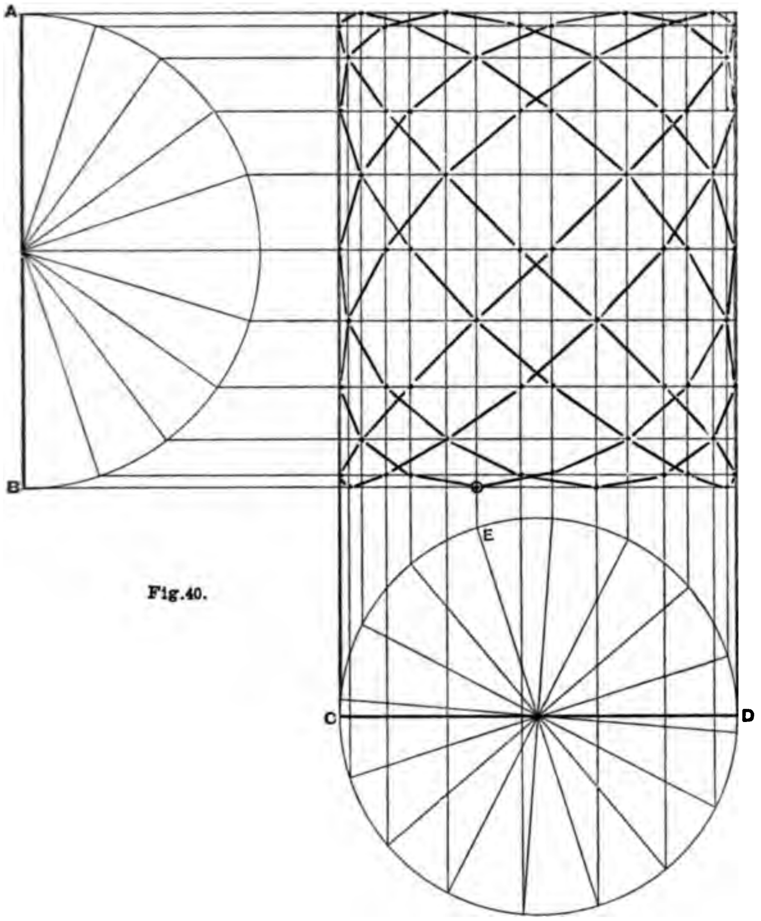
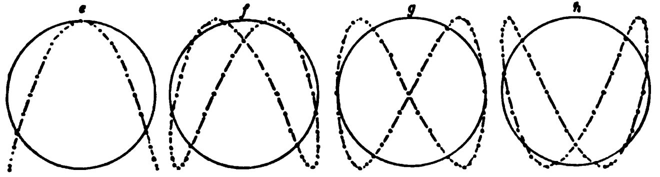


Fig. 40.

Fig. 41.



resultant curve may be, at a given moment, practically the same as that of (*a*) in Fig. 41. The moving body cannot, however, continue to maintain this parabolic path, for the want of exact aliquot proportion of the two periods causes one of the two S.H.M.'s to pass in advance of the other, which, as it were, lags behind, and thus it establishes an increasing difference of phase. When this accumulated difference of phase amounts to $\frac{1}{4}$ the period of the more rapid oscillation, the path described is approximately that of (*b*) in Fig. 41. In this way, by continuous modification, the curve passes successively through all the forms shown in Fig. 41.

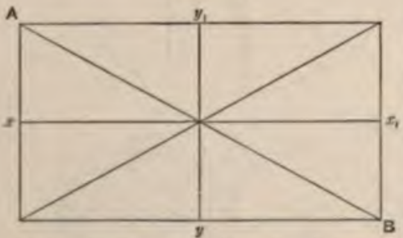
If the respective periods be as 10 : 21, their ratio is approximately 1 : 2, but not exactly so; when the slower S.H.M. has been effected 5 times, the quicker has been effected not 10 but $10\frac{1}{2}$ times, and consequently the quicker motion is a $\frac{1}{4}$ period in advance, and the form of the path has been modified from nearly that of (*a*) to nearly that of (*c*) in Fig. 41. When the slower S.H.M. has been performed 10 times, the form of the path of the body has become approximately (*e*) in that figure; and when 20 of the slower S.H.M.'s have been effected, and of course in the same time 42 of the more rapid ones, the path resumes for an instant its original form (*a*).

If the periods were as 1000 : 2001, in the same way it will be seen that the path regains its original form, but not until 2000 of the more slowly-performed S.H.M.'s have been executed.

Hence the less the proportionate divergence from the simple aliquot ratio to which the actual ratio approximates, the greater the number of oscillations that must be performed, and hence the longer the time that must elapse before the original form of the path recurs, as it will do, approximately if the periods be non-commensurable, perfectly if they be commensurable.

Resolution of S.H.M. into two rectangular components.—We have seen that two S.H.M.'s at right angles to one another, and having the same period and phase, may be compounded into a single S.H.M. by a construction precisely the same as that of the rectangular parallelogram of velocities. Conversely, just as a velocity may be resolved into two component velocities in any two directions at right angles to one another, so may any S.H.M. be resolved into two S.H.M.'s in any two directions at right angles to one another. If in Fig. 42 the S.H.M. be in AB, it may be resolved into two in $x_{,}x$ and $y_{,}y$ respectively.

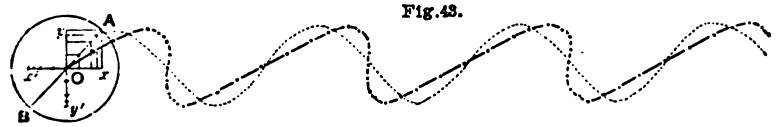
Fig.42.



Any number of S.H.M.'s in any direction may be resolved into their components in three rectangular axes, and these may then be compounded.

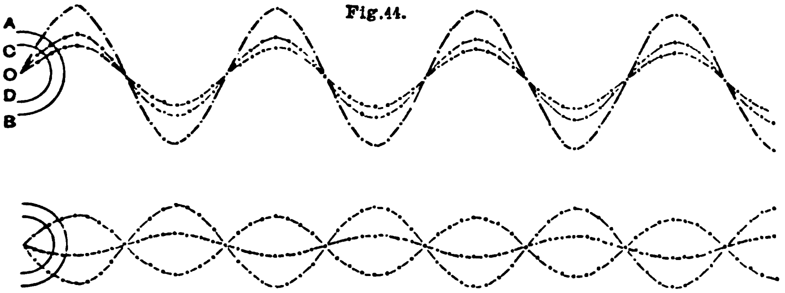
Composition of any S.H.M. with a uniform movement in any direction.—If we wish to compound a S.H.M., which is effected in a line neither parallel nor at right angles to the axis Ox, with a uniform motion in the direction Ox, we must first break the S.H.M. up into its components in the respective directions Ox and $yy_{,}$. We may then compound the component S.H.M. in the direction $yy_{,}$ with the uniform movement in the axis of X, thus producing (as was done in Fig. 31) the Harmonic Curve or Curve of Sines, indicated by a dotted line in Fig. 43. We may then from point to point compound this harmonic curve with the component S.H.M. in $x_{,}x$ by determining point after point in advance of or behind the dotted harmonic curve to an extent corresponding to the displacement produced by that component. The resultant path, indicated by the thick dotted line, is compounded, then, of a uniform motion in the axis of X, a S.H.M. in the same axis, and another S.H.M. of the same period and phase in a line at right angles to that axis. The form of the resultant curve varies according to the speed of that uniform motion which is compounded with the oblique S.H.M.

Fig. 43.



Composition of two S.H.M.'s in the same plane.—If two S.H.M.'s in the same line be compounded, the resultant motion will also be in the same line, and it is best studied by reference to the harmonic curve. Let two S.H.M.'s, which have the same periods and phases, and which are in the same straight line AB, have the amplitudes AB and CD, and let a corresponding Harmonic Curve be traced for each. Then the corresponding curve produced by the superposition of these two motions may be traced from point to point by adding the displacements separately indicated by the harmonic curves. This resultant is found to be a Harmonic Curve, and on careful drawing to scale it may be shown absolutely to coincide with the Curve of Sines derived from a S.H.M. in a line whose direction is the same as that of

Fig. 44.



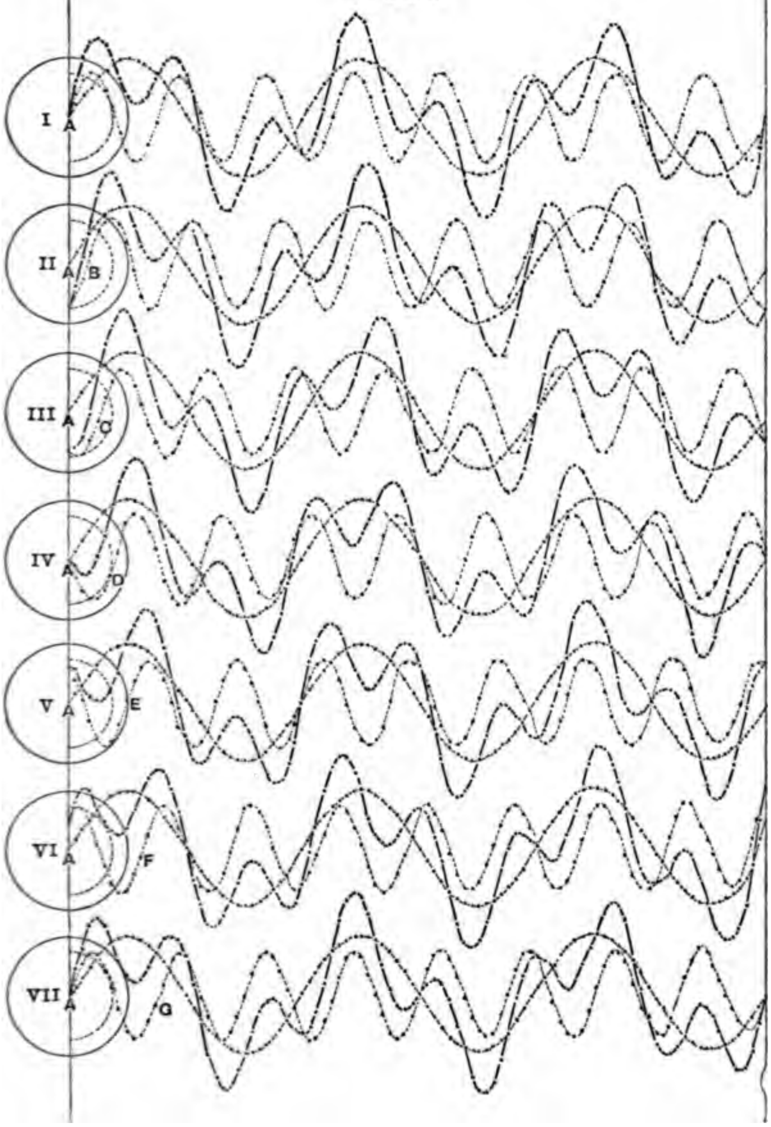
AB, and whose amplitude is equal to the sum of AB and CD. On the other hand, when these two S.H.M.'s are in opposite phases, differing by half a period, so that while one raises the body above the point O, the other depresses it below that point, the resultant curve is also found to be a Harmonic Curve corresponding to a S.H.M. whose amplitude is equal to the difference between the amplitudes of the two components. Hence two S.H.M.'s of the same period and in the same straight line will, when compounded, produce a single S.H.M. of the same period and in the same line, whose amplitude is the sum of the amplitudes of the components if they agree in phase, and their difference if their phases be opposed. Manifestly, if the two component S.H.M's be equal to one another, the resultant will be in amplitude, double of either of them if they agree in phase, and will be zero—that is, the body will be at rest—if they be opposed in phase, the corresponding harmonic curve being in this latter case reduced to a straight line.

If the phases be neither in exact accord nor in exact opposition, the resultant curve is still Harmonic, but the amplitude is found by a construction the same as that of the parallelogram of velocities: lines representing the two component amplitudes are laid down at an angle representing the difference of phase, and the diagonal thus found represents, in length, the amplitude sought.

If the two S.H.M.'s have different Periods, the result is more complex. Let the periods of the S.H.M.'s bear the ratio 3 : 8. Then the Harmonic Curve corresponding to the more rapid S.H.M. will present eight undulations for every three of the less rapid ones. These are drawn in Fig. 45 (I). On adding the displacements represented by these curves, the resultant may be traced from point to point, and is found to form a comparatively complex curve. Obviously there may be an indefinite number of forms of this resulting curve, for the ratio of the amplitudes may vary indefinitely.

In Fig. 45 the curves show the change produced in a compound harmonic curve by a **difference of phase** in the component S.H.M.'s. The curve (1) is that corresponding to the composition of two S.H.M.'s whose periods are as 3 : 8, whose amplitudes are as there shown, and whose phases at the point A coincide. In the next, the more rapid S.H.M. is seen to be, in respect of its phase, in arrear by an interval of time represented by the length of the line AB, and the superposition of the two curves now produces a resultant slightly differing from its predecessor, but, on the whole, similar to it. A similar construction produces the succeeding curves, in which the difference of phase corresponds to

Fig. 45.



the respective intervals AC, AD, etc. It will be plain that if the differences of phase be intermediate between those chosen in the figure, there may be drawn any number of resultant curves intermediate in form between those shown. The time taken by the moving body to go once through its periodic movement, or, in other words, the period of the resultant complex harmonic motion is unaltered by variations in the amplitude and phase of the component S.H.M.'s, and depends only on their relative periods.

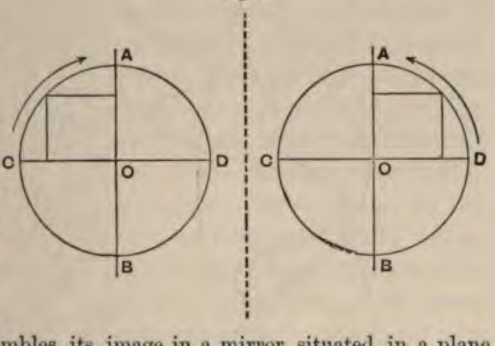
If in Fig. 45 the difference of phase were not constant but continuously increased, the curve would successively assume all the forms there shown, and it would naturally pass through all the possible intermediate forms, returning at intervals to the form (I).

Beats.—If two harmonic curves be compounded, of which one corresponds to a more rapid vibration than the other, the periods being approximately equal, the resultant curve will be one which at any one spot approximates in form to the curve of sines, but alternately waxes and wanes in amplitude. If the respective periods be as 2000 : 2001, the quicker oscillation gains $\frac{1}{2000}$ period on the slower at each complete S.H. movement, and at the end of 1000 of the slower S.H.M.'s the quicker is in complete disaccord with the slower; then, if the two amplitudes of the oscillations be equal, the particle affected is at rest; thereafter the quicker oscillation comes more and more completely into renewed accord with the slower, and at the end of 2000 of the slower oscillations or 2001 of the more rapid, the amplitude of the compound vibration is equal to the sum of those of the components. This is the cause of beats in music. If the periods approximate to any other whole-number ratio than that of equality, similar phenomena occur; at any one instant the curve resembles the corresponding compound harmonic curve, but alternately waxes and wanes; the deficiency of amplitude occurring once for each complete oscillation gained by the more rapidly vibrating body. Thus oscillations whose frequencies are 500 and 751 per second give one beat or period of relatively small amplitude during each 751 of the more rapid vibrations—that is, for every occasion on which it gains one oscillation on that number, 750, which would make the ratio of frequencies exactly the ratio 2 : 3. If the oscillations had been 500 and 750½ per second, there would have been a beat every two seconds.

If the periods of the S.H.M.'s be non-commensurable, the resultant curve approximates in form to that of the nearest commensurable ratio, and successively assumes forms nearly resembling those assumed by that curve when the difference between the phases of its components gradually changes.

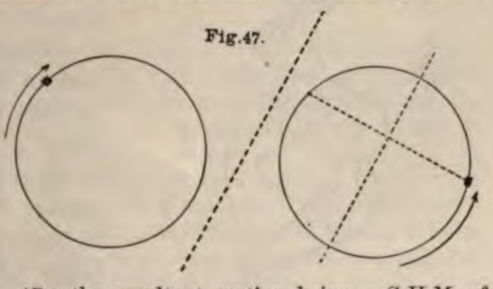
As a particular case of the composition of harmonic motions we may take the following problem, which is of importance in the Theory of Light. A particle is acted upon by two simultaneous circular vibrations. These, considered singly, act in opposite directions, as in Fig. 46. Let them be identical in period and in amplitude. Let them be resolved into components at right angles to one another, which lie respectively in the lines AB and CD. Let the S.H.M.'s in AB coincide in phase, while those in CD differ in phase by half a period. In such a case the one circular vibration resembles the other, as an object resembles its image in a mirror situated in a plane parallel to AB; and the S.H.M.'s in CD will neutralise one another, while those in AB will reinforce one another; so that the result will be a S.H.M. in the plane AB, and of double amplitude.

Fig.46.



If, however, the components in CD do not differ in phase by half a period, the same considerations of symmetry do not apply in reference to the plane of AB; and the resultant motion cannot be a S.H.M. in that plane. If the circular motions be identical in period and amplitude, there must be some plane in respect to which the circular motions are symmetrical, as in Fig. 47; the resultant motion being a S.H.M. of identical period parallel to that plane and of double amplitude.

Fig.47.



If the two circular motions differ in period, they will continuously differ in relative phase, and the resultant will be a S.H.M. of double amplitude effected in a plane which is constantly changing—a plane at right angles to that of the paper, and rotating round O, the point of rest in Figs. 46 and 47.

Hence a S.H.M. can always be considered as compounded of two circular oscillations; and if one of these be retarded or accelerated, whether suddenly or continuously, the plane of the S.H.M. will be rotated, suddenly or continuously.

Composition of several S.H.M.'s in the same Plane.—This may be geometrically effected by the same method as that employed in the construction of the curves of Fig. 45, viz., by drawing separately the Harmonic Curves corresponding to each

S.H.M., and adding from point to point all the respective displacements indicated by each of these curves. Fig. 48 shows the harmonic curves corresponding to five S.H.M.'s, each of which is drawn so as to represent its proper phase, period, and amplitude relatively to the others. The resultant curve is periodic—that is, the complex form is repeated at regular intervals—if the periods of the component S.H.M.'s be commensurable; it cannot be if they are not so.

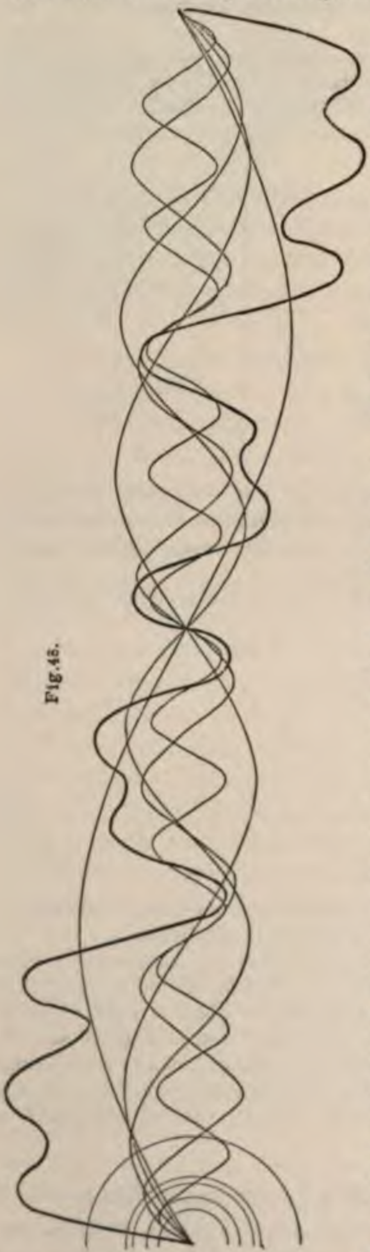


Fig. 48.

In all cases of a body affected by several simultaneous S.H.M.'s, in which the component S.H.M.'s have been in the same line, the **real resultant motion** of the particle may be studied by finding the complex harmonic curve produced by compounding these S.H.M.'s with a uniform movement, and on this curve laying a card in which a slit is cut which is laid at right angles to the axis of the curve. The card is then moved uniformly along this axis, the slit being kept at right angles to it. At any one moment only one point of the resultant curve can be seen in the slit, if that slit be made narrow enough. As the card is moved along, this point appears to move up and down in the slit with greater or less regularity, and the way in which it so moves is the way in which the body really moves when affected with the given simultaneous S.H.M.'s in the same line.

Fourier's Theorem.—The great variety in the forms of the resultant curves drawn to illustrate the previous discussion will prepare the reader to accept the positive statement that by properly choosing a number of harmonic curves, their amplitudes, their periods, and their phases, and by compounding these, any Periodic Curve of any complexity may be built up, provided that the curve required never goes off to an infinite distance from the axis. Conversely, any complex periodic motion must be compounded of and may be resolved into a definite number of S.H.M.'s of definite periods, definite amplitudes, and definite phases. In order, however, that such motion may be periodic—that is, that the complex resultant motion may accurately repeat itself at regular intervals —it is necessary that the periods of the component S.H.M.'s should be exactly commensurate; for if they were not so, the resultant motion could not exactly repeat itself, and would not be periodic. Granted, however, the periodicity of the complex motion and the limitation mentioned above as to the form of its curve, Fourier's theorem states that any such motion is compounded of a definite number of *commensurate* S.H.M.'s; and this is true not only of motion represented by the curve, but also, with wider interpretation, of any phenomenon which the curve may represent.

Tide Calculating Machine.—A number of wheels may have prearranged velocities imparted to them by being separately connected with cranks, of which a number are actuated by the same clockwork, through the intervention of toothed wheels. Thus the mechanism of Fig. 33 may be multiplied and any number of S.H.M.'s simultaneously produced. If the extremities of the oscillating rods be connected by a tense and flexible cord, that cord will be drawn upon or tightened to an extent depending at each instant upon the position of the wheels. One extremity of this cord being fixed to an immovable point, the other may be connected with a spring, and the varying distortions of that spring will indicate the varying tensions of the cord. To some point of the spring a pencil or pen may be attached, and under this writing-point a piece of paper may be unrolled at a rate proportioned to the velocity of the clockwork. When the mechanism is set in motion there is recorded upon the unrolling paper a curve which represents the summation of all the S.H.M.'s which are being executed. The preliminary adjustment consists in adjusting for each wheel the position of the pin which works in the slot—this regulating the amplitude, and also in modifying the angular position of each crank so as to represent the appropriate epoch of each S.H.M. at the moment of starting. A machine of this kind enables a curve to be drawn after preliminary adjustment, which represents the height of the tide for every moment of a considerable period, such as a year, without the aid of further calculation than that involved in deducing from astronomical considerations, and from the tidal record of a place, a knowledge of the component S.H.M.'s, their respective periods, amplitudes, and epochs.

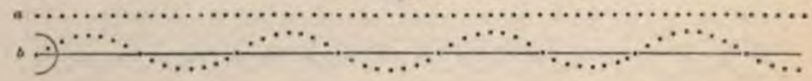
The results shown in this case by drawing the wave in angular outline, are—(1) that there are twice as many waves, the wave-length being half as great as in the previous figure; and (2) that the speed of travelling of the wave-form is the same in both cases; for if a sufficient time be supposed to have elapsed to permit each particle to have performed half a revolution (this corresponding to the time allowed in Fig. 49 for the accomplishment of a quarter revolution), and if the then position of the chain of particles be investigated, the wave-form will be found to have travelled forwards through a space which is the same as in Fig. 49. If, then, the relative retardation of the particles be independent of the speed of the particles, the rate of propagation of the wave-form will be constant, and the only effect of a change in the rate of the oscillation of the particles will be a change in the wave-length and in the corresponding curved form assumed by the chain, and in the number of waves which pass any given point during a given interval of time.

The circular form is not a necessary attribute of the path of each particle: the path may be elliptical with a similar result, the difference being one in the form of the resultant wave.

The two limiting cases are of great interest. These are (1°) the case in which the ellipse is reduced to a straight line at right angles to the chain of particles; and (2°) that in which it is reduced to a straight line in the same direction as that chain. The former gives rise to Transversal vibrations; the latter to Longitudinal vibrations.

Transversal Vibrations.—Each particle effects a S.H.M. in a direction at right angles to the chain of particles. In Fig. 51

Fig. 51.



are shown (*a*) the series of particles unaffected by vibration; (*b*) the same particles affected by transversal vibrations, executing S.H.M.'s, the phases of which differ to an equal extent in equidistant particles. The form assumed by the system of particles in this case is exactly the curve of sines. It is an easy matter to show in this case, as in those previously discussed, that as the particles perform their several S.H.M.'s the wave-form travels along the cord.

Composition of Transversal Vibrations.—It is quite possible for an indefinite cord or series of particles, such as the one considered in these paragraphs, to have several wave-motions running along it simultaneously, each producing its own effect, and the total effect of their united action should be traceable by some process of composition analogous to our previous composition of simultaneous movements. There are two main cases to be considered—(1) that in which the vibrations are in the same plane, and (2) that in which they are not in the same plane.

Transversal Vibrations in the same plane: their Composition.—Since the effect of transversal vibrations on an indefinite straight cord is to cause it to assume the form of the curve of sines, and since each vibration acts independently in this sense, the effect of compounding such movements is reduced to exactly the same problem as has been considered on pages 95-101, and there illustrated. If, for instance, we refer to Fig. 48, the resultant wave on a perfectly flexible and extensible cord on which the five wave-systems there represented were simultaneously travelling, would, for the instant at which the phases happened to be as there shown, assume the form there drawn; but when that wave had travelled a little way along the cord, the relative phases of the component transversal vibrations would have altered, and the wave would thus continuously alter its form from instant to instant, returning, however, nearly to the form shown in Fig. 48, as often as the same coincidence of phases recurred.

Their Resolution.—If a changing wave-form run in this way along a cord, and if the same form recur at regular intervals, the wave-form passing at every recurrence through the same changes, then Fourier's theorem applies, and the most complex phenomenon of this kind may be analysed or resolved into a number of separate waves, whose periods are commensurable, running simultaneously along the cord.

Transversal Vibrations not in the same plane: their Composition.—Here we have to consider two cases—(1) that in which the vibrations are at right angles to one another, and (2) that in which they are not so. The latter case differs from the former only in the form of the curve described by each particle.

When the simultaneous vibrations are in planes at right angles to one another, the motion of each particle is confined to a plane at right angles to the line of the cord. In the plane in which it moves, each particle describes paths such as those exemplified in figures 35-40. In these figures AB may be taken as representing the amplitude and direction of the S.H.M. in one plane, and CD wave progressively increases, it acts upon material whose mass increases with the radius; the energy imparted to each particle varies inversely as the radius; and the amplitude of movement of each particle in its S.H.M. varies inversely as the square root of the radius. In this way, the farther the wave has travelled from its centre of disturbance, the shallower it becomes.

At two instants of time, the respective radii of the circular wave are r and $r_{,}$, and the maximum velocities of the particles affected v and $v_{,}$. Let m be the amount of mass affected per unit of length of the wave-front. The whole mass set in motion is, at the two successive instants, $2\pi r.m$ and $2\pi r_{,}.m$. But the energy is constant, and

$$2\pi rm.\frac{v^2}{2} = 2\pi r_{,}m.\frac{v_{,}^2}{2}.$$

Whence $$\frac{r}{r_{,}} = \frac{v_{,}^2}{v^2} \text{ and } \frac{v_{,}}{v} = \sqrt{\frac{r}{r_{,}}}.$$

But the amplitude of a S.H.M. is proportional to the velocity with which the particle passes its mean position; for the velocity in the circle of reference, which is equal to the velocity at the middle point of the S.H.M., is $\frac{2\pi \times \text{radius of circle}}{\text{Time}} = \frac{2\pi}{\tau} \times$ amplitude, whence the amplitude $= \text{V} \times \frac{\tau}{2\pi}$, and therefore varies as the velocity. The amplitudes therefore vary inversely as the square root of the radius of the wave.

Waves propagated in a tridimensional substance.—The solid figure whose surface is everywhere at equal distances from its centre is a globe or sphere. If a disturbance at a point be propagated with equal velocities in all directions in space, the form of the wave-front will be spherical. If the velocities be unequal, the wave-front will be ellipsoidal or spheroidal. As the distance from the centre increases, the amplitudes of oscillation of the particles will diminish, for they vary inversely as the distance from the centre of disturbance.

When the radius of the spherical wave changes from r to $r_{,}$, the mass set in motion becomes greater in the proportion of r^2 to $r_{,}^2$; but the energy, which is equal to $2\pi r^2 v^2$, or $2\pi r_{,}^2 v_{,}^2$, remains constant. Therefore $r^2 v^2 = r_{,}^2 v_{,}^2$, and $\frac{v_{,}}{v} = \frac{r}{r_{,}}$, or the velocity of the particles varies inversely as the radius of the wave-front. But the amplitude varies as the velocity, and therefore varies inversely as the radius or the distance of the wave-front from the centre of disturbance; and the energy of motion of each particle—that is, the **Intensity** of its vibration—varies as the square of the amplitude, and therefore inversely as the square of the radius. This corresponds to the statement that the intensity of Light varies inversely as the square of the distance from the illuminating point.

Concentric Waves.—In all these cases, if the primitive disturbance be repeated at regular intervals, the wave system will assume the form of equidistant and concentric waves of circular, elliptical, spherical, ellipsoidal, spheroidal form, as the case may be. If the primitive disturbance be repeated at irregular intervals, the waves will still be concentric but not equidistant, and they will arrive at any point in an order of irregular sequence exactly reproducing the irregularity of the central disturbance.

If the central particle be affected by complex periodic disturbances, these will be, as regards the period and the phase as well as the relative, but not the absolute, amplitude of every component motion, faithfully reproduced in the motion of any particle affected by the resultant complex wave-motion; and this motion of such a particle may in many experimental instances be taken cognisance of by an observer.

Direction of the wave-front.—When a wave is said to be at a certain time and place travelling in a certain direction, in an isotropic medium, it is meant that the normal to the wave-front, a straight line drawn at right angles to the wave-front—*i.e.*, at right angles to its tangent or tangent-plane—takes the direction said to be that of the wave itself. In Fig. 54 the lines *qr* and *st* indicate the directions of the circular wave at the points *q* and *s*.

Flat wave-front.—The nearer the centre of disturbance, the more marked the convexity of the wave-front; the farther the centre, the flatter the wave-front: when the centre of disturbance is very far, the wave-front may for any small area be regarded as approximately plane, just as any small portion of the surface of a very large sphere may be.

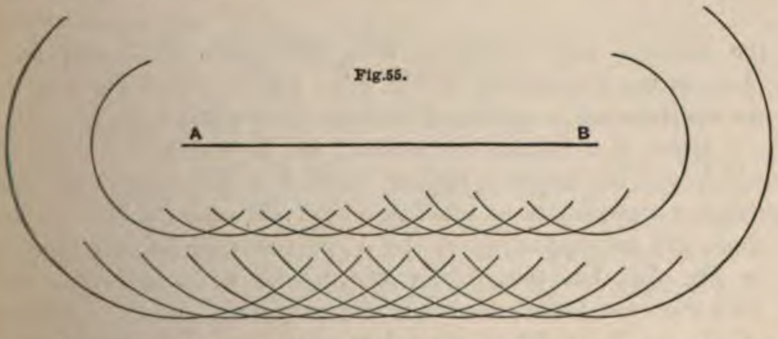


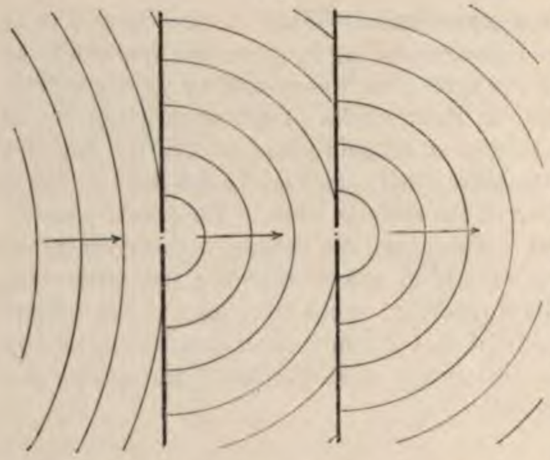
Fig. 55.

If, again, all the points of a plane surface act as centres of disturbance, then in the immediate proximity there may be a flat-fronted wave. If the disturbed surface be represented in section by AB in Fig. 55, the wave-front will be flat opposite its centre.

Modifications in the Form of a flat Wave-Front.—Through the upper and cooler layers of the atmosphere waves travel with less rapidity than they do through the lower. A flat wave-front is thus distorted, its upper part travels with the least velocity, and the wave-front comes to converge upwards. Points on a level with the point of disturbance may remain unaffected, for the wave-front is, in the main, restricted to its normals, and the sound ascends. If, however, there be a movement (such as that due to wind) in which the upper strata move more rapidly than the lower, it is not difficult to see that the wave-front may come to bear down, and that sound-waves may thus appear to travel with the wind, and to be best heard at certain distances, which depend upon the speed of the wind.

Wave passing through an aperture.—If a flat wave impinge upon an obstacle containing an orifice, it will, in part, be propagated through that orifice. If the orifice lead into the lumen of a cylindrical tube whose diameter is the same as that of the orifice, no lateral expansion of the wave-front is possible in that tube. If there be no such tube, there may or there may not be expansion of the wave beyond the orifice. Such expansion, if it take place at all, will take place in the way shown in Fig. 56. The disturbed particles in the aperture act as centres of disturbance to those lying beyond.

Fig. 56.



There is a curious proposition, the nature of the proof of which will be indicated further on, that if the *aperture* through which a wave passes be *small in comparison with the wave-length*, there will be expansion of the wave-front, such as that shown in Fig. 56; but that if the aperture be wide in comparison with the wave-length, the wave will only travel in the direction of all the lines drawn normal to that part of it which passes through the aperture, the wave therefore travelling with a correspondingly limited amount of expansion or none at all; while for conditions intermediate there will be a certain amount of expansion beyond the limitation indicated by the normals.

When a wave-motion passes through an aperture relatively wide, then the cases are three :—

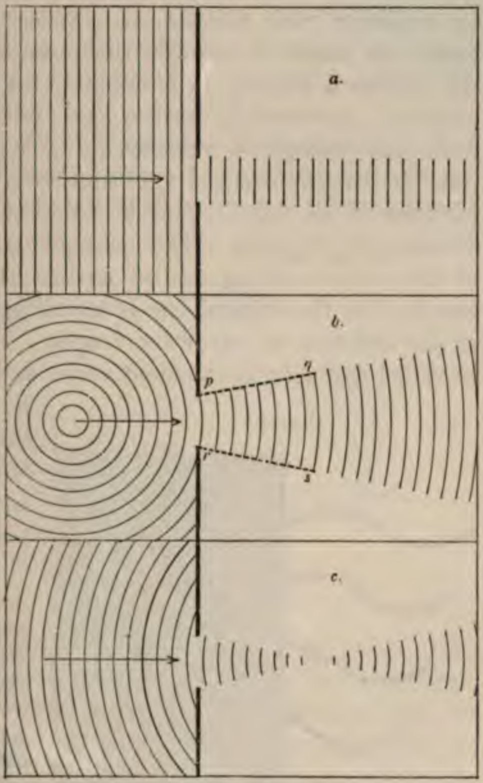
(*a*.) The wave-front may be *flat*, as in Fig. 57*a*, in which case it does not expand: this is the condition of a "parallel beam" of light.

(*b*.) It may be *convex*, as in Fig. 57*b*, in which case the wave-front expands, being limited by the normals *pq*, *rs*. This is the condition of a "divergent beam" of light.

(*c*.) It may in some cases be *concave*, in which case the wave-front first contracts and then expands: this is the condition of a "convergent beam" or "convergent pencil of rays" passing through a "Focus."

When a wave-front passes through a focus, exactly or approximately, the energy, constant in amount, is distributed over a comparatively small field, and the intensity of disturbance is, at the focus, correspondingly great.

Fig.57.

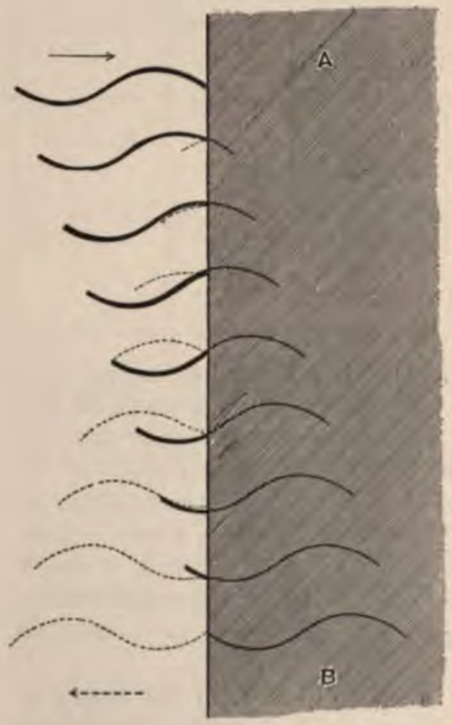


Reflexion of Linear Waves.—If a linear longitudinal wave of compression be incident, or impinge, on an obstacle so firm that the first particle of it at which the wave arrives does not move, there is then produced between this first particle of the obstacle and the nearest particle of the vibrating cord (which we shall represent as particle *i*) a Compression, which results in particle *i* rebounding at a rate equal to that with which it struck the obstacle—that is, with velocity $v_{\prime}$—but in the opposite direction, and in its then meeting the next particle, *ii*, as it comes up with velocity $v_{\prime\prime}$.

We may here borrow a proposition from the theory of Elasticity, which shows that if two equal elastic bodies meet one another and rebound, they will do so with exchanged velocities.

Particle *i*, meeting particle *ii*, exchanges velocities with it; *i* acquires velocity $v_{\prime\prime}$ and returns towards the obstacle; *ii* acquires $v_{\prime}$. Particle *i* strikes the obstacle with velocity $v_{\prime\prime}$ and rebounds; in the meantime particle *ii* has acquired velocity $v_{\prime\prime\prime}$ by exchange with particle *iii*. When particles *i* and *ii* again meet, *i* is impelled towards the obstacle with velocity $v_{\prime\prime\prime}$, and the backward velocity $v_{\prime\prime}$ is imparted to particle *ii*. So on: the particle *i* successively strikes and rebounds from the obstacle with each successive velocity, $v_{\prime}$, $v_{\prime\prime}$, $v_{\prime\prime\prime}$, $v_{\prime\prime\prime\prime}$, etc.; at the same time the backward speed $v_{\prime}$ is transferred successively to all the particles *ii*, *iii*, *iv*, etc., and is followed by the successive velocities $v_{\prime\prime}$, $v_{\prime\prime\prime}$, $v_{\prime\prime\prime\prime}$, etc. The consequence is that, just as the end of the wave is being dashed against the obstacle, a wave-front exactly like the original one is travelling away from the obstacle, at the distance of one wave-length. The "reflected wave" has travelled through the incident wave, and then, becoming clear of it, travels alone, equal to the incident wave in wave-length, in period, in phase, and in amplitude, but opposed in direction.

Fig.58.



In Fig. 58 a single wave is shown, running along a cord against a fixed obstacle AB. Within the obstacle thin lines show the course which the wave would have taken, had the cord not been interrupted. To the left of AB, light dotted lines indicate the course of the reflected wave. The light dotted lines are seen to be of exactly the same form as the thin lines within AB, but turned sharply in the reverse direction at the surface of impact. The reflected wave is then a direct continuation of the incident wave in all but direction.

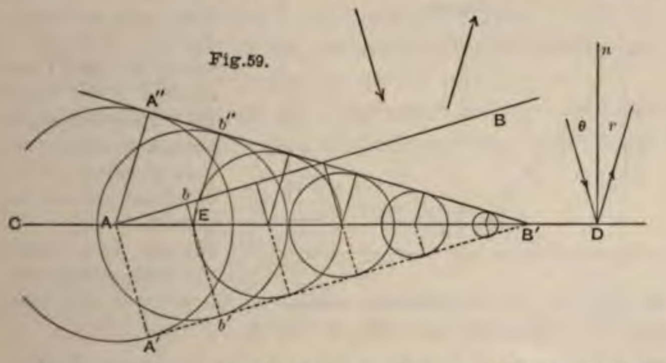
If of incident waves there be a succession, simple, complex, regular, irregular, all these peculiarities will be faithfully reproduced in the reflected wave.

What has been said of a wave longitudinal and commencing with a compression may be easily modified so as to become generally applicable to the explanation of any kind of linear undulatory disturbance. If the obstacle stand fast, it is a matter of indifference whether it be composed of matter whose particles lie more or less closely together.

Special consideration of the reflexion of waves traversing space of two dimensions may be omitted.

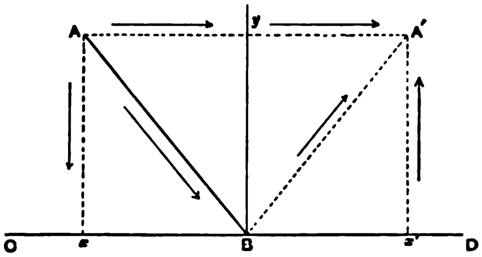
Reflexion of a plane wave-front at a plane surface.—If a plane wave-front meet a plane surface, any section through the wave and surface will present a condition such as that shown in Fig. 59. The line AB represents the wave-front advancing; CD represents the surface on which it impinges. Every point in the wave-front acts as a centre of disturbance. Thus the wave-front advances parallel to its former plane forms. After the lapse of a certain time the whole of the wave-front has impinged on the obstacle: what is then its condition? The part of the wave corresponding to the particle A would have travelled as far as A′, if there had been no obstacle. After reflexion it has travelled to a corresponding extent in some direction tending away from the surface—that is, to some point on the circumference of a circle, the centre of which is at A, and the radius of which is AA′. So the part of the wave indicated by *b* would have reached *b*′; the line *bb*′ crosses the surface at E; that part of the wave is reflected to a distance limited by a circle whose centre is the point E on the surface, and whose radius is the distance, E*b*′, between the surface and the position at which the wave-front would have arrived if there had been no obstacle. By drawing a sufficient number of circles in this way, we see that the aggregate disturbance produces a plane wave-front A″B′, receding from the surface CD. If, as it approached the plane surface, it had been parallel to that surface, it would retrace its path. If it had approached the surface obliquely, so that the direction of the wave makes

Fig.59.



an angle θ with the normal to the surface, the direction of the receding wave will make an equal angle θ with the normal, but on the other side of it. This is expressed by saying that the Angle of Incidence, θ, is equal to the Angle of Reflexion, r; these being understood to be angles made between the direction of the wave and the normal to the surface, or, what amounts to the same thing, between the plane of the wave and the plane of the surface.

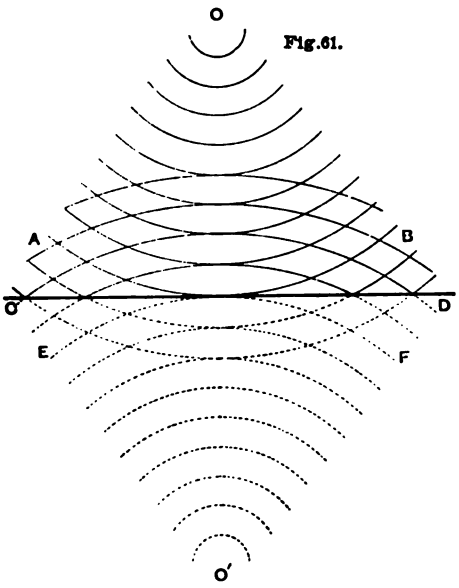
Fig. 60.



The same proposition may be otherwise demonstrated. In Fig. 60 let AB represent the direction of a wave, and CD the reflecting plane surface. Every movement of the vibrating body with reference to the direction AB may be resolved into two, these being referred to the axes By and Bx. On reflexion, the component in yB has its direction reversed; that in the direction xB is not thus interfered with. After reflexion, on recompounding the components, the resultant is found to be a disturbance similar to the original one, but in the direction BA′, while the angle ABy is equal to the angle yBA′.

Reflexion of a curved wave-front at a plane surface.—If a wave-front be curved, it may be considered as consisting of a very large number of very small plane surfaces. To each of these a normal may be drawn; each such normal indicates the direction of the corresponding part of the wave-front; the angle which each such normal makes with the reflecting surface, when its own part of the wave-front strikes the obstacle, is the angle of incidence for that part of the wave-front; to this angle the angle of reflexion for that part of the wave-front must be equal.

Fig. 61.

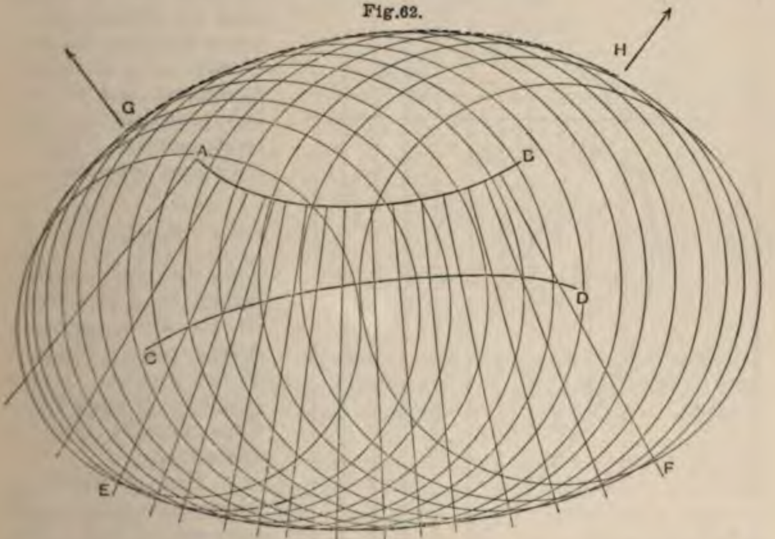


Let the convex spherical wave-front AB strike the surface CD. Each part of the wave-front is reflected at its own angle. The result is the reflexion of a convex wave which is of the same form as AB would have assumed in the time, but which travels in the opposite direction. Such a wave is, in effect, exactly such a wave as would have travelled from the point O′ as far behind the reflecting surface as O is in front of it.

In the same figure, if the directions be reversed, so that a concave wave-front travels towards the reflecting surface, converging upon O′, it will, when reflected, become reversed, and on receding from the reflector, it will converge upon O, which is as far on the one side of CD as the point O′, upon which the wave had originally been converging, is on the other.

General construction of a reflected wave.—Let AB (Fig. 62) be a wave-front, and CD a reflecting surface, both of any form.

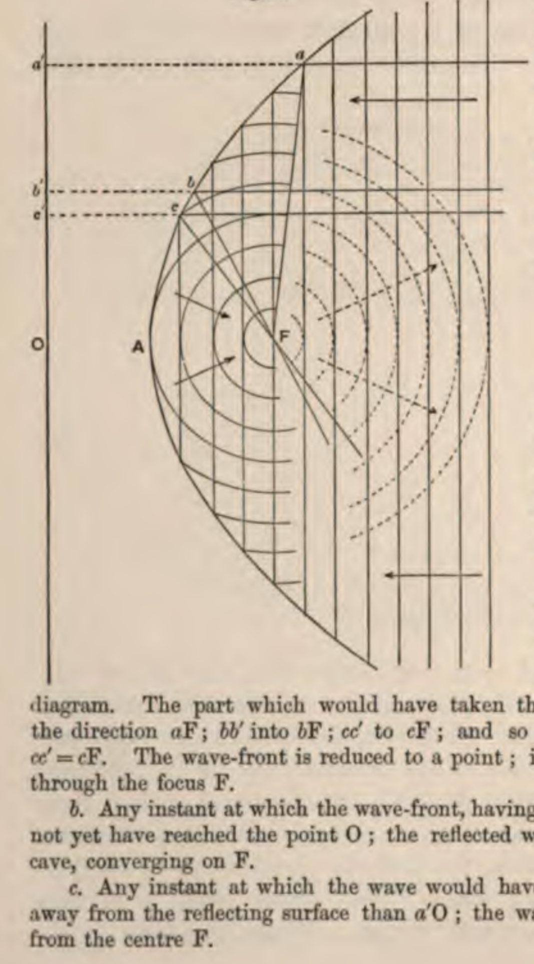
Fig. 62.



Draw normals to AB of such lengths that they may all cut CD; from these normals cut off equal portions, and join the extremities of these portions; the line EF is thus obtained, which represents the form that the wave would have assumed but for the reflecting obstacle. Draw a number of circles; the centre of each of these is a point at which one of the normals to AB cuts CD; the radius is the distance along the normal in question from the surface CD to the surface EF. These circles have a common tangent, the curved line GH, which indicates the form of the reflected wave. Normals drawn to this, of equal length, may indicate the form of the reflected wave at any subsequent instant; if these be drawn backwards, all the previous positions, real or apparent, may be investigated.

This is the general construction; but it very frequently leads to difficulties where different parts of the wave cross one another. In most cases, however, the following method is effective. Consider a part of an incident wave and the point at which it impinges; from the point of incidence draw a line indicating the direction in which that part of the incident wave will be reflected. Find to what distance behind the reflecting surface the incident wave would have travelled in a given time if there had been no obstacle; measure off along the direction of the reflected wave a distance equal to this; repeat this operation for several parts of the wave-front; join all the points thus obtained. This gives the form of the reflected wave at the time chosen. Equal distances, measured forwards or backwards upon the normals to this wave, will give the form of the wave at instants subsequent, and its true or hypothetical form at instants previous. The following are examples :—

Fig.63.



Problems.

1. Let the reflecting surface be a paraboloid: let the advancing wave-front be plane. The focus of the parabola is at F (Fig. 63). We may choose three instants for consideration.

a. That at which the whole wave-front would have arrived at O. Each part of the wave-front is reflected as shown in the diagram. The part which would have taken the course aa' is turned into the direction aF; bb' into bF; cc' to cF; and so on. $aF = aa'$; $bb' = bF$; $cc' = cF$. The wave-front is reduced to a point; it is at that instant passing through the focus F.

b. Any instant at which the wave-front, having passed the point A, would not yet have reached the point O; the reflected wave is spherical and concave, converging on F.

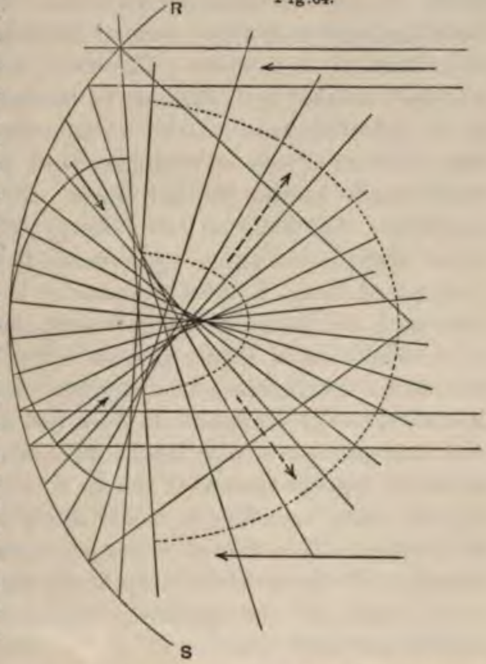
c. Any instant at which the wave would have reached a plane further away from the reflecting surface than a'O; the wave is spherical, divergent from the centre F.

2. In the same figure, the wave-front is one which starts from F as a centre; it meets the paraboloid reflector; it is reflected with a plane wave-front.

3. The reflecting surface is spherical and concave, the incident wave-front flat.

In Fig. 64 the reflecting surface is represented by the line RS. The wave travels from right to left and meets RS. Each part of the front of the wave is turned back at its own angle of reflexion: the wave-front becomes convergent. It does not, however, converge on any one point; it is not spherical. The figure shows that there is a curved line, a "**caustic by reflexion,**" in which lie all the foci of all the separately considered parts or elements of the wave-front. In each wave the element reflected from the outer part of the surface RS will sooner come to focus than that reflected from the centre of that surface; hence if the wave be single a spot of maximum disturbance will appear to run along each limb of the caustic, and to disappear in a diverging wave at its apex. A succession of waves will keep the whole of the caustic in a state of maximum disturbance.*

Fig. 64.



4. A reflecting mirror a segment of a sphere: a centre of disturbance midway between the centre of the sphere and the reflecting surface. With ruler and compass draw the form of the reflected wave as in Fig. 64. Approximately plane at its centre.

5. A complete spherical surface used as a reflector; a centre of disturbance at the centre of the sphere: a divergent spherical wave produced. Prove that after reflexion this becomes a convergent spherical wave, converging on and passing through the same centre, and then repeatedly reflected and alternately converging on and diverging from the original point of disturbance.

6. Prove that a spherical wave starting from one focus of an ellipse converges after reflexion on the other focus. Hence prove that if the centre of disturbance be at one focus, and if it be surrounded by a complete ellipsoidal reflecting surface, a wave passes back and fore between the foci, alternately converging and diverging, first at one, then at the other focus.

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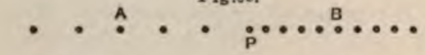
* Take a long strip of bright tinplate; bend it into a semicircle; place it on its side on a sheet of paper in the sunlight, exposing the concavity to the sun: a brilliantly-illuminated Caustic Curve will be seen on the paper. The form of this curve may be varied by altering that given to the tinplate.

Transmission of a linear wave into a denser medium.—If, as in Fig. 65, the particles be more closely placed in B than in A, B is the denser medium. A linear wave-motion travels to the right in A; it arrives at P. It meets a relative obstruction. P, the first particle of the dense substance, is more resisted than the preceding particles set in motion by the wave. The wave is not entirely obstructed, and goes on into B; but there is, to some extent, the production of a reflected wave in A. This reflected wave, like that of Fig. 58, is of the same phase and period, and of the same wave-length as the original wave. It cannot be of the same amplitude, for some of the energy of the wave-motion has been spent in setting up a wave in B. The wave-motion propagated along B must necessarily be of the same period as that in A, for the particles in B must move in unison with those of A, which impel them; it must be of the same phase, for it is the direct continuation of the wave in A. Since the particles are more crowded together in B (a less distance corresponding to the same number of particles), a wave cannot propagate itself in B so far in a given time as it can in A, for its doing so, still retaining the same wave-length, would imply its setting a greater mass in motion. This would, however, require a greater amount of energy. If the latter be definite in amount, as it must be, the wave-length and the speed of propagation must be less in the denser medium.*

Fig.65.



As to the relative amplitudes of the respective vibrations, the original, the reflected, and the transmitted, the amplitudes of the two latter taken together are not necessarily equal to that of the first; but in every case the energy of vibration of the original wave is equal to the sum of the energies of the reflected and the transmitted waves.

If in Fig. 65 a wave beginning with compression be supposed to run through B towards the left, when it comes to the particle P—which may be considered as the last of B or the first of A—that particle, meeting less resistance than its predecessors in B had encountered, plunges into the rarer medium and sets up in A a wave depending on the original wave in B for its period and phase, but of greater amplitude; and the wave in A will also have a greater wave-length than that in B, for a reason the converse of that stated in the last paragraph. The effect on the denser body is, however, singular. The particle P, plunging away from the rest of the particles of B, produces in that part of B a dilatation which is propagated backwards, and there then travels in B a reflected wave, agreeing with the incident wave in period and in wave-length, necessarily not in amplitude, and opposed in phase. A maximum compression arriving at P causes that particle to yield to the greatest extent, and to produce a maximum dilatation in B; hence, when the incident wave produces a maximum compression among all the particles of A in the neighbourhood of P, P itself starts a wave in B, commencing with a maximum dilatation, and the incident and reflected waves are not continuations of one another as in Fig. 58, but there is *loss of half a wave-length.**

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* To avoid misconception it may be remarked here that in concrete cases, while the density of a body is a powerful factor in determining the velocity of vibration in a given body, this also depends greatly on the peculiar molecular properties—the elasticity—special to each substance.

A comparison of the diagrams in the preceding discussion shows that, in every case where the medium in which the wave has travelled is the same, the space traversed by every part of the wave, reflected or not reflected, or sooner or later so affected, must necessarily be the same in a given time; and hence, counting from any initial condition to any final wave-front form, the space traversed before reflexion + that traversed after it = a constant quantity for a given time, and that for every element of the wave-front; or, as it is often expressed, the "incident ray" + the "reflected ray" = constant for the whole wave.

Refraction of a plane wave at a plane surface.—If the incident wave strike the plane surface simultaneously at all parts of its own front, it will simply pass more slowly through the denser medium, while a reflected wave is sent back; but if it strike it obliquely, there are some changes in the wave, which result from one part of it being hampered in the retarding substance while the rest is still moving with comparative rapidity in the rarer medium.

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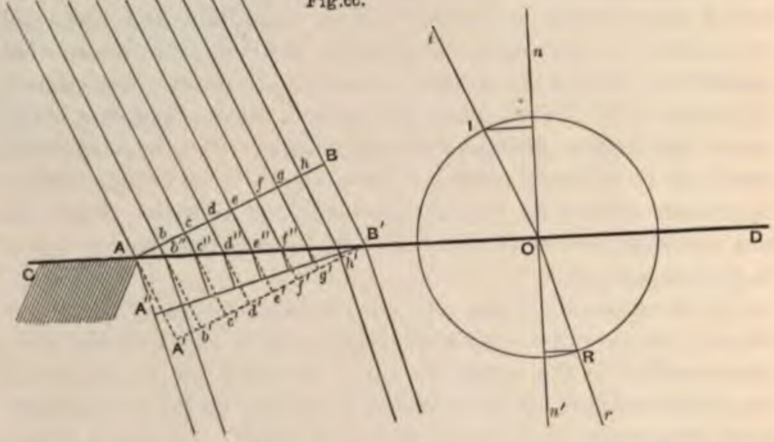
* This curious result has an interesting bearing on the Conservation of Energy. Both the amplitude and the length of the wave in the rarer medium are greater than in the denser. The body B, if the metaphor may be allowed, finds itself to have done more work on the body A, and therefore to have transmitted more energy to it than it had intended: the particle P has compromised the body B by giving a greater dash forward than was expected. Under the circumstances, matters are adjusted by the propagation of a wave of opposite phase in the body B. If each of these waves, the reflected and the transmitted, be regarded separately as containing so much energy, the sum of their energies may appear to exceed that of the original wave. The whole vibrating matter must, however, be regarded as forming one system. In this system, a compression in one wave, and a dilatation in another, produce a relative motion amounting only to their difference; and this is the true motion of the system, the energy corresponding to which is equal to the energy of the original vibration.

In Fig. 66 let AB be the wave-front in the rarer medium; CD the surface separating the denser from the rarer medium; A′B′ a position at which the wave-front would have arrived if it

Fig. 66.



had not encountered the denser substance. The lines *bb*′, *cc*′, *dd*′, etc., are normals to the incident wave-front, meeting the line CD in *b*″, *c*″, *d*″, etc. The angle BAB′ between the wave-front and the surface, or *i*O*n* between the direction of the incident wave and the normal to the surface, is called the *angle of incidence.*

Let us suppose that the velocity of propagation in the denser medium is $\frac{2}{3}$ of that in the rarer. Then with centres A, b'', c'', d'', etc., and radii $= \frac{2}{3}\,\mathrm{AA'}$, $\frac{2}{3}\,b''b'$, $\frac{2}{3}\,c''c'$, etc., draw circles. The line A″B′, which is their common tangent, indicates the position of the wave-front at the end of the time during which it would have advanced to A′B′. The wave has been rendered somewhat broader, and has changed its direction. The angle AB′A″ or *n*′O*r* is called the **"angle of refraction."**

In the figure, $\mathrm{AA''} : \mathrm{AA'} :: 2 : 3$; but in the two triangles AB′A′, AB′A″, we see that $\mathrm{AA''} : \mathrm{AA'} :: \sin \angle \mathrm{AB'A''} : \sin \angle \mathrm{AB'A'}$.

$$\therefore \frac{\sin \angle \mathrm{AB'A''}}{\sin \angle \mathrm{AB'A'}} = \frac{\text{sin ang. refr.}}{\text{sin ang. incid.}} = \frac{2}{3} = \frac{\text{velocity in denser medium}}{\text{velocity in rarer medium}}.$$

Generally, if $\dfrac{\text{vel. in denser med.}}{\text{vel. in rarer med.}} = \dfrac{1}{\mu}$, a fraction,

sin ang. incidence $= \mu \times$ sin ang. refraction, and μ is called the **"index of refraction"** of the denser substance as compared with the rarer one.

This formula shows that in Fig. 66 if *i*O indicate the direction of the incident, O*r* that of the refracted wave; *n*O*n*′ the normal to the plane refracting surface; if a circle be drawn with centre O and any radius,—the lines *i*O and O*r* will cut it in I and R; from I and R draw lines at right angles to *nn*′, as in the figure. These lines always bear to one another, whatever the angle of incidence, the same ratio as the Velocities in the respective media, and this law defines the relation of the angle of refraction to the angle of incidence. An equivalent construction is given in Fig. 182.

Refraction of a wave at a surface: General construction.—Let AB be an advancing wave-front, CD the bounding surface of a denser medium. Let the assumption be made that each several element of the wave-front, as long as it is in the same medium, travels mainly in the direction of the normal drawn to it. In this way the whole wave-front is always simply related to all its previous forms, all the parts of it having at any instant travelled along their respective normals to an equal extent during any given interval of time; and a line once normal to the wave-front is, if produced, always normal to it as long as it travels in the same isotropic medium.

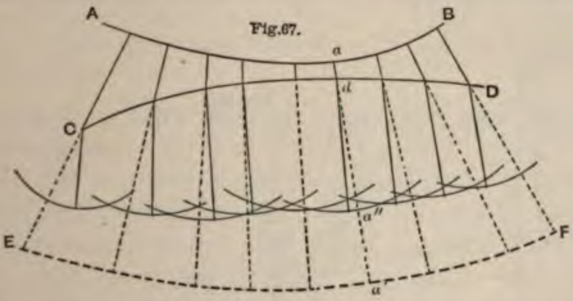


Fig.67.

Then a number of these normals to the incident wave, such as *aa*′ in the figure, are drawn equal to each other, and of length sufficiently great to enable the surface to cut them all. The extremities of these equal normals are joined; in this way a curve EF is produced which indicates the form that the incident wave would have assumed had it travelled thus far in the original medium. Lines normal to AB are also normal to EF. We see segments of these normals, such as *da*′, cut off between CD and EF. The fraction $\frac{1}{\mu}=\frac{\text{velocity in denser}}{\text{velocity in rarer}}$ must now be known. From

da' cut off da'', which is to da' as $\frac{1}{\mu} : 1$; with d as a centre, and da'' as radius, draw an arc of a circle through a''. Treat similarly all da''s fellow-normals. A number of arcs are thus obtained, to which the common tangential curve must be drawn. This gives the form of the refracted wave-front.

This having been obtained, normals may now be drawn to it; these will not in general coincide with aa' and its fellows. By measuring off equal distances along these normals to the refracted wave-front, all the future forms of the refracted wave and all its apparent past forms may be ascertained.

Refraction of a spherical wave at a plane surface.—Let a spherical wave whose centre is at F strike the plane surface RS

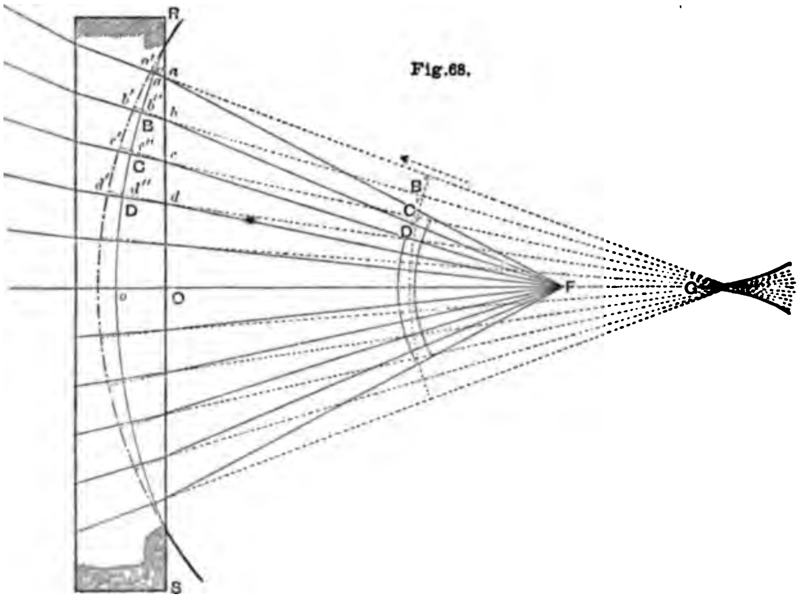


Fig. 68.

and enter a denser medium. The wave would at a certain instant have arrived, say at $a'b'c'd'$. According to the preceding construction, lines aa'', bb'', etc., are cut from aa', bb', etc., to which they respectively bear the constant ratio $1 : \mu$. Arcs are drawn with centres a, b, c, d, etc., and radii aa'', bb'', cc'', dd'', etc.: the common tangent BCD—a curved line—is found: this gives the form of the refracted wave in the denser medium; it is hyperbolic.

Normals may now be drawn to this, by means of which the future and the apparent past forms of the wave may be traced out. In the denser substance, as it travels onward it retains the hyperboloid form; and if the normals *b*B, *c*C, etc., be traced backwards, and such equal lines as BB′, CC′, etc., measured off along them, all those hypothetical wave-fronts may be drawn from which the wave-front as it travels through the denser medium presents the appearance of having been developed. On tracing back far enough we find that the wave appears to be developed from a wave-front convergent not upon the centre F, but through a caustic the apex of which is at G, where OG is to OF as $\mu : 1$.

Problem.

A spherical convergent-wave meets the plane surface of a refracting substance. By construction show that the wave converges through a caustic, the distance of whose apex from the surface is less than that of the original centre in the ratio of $1 : \mu$.

Passage of a wave through a parallel-sided sheet of a denser substance.—At its entrance into the denser substance, the wave-front becomes hyperbolic; at exit every part of the wave resumes the direction which had pertained at the instant of the first refraction to that part of the wave-front from which it had been developed, and thus the wave-front again approximately, but not exactly,[*] resumes its original spherical form. In Fig. 68 the part of the wave-front which passes through *a* is, when the wave approximately resumes its spherical form, again refracted, so that that element assumes a direction parallel to its original direction F*a*.

Approximate Foci.—In Fig. 68, if of the hyperbolic wave in the denser medium only a limited portion in the centre be considered, it will be found to approximate very closely to a small arc of a circle whose radius is G*o*. We have already learned to express this by saying that its radius of curvature is G*o*. If accordingly the incident wave-front be narrow, the refracted wave-front appears to have diverged from a group of points in the immediate neighbourhood of G, the apex of the caustic; and the narrower the incident wave, or the less its divergence, the more nearly will it appear to come from a single point, the very apex of the caustic itself. Similarly in Fig. 64, the narrower the incident wave is in comparison with the breadth of the curved reflecting-surface, the more nearly will the reflected wave converge on the very apex of the caustic, the centre of the sphere. Consequently these points, the apices of the caustics, are approximate foci for comparatively narrow-fronted waves.

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[*] An object seen through a pane of glass is never as distinct as the same seen through the intervening air alone.

Utility of the idea of "Rays" in geometrical construction.—All the preceding discussions have been grounded on consideration of the various forms assumed by the wave-front: we have shown that any form may be developed from any of its predecessors by taking each point in that predecessor as a centre of disturbance, and drawing equal circles of appropriate radii which indicate the extent to which the disturbance has travelled; then of these circles the common tangential line denotes the developed form of the wave-front: we found that in simple cases every line normal to any wave was normal to all those developed from it, and to all those forms through which it had passed; in all this it being supposed that the medium was one in which the velocity of transmission was the same in all directions.

We also made an assumption that when the form of the original wave was complex and the medium *isotropic*, the same law applied; that the maxim once a normal always a normal was true; that there was no lateral expansion of the wave-front beyond the limits indicated in a diagram by lines set down to represent such normals. The assumption is approximately true only in special cases—in general there is lateral expansion of the wave-front beyond such limits; but on reference to what was said in connection with Figs. 56 and 57, we are reminded that it is possible for us to conceive a point of disturbance on a wave-front as one in a *wide aperture*; and hence if the wave-length be very small, the wave-front propagated from each little element of the wave-surface travels along the normal to that element. In any case this is never an absolute statement, and there is always more or less lateral divergence; but as a first approximation of sufficient value for most purposes, it may be said that in an isotropic medium (where the velocity of propagation is equal in all directions) the wave-front is developed from all its predecessors along their common normals; that this is nearly the case when the wave-length is comparatively short: but the greater the proportionate length of the wave, the more lateral expansion is there, and the less able would we be to find the form of the wave-front at any moment by exclusively considering the normals to its previous forms. If, however, the wave-length be comparatively short, we may, by considering the normals only, erect in a diagram the scaffolding on which the form of the wave-front may be constructed. In reflexion, for instance, as in Fig. 64, normals may be drawn to the front of the incident wave; the reflected wave is of such a form that each normal to it makes, with the corresponding normal to the reflecting surface, an angle equal to that made by a normal to the incident wave. But the wave-front itself might have been omitted from the diagram, and the same results as regarding focus and caustic would have been obtained. Then attention might be fixed on the normal lines, and on the way in which these change their direction on reflexion or refraction. They might be treated as if they were physical entities, and might receive special names. This has actually happened. The imaginary straight line drawn at right angles to the wave-front at any point has been called a "*ray*" passing through that point. Each ray is straight, for the normals preserve always the same direction; and of all wave-motion—such as light—which does not expand perceptibly beyond the limits laid down for it by its normals, as in Fig. 57, the progress is described by saying that its rays travel in straight lines as long as it is in the same medium. This mode of expression has both advantages and disadvantages. It leads to the assumption that a divergent wave-front is a divergent "pencil" of rays, each of which is somehow distinct from its fellows; it leads to these rays being conceived as themselves reflected, refracted, etc.; it isolates the physics of those phenomena—those of Light—in which waves approximately follow their normals only from those in which this approximation is much less complete, as in the case of Sound. On the other hand, it presents certain advantages; it simplifies diagrams; it enables any problem to be reduced to its simplest elements by an absolute rejection of all lateral disturbances, and of the effects produced by any parts of the wave other than those at the points of the fronts crossed by the normals; and it gives results which in the theory of Light are, up to a certain point, of sufficient accuracy. This advantage persists, however, up to a certain point only, and, as a whole, a habit of referring the phenomena of wave-motion to the form of the wave-front is to be preferred, though hereafter we shall make free use of the device of reference to rays whenever it is found convenient to do so.

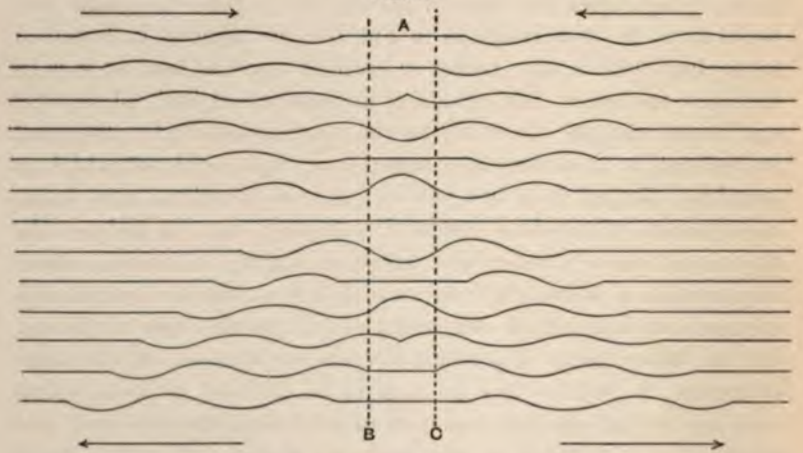
The law can easily be verified that the path traversed by every element of the wave in the rarer medium, together with $\mu \times$ that traversed in the denser, is a constant quantity.

Ptolemy's Law.—If a ray pass from A to B, striking some point of a plane reflecting surface in its course, and being thence reflected to B, there is no path from A to B *viâ* any point of the mirror, so short as that actually traversed by the ray, under the law of reflexion.

Fermat's Law.—If a ray pass from A in one medium to B in another, there is between these points no path which, the relative velocities in the two media being taken into account, could be traversed in so short a time as that actually traversed under the law μ sin ang. refr. = sin ang. incid. If the construction be attempted, it will be seen that the actual law allows the ray the greatest possible proportion of time in the rarer medium.

Superposition of simultaneous wave-motions on an indefinite cord.—We shall here simply discuss the single case in which two equal waves travel in opposite directions on the same cord. In Fig. 69 are seen two waves approaching one another on a cord of indefinite length; they meet at A, and pass through one another. Where crest meets crest and trough trough, as at A, the amplitude is doubled; where crest meets trough, as at B and C, there is no movement as long as the waves are passing through one another: B and C are, during this mutual interference of the waves, **non-vibrating** or **"nodal" points,** between which the cord vibrates.

Fig. 69.

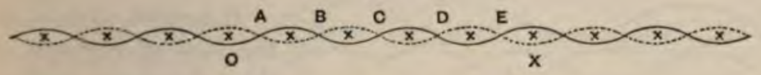


If the waves had been equal, but opposite in phase, so that crest met trough at A, then A, the point of meeting, would have been a nodal point.

The two nodal points, B and C, are seen to be at a distance of *half a wave-length* from each other.

If the waves meeting each other had been indefinitely numerous, the interference would occur in every region of the indefinite cord, and there would be an indefinite number of nodal points, half a wave-length distant from each other. In Fig. 70 such waves are seen running on an indefinite cord, and the nodal points, where crest meets trough, are marked with crosses.

Fig. 70.



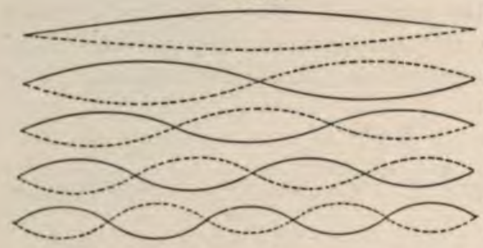
Cord of definite length.—In Fig. 70 let us limit our attention to a part of the string comprised between two nodal points, say between O and X. Within this limited part of the string we observe two waves running in opposite directions, the crest of one meeting the trough of the other at four points: there are five loops, each of which is equal to half a wave-length; the centres of the loops, the points of greatest vibration, are the points A, B, C, D, E; the wave-length is here equal to $\frac{2}{5}$ of OX.

Nodes and Loops.—If the points O and X in Fig. 70 had been the ends of the string, and if it had been possible in OX to establish two waves each of wave-length $= \frac{2}{5}$ OX, these meeting one another so that trough always coincided with crest, the necessary result would be a continuous vibration of the cord in five segments, marked off by four non-vibrating points, and these segments would always be in opposite phases of vibration. The vibration would in such a case be said to be **Stationary Vibration.**

This is precisely the case where a wave running from O to X meets its own reflexion returning from X to O. If the wave-length be $\frac{2}{1}$, $\frac{2}{2}$, $\frac{2}{3}$, $\frac{2}{4}$, $\frac{2}{5}$, etc., of the length of the cord OX, the result of the composition of the wave and its reflexion will be a stationary vibration, in which the string will vibrate in the first case as a whole, or in the others in two, three, four, five, etc., vibrating segments or Loops, separated by non-vibrating points or Nodes.

Vibrations of a cord whose extremities are fixed.—A string acted on by any force tending to bring the particles back to their mean position, and varying as the displacement—a criterion, as we have seen, of harmonic motion—will thus enter into vibrations of a type obeying Fourier's law, and in the general sense any periodic disturbance of such a cord will be compounded of vibrations such as those shown in Fig. 71. These simultaneous vibrations will, as regards amplitude, be independent of one another, and will also from moment to moment necessarily differ in their relative phase. The whole motion is, however, periodic.

Fig. 71



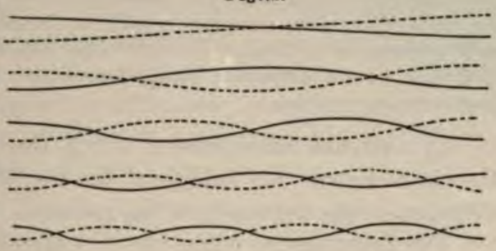
If a point situated in the loop of any one of these harmonic components be held fixed, the corresponding oscillation is prevented. If the centre of a vibrating string be touched, the oscillations corresponding to the whole string, to one-third, to one-fifth, etc. —all the odd components—are suppressed, and only the even components—those, namely, which already have a node at the point fixed—are allowed to go on. If the string be touched at $\frac{1}{3}$ of its length from the end, all vibrations except those corresponding to $\frac{\text{string}}{3}$, $\frac{\text{string}}{6}$, $\frac{\text{string}}{9}$, etc. cease; these still continue, for the effective fixing of their nodes does not affect them. Similarly, if the string be held steady at a point $\frac{1}{4}$ of the string-length from the end, the 4th, 8th, 12th, 16th, etc., components remain unaffected, while all the rest are stopped.

Longitudinal vibrations of a string or rod—for a rod acts in this case like a bundle of parallel strings—whose ends are held fixed obey the same principles as transverse vibrations. Fourier's law holds good; and if any point be held steady, those component vibrations which have a node of displacement at the point held steady, and those components only, will remain unaffected. In longitudinal vibration, where there is the greatest displacement there is least actual change of density; and at the extremities of a rod fixed at both ends, and at the nodes, while there is no displacement, there is a maximum change of density.

If longitudinal vibrations occur in a string or rod, or in a cylindrical mass of gas—such as the air in an open organ-pipe—which is free at both extremities, it is plain that at the free ends there can be no change of density, but that there is freedom of movement; hence each extremity must, as regards displacement, be the centre of a loop. The component vibrations which make up the Fourier-motion in such a case are such as those shown in

Fig. 72. In this case, as well as in the preceding, all the components, even and odd, are possible, and the wave-length of the slowest or fundamental vibration is equal to twice the length of the rod or string vibrating longitudinally.

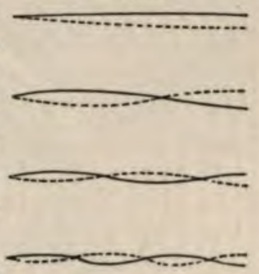
Fig. 72.



In these cases we see that the wave-length of the fundamental as well as of the concomitant vibrations is determined by the length of the vibrating string or rod itself. We have seen that $v = \frac{\lambda}{\tau}$; λ, the wave-length, may easily be found from l the length of the rod, for $\lambda = 2l$; τ, the period, may be measured by acoustical or graphic methods; these being experimentally known, we may find the quotient $\lambda/\tau = v$, the velocity of propagation of an undulatory disturbance in a vibrating string or rod.

If the central particle of the system of Fig. 72, vibrating longitudinally, be held fixed, those vibrations (2, 4, 6, etc.) are suppressed which have not their nodes at the centre of the rod. Thus only the odd components are left; but the rate of these is unaffected. If now one half of the rod were removed altogether, we would have remaining a rod fixed at the one end, free at the other. This rod would have component vibrations, as shown in Fig. 73. A rod thus vibrating longitudinally will have a fundamental vibration whose wave-length will be four times the length of the rod; the concomitant components will have wave-lengths equal to $\frac{4}{3}$, $\frac{4}{5}$, $\frac{4}{7}$, etc., of that length.

Fig. 73.



If the same rod be supposed to be set in longitudinal vibration, first with both ends free, and next with one end fixed, the fundamental wave-length will in the latter case be doubled, and the period of vibration will also be doubled.

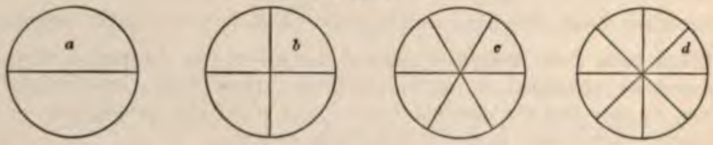
Nodes and Loops in a vibrating membrane.—A membrane may vibrate in such a way that certain lines may be at rest. The number of these lines, if they extend from the centre to the circumference, must be even, for on each side of a node the directions of movement are opposite, and there cannot be an uneven number of nodes.

The forms of these lines vary according to the shape of the membrane and the mode of disturbance. In a square membrane, for example, the nodal lines may be one diagonal—two diagonals—lines joining the centres of opposite sides—lines more numerous parallel to these—curved lines symmetrical with reference to the centre—complex lines obtained by the superposition of these. In a circular membrane we may have concentric circles, or radial lines even in number.

In a circular membrane of which the centre and one point of the circumference are held fixed, the frequency of the fundamental vibration varies inversely as the radius.

The frequency of vibration of a circular membrane vibrating as in Fig. 74 (*a*) being taken as 1, that of the same membrane

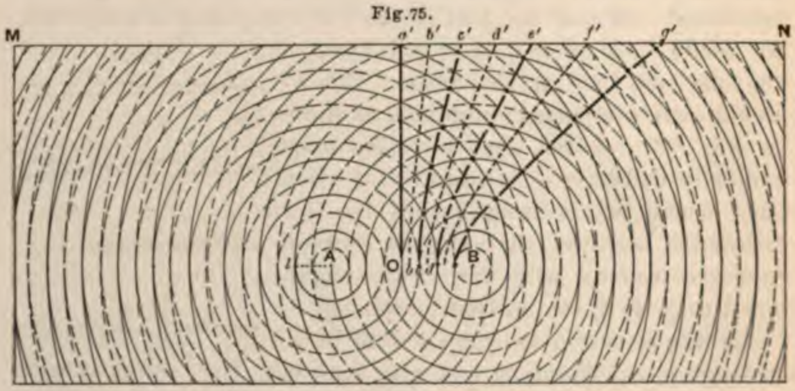
Fig. 74.



vibrating as in (*b*) is $\frac{4}{3}$ nearly;* as in (*c*) $\frac{5}{3}$ nearly; as in (*d*) 2 nearly.

Waves from two different centres—Interference.—In Fig. 75 let A and B be the centres of disturbance; A*l* the wave-length: the dotted circles indicate troughs, the plain circles crests of waves. Where crest coincides with crest, the elevation or compression produced will be the sum of those produced by each wave; when trough meets trough, the converse will hold; but where the trough of one wave coincides with the crest of another, if that crest be equal, the resultant motion at that point is null.

Fig. 75.



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* Lord Rayleigh, *Theory of Sound*, i. 275.

This is the result of the mutual interference of waves. Join the points at which there is maximum movement, whether of crest or trough; join also those at which crest and trough coincide: we thus obtain a series of hyperbolas indicated in the figure. Along O*a*′ there is motion due to the concurrent effects of the disturbances at A and B; along *bb*′, or a line very closely approximating to it, there is rest; along *cc*′ there is concurrence; along *dd*′ approximate rest, and so on.

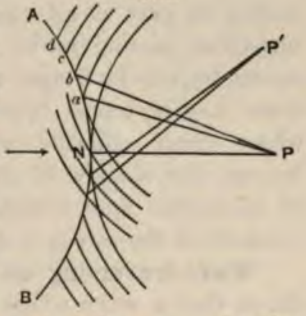
The hyperbolic lines *bb*′ and *dd*′ would be lines of perfect rest if it were not that the one wave is half a wave-length, one-and-a-half wave-length, etc., behind the other, and hence the amplitudes are not equal. The divergence of the true lines of rest from the true hyberbola occasioned by this could not be indicated in the diagram, is less the greater the distance from the centre, and, if the wave-length be very small, will approximately vanish.

If these two points were the only centres of disturbance, and if a screen MN were placed in the field of the wave, there would be movement at *a*′, *c*′, *e*′, rest in the neighbourhood of *b*′, *d*′, *f*′.

Propagation of waves along normals.—The principle has been already stated (see Fig. 54) that the propagation of any wave-front is due to the sum of the effects produced by all the points of it acting as centres of disturbance.

Let AB in Fig. 76 be a wave-front whose normal at the point N is NP. Trace the effect of the wave-front when the wave-length is comparatively small. The point P is situated on the normal; P′ is situated laterally. From P as a centre draw circular arcs, whose radii, PN, P*a*, P*b*, P*c*, P*d*, etc., differ from one another successively by half a wave-length. The disturbance caused at P by the movement of N*a* is to some extent counteracted by that derived from *ab*, aided by that from *bc*, counteracted by that from *cd*, and so on. But N*a* is greater than *ab*, *ab* than *bc*, *bc* than *cd*, and so on. On the other side of N the circumstances are similar; and thus on the whole P is disturbed by the wave-front on both sides of N.

Fig.76.



If on the other hand the point P′ be considered, it will be seen that if the wave-front be wide enough in comparison with the wave-length, the disturbances radiating from it interfere with one another; for points on the wave-front can always be chosen and set off in pairs, differing in distance from P′ by half a wave-length; and consequently there is no disturbance produced at any such lateral point as P′ *by the wave-motion at N*, and the wave-front travels along the normals without any lateral expansion.

The narrower the wave-front or the greater the wave-length, the greater will be the difficulty in this construction, and the greater will be the lateral divergence of the wave. The wave-front must be at least one wave-length in breadth before this construction begins to become possible.

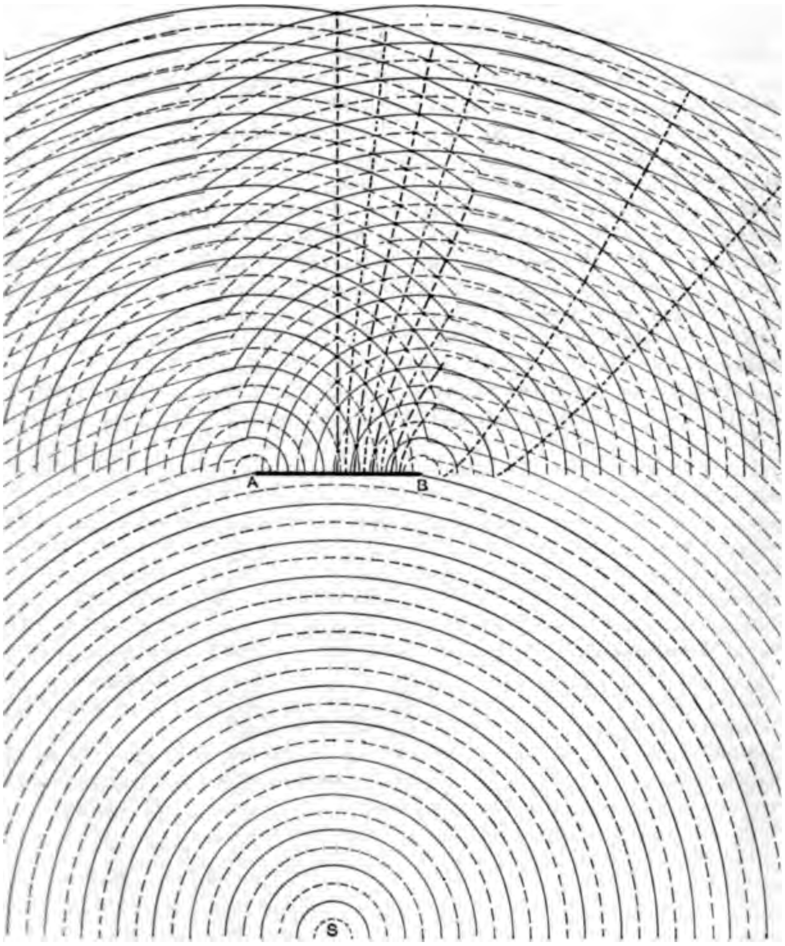
Effects of a screen.—In the figure 76 we may neglect the influence on P of the part of the wave-front beyond *d*, for the extent to which it disturbs P is very small. If a screen were thrust between the wave-front and the point P, so as to cut off the influence of the part *cd*, the disturbance of P would be *increased*: if the screen come to *b*, the motion of P will be *less* than at first: if it come to *a*, it will be greater than if there were no screen: if to N, it will be less, being about one half of the original amount. If the screen be pushed still farther, P will, in the same way, be in more or less active motion according to the position of the screen. The waves therefore pass round the edge of the screen, producing fringes of alternate maximum and minimum motion. The possibility of this result depends on the smallness of the wave-length.

Effect of a very small screen.—If a wave-front be interrupted by a very small screen placed so as to allow the wave motion to pass it all round its edge, Fig. 77 shows that disturbances passing round this obstacle produce, even behind the screen, hyperbolic fringes of maximum disturbance, between which there may be traced hyperbolic fringes of minimum disturbance where crests coincide with troughs; and that, further, even beyond the shadow of the screen, there are hyperbolic fringes of maximum and minimum disturbance. The centre of the shadow of the screen is a spot of maximum disturbance.

Wave traversing an aperture.—The same kind of diagram shows that a wave whose wave-length is small, passing through an aperture, gives fringes of maximum and minimum disturbance beyond the edge of the aperture. A very curious result is, that a wave-front passing through an aperture may produce no movement in a point situated immediately opposite its centre, for there may be complete interference between the waves proceeding from the edges and those from the central regions of the aperture.

Relation of the wave-length to Fringes.—The position of the fringes depends on the wave-length. The smaller the wave-length, the nearer to one another will the fringes be. If the incident wave, diverging from a point, be compound, each component will form its own set of fringes without reference to

Fig. 77.



the others; and any particle within the field of fringes may be stationary as regards one of the component vibrations, while at the same time it is affected by the others.

Energy of S.H.M.—The Energy of a S.H.M. is proportional to the Square of its Amplitude. The angular velocity is constant, for S.H.M.'s are isochronous; the velocity in the circle of reference varies as the radius, *i.e.*, as the amplitude; the Energy varies as the square of the velocity with which the body executing the S.H.M. passes the midpoint; this velocity is the same as the velocity in the circle of reference; therefore the Energy varies as the square of the amplitude.

If a S.H.M. be wholly in one plane, its energy is wholly kinetic as the moving body passes the mid-point with velocity v; it is equal to $\frac{1}{2}mv^2$.

If another S.H.M. of equal amplitude and period, and in the same plane with the former, be compounded with it, the amplitude is doubled, and the energy therefore quadrupled; there must therefore be a draft of energy from elsewhere before this superposition can actually occur.

Energy of Conical Pendulum.—If with one S.H.M. there be compounded another S.H.M. of equal amplitude and period, but in a plane at right angles to the former, the energy of the compounded movement is simply double that of either of the components. In circular motion these two S.H.M.'s differ by $\frac{1}{4}$ period; when the energy of the one is wholly kinetic that of the other is wholly potential, and *vice versâ;* while in intermediate positions the one has gained as much potential or kinetic energy as the other has lost; so that the amount of kinetic energy is continuously equal to the amount of potential energy, each of these being $\frac{1}{2}mv^2$. The whole energy of a circular movement of velocity v, when the moving body is attracted towards the centre by a force which varies directly as the distance from the centre, is thus mv^2; of which half is kinetic, half potential.

Energy of wave-motion.—The energy of a wave-motion is equally divided between the potential and the kinetic forms. Let us first consider a linear wave: at the crest and at the trough the whole energy is potential; midway between crest and trough the energy of the particles as they pass through their mean position is wholly kinetic; elsewhere all the particles are symmetrically and continuously losing potential and gaining kinetic energy, or gaining potential while losing kinetic energy; the gains must be equal to the losses, for there is no change in the type of vibration, and no change in the amount either of potential or of kinetic energy (friction being imagined absent). Hence the whole energy of the wave is divided into two equal moieties, kinetic and potential in their respective forms. In a circular wave the kinetic energy is, under the same supposition, invariable in its absolute amount, and the potential energy bears to it the same symmetrical fixed ratio of equality. So for tridimensional waves.

The Energy of tridimensional waves per cubic cm. is numerically equal to the pressure per square cm., which is exerted on the bounding surface in consequence of the continued propagation of wave-motion. This is equal, if ν be the velocity of propagation, ρ the density, v the maximum velocity of vibration, to $\frac{1}{2}\rho\nu . v^2$ dynes per square cm., or ergs per cubic cm.

CHAPTER VI.

KINETICS.

GENERAL PROPOSITIONS relating to the possible forms of Motion find their parallel in those relating to Forces. The formula $f=ma$, already established, shows that every force is measured by the quantity of motion produced in unit of time. In this way we see that the truth of the propositions entitled the Parallelogram of Velocities and that of Accelerations involves that of a similar proposition in regard to Forces. Two forces acting on a single particle produce the same result as would be produced if a single force were acting on it represented in magnitude and direction by the diagonal of a parallelogram, the adjacent sides of which represent, in the same respects, the two simultaneous forces. This is the proposition of the **"Parallelogram of Forces."**

This proposition shows us that if we have two component forces at right angles to one another, the square of the resultant force will be equal to the sum of their squares. Refer to Fig. 11; let it be desired to apply force to a particle lying at A, so as to make it move in the direction AC: the available force is represented in magnitude and direction by the line AD: then obviously this force cannot exert its full effect in the direction AC; it is only its effective component in that direction that can produce any such effect: this component is represented by the line AC. The other component AB, at right angles to AC, can produce no such effect. This is an example of the **Resolution of Forces.** Plainly, we may resolve any force into two components at any angle to one another, just as we may so resolve a velocity. In tridimensional space we may resolve a force into three components. Suppose, then, such a question as the following:—A pull is made in the direction AD; this pull is designed to draw an object through a tube whose direction is AC: what proportion do the component effective in pulling down the object and the lateral pressure on the walls of the tube respectively bear to the force applied? If Fig. 11 (*a*), were drawn with the proper angle $CAD = \theta$ between the direction of application of the force and the line along which the object is to be drawn: then the effective component would be represented by AC, and the component producing lateral pressure by AB. But $AB = AD \sin\theta$; $AC = AD \cos\theta$: or—

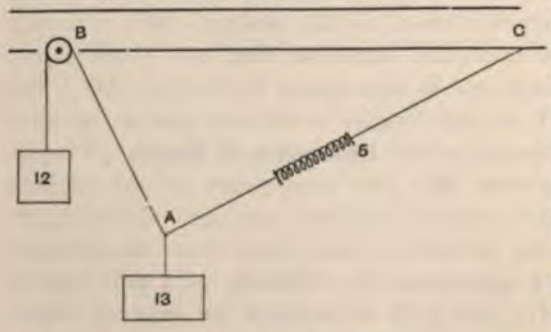
$$AB : AD :: \sin\theta : 1$$
$$AC : AD :: \cos\theta : 1.$$

Hence, if AD be taken as unity, AB and AC may easily be found, if the angle θ be known, by finding the value of $\sin\theta$ and $\cos\theta$ in a table of trigonometrical ratios. If AD have any other value than unity, the values of $\sin\theta$ or $\cos\theta$ derived from the tables must be multiplied proportionately.

Problem.—A force of 50 lbs. is applied to a solid body drawn down a fixed canal which the solid body exactly fits; the angle θ which the direction of traction makes with the axis of the canal is 29°. What is the effective component available in pulling the solid body? and what is the pressure produced by the traction-force on the walls of the canal? In Fig. 11 (*a*), if θ be 29°, AD represents the force applied, equivalent to the weight of a 50-lb. mass; AC represents the effective component $= AD \times \cos\theta$; AB represents the component at right angles to AC—that is, the detrimental pressure $= AD \times \sin\theta$. But $\cos 29° = \cdot 8746197$; $\sin 29° = \cdot 4848096$. The effective component is thus ·8746197 of the force applied—*i.e.*, it is equal to the weight of 43·731 lbs.; the detrimental pressure is 24·2405 lbs.

It seems rather surprising that the effective and lateral components together should appear to be so much greater than the original force applied. But geometrically this is the same thing as to say that two sides of a triangle are greater than the third side; and further, we have already seen that there is no law of the Conservation of Force, though there is a law of the Conservation of Energy. The principle of the Conservation of Energy is maintained, for the energy imparted in a given time is proportional to squares of the forces acting, and the squares of AB and AC are together equal to the square of AD.

Fig.78.



Experimental proof of the Parallelogram of Forces. — A cord, two masses of 12 and 13 lbs. respectively, a pulley, and a dynamometer, are arranged on a beam as shown in Fig. 78. The string can be adjusted so that the angle BAC may have a wide range of values: for every position there is a corresponding stress on the dynamometer. When AB and AC are at right angles to one another, the spring of the dynamometer is pulled out to exactly the same extent as it would have been by the weight of a 5-lb. mass. The dynamometer may be replaced by a 5-lb. mass suspended over a pulley; in that case the cord would so adjust itself that the angle A would be a right angle. Here $5^2+12^2=13^2$, or $25+144=169$; and the law is confirmed.

In a similar way, the propositions known as the Triangle of Velocities, the Polygon, etc., are replaced in Kinetics by the **Triangle,** the **Polygon,** etc., of **Forces**; and the resultant force is the missing side of the triangle or polygon, of which all the sides except the missing one represent the various forces acting, in precisely the fashion already studied under Kinematics.

In tridimensional space we have the propositions of the parallelepipedon, the skew-polygon, etc., of forces, parallel to similar propositions under Velocity.

Equilibrium of Forces.—There here emerges an important and difficult question of nomenclature. It may appear not strictly consistent with the definition of Force to speak of two forces, equal and opposite, balancing one another and producing no effect, as being still two distinct Forces; for the essence of Force is acceleration observed. But it is not more inconsistent than it is to speak of two equal and opposite simultaneous velocities being equivalent to rest, or of two equal and opposite simultaneous accelerations resulting in no change of velocity. There is no doubt that we are entitled so to speak; and if so, then we are entitled to speak of equal and opposite simultaneous forces balancing one another, for propositions concerning forces are strictly parallel to those concerning velocities or accelerations.

The inconsistency finally vanishes when we observe that the Forces which apparently destroy one another are not physical entities, but mental artifices, "paper bullets of the brain;" and that we do not really think of them as simultaneous: we look first to the one and then to the other, and see that what the one does the other undoes; in the end there is thus no result. The imaginary Forces are in equilibrium; the actual body is in a condition of Stress of some kind. But "Force" is a compendious phrase; its use saves many words; and bearing this in mind, we may admit that it is often more convenient to study the balanced forces, each in its turn, than it is to consider the actual stressed condition of the body: and thus while we cannot speak of a single force unless there be actual motion produced or checked, we may allow ourselves to speak of the equilibrium of the several forces acting upon a body at rest.

If we suppose a stone perched upon a height, we say that the earth attracts it with a definite and measurable Force. Yet there is no movement. Here, however, we have really a downward Action of the earth, an upward Reaction of the support; and the phenomenon is an equilibrium of two "forces," of which we may confine our attention to one only, the downward Force of Gravitation.

So we may say that an electrified body, freely suspended at a certain distance from another electrified body, is attracted or repelled with a definite Force; and so it is, as we may see if it be free to move; but if it be not free to move, the attraction or repulsion is still the same, but is now balanced by an equal and opposite Reaction of the support; and this Force and Reaction together correspond to a Stress, a mutual Pressure or Tension, between the body attracted or repelled and its support. Hence we may say that the body is in every such case impelled to move (repelled or attracted) with or by a certain definite Force.

Centre of Figure.—We have already seen that a rigid body, of which the several particles are subject to accelerations which are equal and parallel to one another, moves as if concentrated at its centre of figure, and as if this were subject to a single acceleration. A body may thus be acted upon by parallel forces affecting its particles, the result being the same as if a single force had acted at the centre of figure; while conversely, if a single force act at the centre of figure, the result is to impart parallel and equal accelerations to all the particles, and thereby to effect a Translation of the body. Hence most of the propositions of Kinematics, which describe the motion of a single point, may be transferred to Kinetics, not only as relating to the movements of single particles, but also as relating to translation of material bodies.

Inertia of Matter.—If in any system of bodies there be no force acting, the formula $f = ma = 0$ shows that $a = 0$, that there is no acceleration: hence if there be no force acting, there is no change in the speed with which a body is moving, or in its state of rest, as the case may be; in other words, Matter has Inertia. This is Newton's first law of motion, and it appears to be here derived from the formula; but it will be remembered that the formula was itself derived by implication from that law.

Examples of Inertia.—Examples of this abound. Collisions between ships and between trains, which do not stop if there be not sufficient retarding-force at command; trains passing stations when their speed is great and the rails are slippery; a person falling off the stern of a boat or the back of a car, when the vehicle makes a sudden movement forwards in which his body does not participate; the onward motion retained by a rider when his horse stops under him; the jerk received by a horse suddenly starting in order to set in motion a heavy waggon; when the waggon is running, if the horse suddenly stop, he is bruised, for the massive waggon does not stop at once; a greyhound chasing a hare is carried forward and cannot stop or turn his path instantly at the spot where the hare doubled or turned abruptly from her course; the inertia of the dust of a carpet, when the carpet is beaten—the carpet moves forwards at each blow, but the dust remains, and is thus separated from the carpet and blown away by the wind; the inertia of dust when it is shaken off a book—the book and the dust are made to describe together a rapid movement in the air—the book is suddenly arrested by a smart blow, while the dust does not stop but moves onwards; the inertia of the snow which in the same way is kicked off one's boots—the boot is suddenly stopped, but the snow goes on, and is thus shaken off; the inertia of loose grain cargo in a ship—it acquires a certain velocity when the ship rolls, and does not stop when the ship arrives at its normal limit, but pours on so as sometimes to make the ship roll beyond the limits of safety; the oscillations of mercury in an ordinary barometer at sea, the mercury being jerked up by each roll of the ship; the inertia of the mercury in a mercury manometer used to investigate fluid-pressure —the variations in the height of the mercurial column being greater than the real variations in the pressure, for the mercury does not stop moving when the fluid-pressure ceases to rise or fall; the inertia of water in house water-pipes if it be set to run and then suddenly stopped—the water is compressed against itself and a violent jerk is produced, which is utilised in the hydraulic ram; the inertia of water in the case of the water-supply of the locomotive engines of passenger express trains on the L. and N.W. Railway system—the engine puts down a tube, the lower end of which acts as a scoop for the water, which tends to remain in its trough on the ground between the rails and at rest relatively to the ground; but this being equivalent to a backward movement relatively to the engine, the water slips up the inclined tube into the tender if the train be moving at sufficient speed.

There are some further remarkable consequences of the inertia of matter: a body may be struck or pressed so suddenly that it stands practically at rest during the time that the blow is being spent on it, and it may be crushed or broken by such a blow. A grain of corn or a granule of gold-quartz, if thrown up into the air and struck a blow by an iron bar moving at the rate of about 180 feet a second, will be crushed by compression, and will be, by a succession of such blows, very effectively pulverised. Milling machinery has been constructed on this principle. A bullet in a gun, though free to move onwards, is crushed against itself before it fairly starts, so that the soft lead is moulded into the grooves of the rifle-barrel by the rapidly-applied pressure, due to the explosion of quick-burning gunpowder.

Another example is afforded by that instrument with the aid of which M. Rosapelly* has investigated the movements of the larynx during the emission of sounds. A heavy mass of metal is suspended in a light framework which is tied over the larynx: as this mass cannot at once participate in the rapid movements which the vibration of the larynx communicates to the light framework, it forms a kind of fixed point, and the light framework, as it vibrates in contact with the skin over the larynx, may strike the heavy mass a series of blows; these may cause an electric current to be alternately made and broken; the number and frequency of these interruptions may be registered on an appropriate recording-instrument.

Further, the inertia of matter is a property of retaining whatever motion an object has, and that in a plane fixed in space, without reference to the movements of surrounding objects, unless these are so connected with it as to be able to affect its motion. A hammock retains its position in space independently, in the main, of the pitching and rolling of the ship. The statement would be approximately accurate that the hammock does not swing in the ship, but that the ship swings enclosing the hammock, which may for any short period of time be regarded as moving onward in space with the average velocity of the ship, but independent of it. A long and heavy pendulum set to swing in one plane, and connected by a very slender attachment to the roof of the building in which it is suspended, will swing in the same plane in absolute space though the earth rotate under it: the apparent result is, that the plane in which the pendulum swings gradually alters its aspect, so that the pendulum swings successively in every possible direction. The real state of the case is not that the heavy pendulum alters its direction of oscillation, but that the earth rotates or has a component of rotation under the pendulum. If a heavy wheel be set in motion, it will in the same way, if it can rotate for a sufficiently long time, show the same phenomenon, for it tends to continue to rotate in the same plane in space.

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* Trav. du Laboratoire de M. Marey, 1876.

Momentum.—The product mv of m, the mass of a moving body, into v, its velocity, is called the *Momentum* of the body.

If a shell explode, its fragments form a system of bodies moving at different velocities. The average velocity of the centre of figure of the whole system is, however, unchanged: some fragments travel with a greater, some at a less velocity than that with which the shell had travelled before the explosion; but the mass m is unchanged though differently arranged, the mean velocity of the system is the same as that of the original shell, and thus the momentum of the whole system is the same after explosion as before it.

Impact.—If there be two *inelastic* bodies of masses $m_{,}$ and $m_{,,}$ respectively, of which the first moves with velocity $v_{,}$, while the second is at rest: if the moving one, whose momentum is $m_{,}v_{,}$, strike the other, it will divide its momentum with that other; it itself will travel more slowly while the other is set in motion; but the two will travel together with a common velocity V.

The whole mass moving with this new velocity V is $(m_{,} + m_{,,})$; its momentum is equal to the original $m_{,}v_{,}$; hence the velocity V may be found by stating this equality of momenta in the form of the equation—

$$(m_{,} + m_{,,})\,V = m_{,}v_{,}$$

$$\therefore V = \frac{m_{,}v_{,}}{m_{,} + m_{,,}}.$$

If the mass $m_{,,}$ be large in comparison with $m_{,}$, the velocity V is much less than $v_{,}$. If a man lie with an anvil on his chest, and if the anvil be struck a blow with a hammer relatively not too heavy, the person lying down, if he can support the anvil, will not be much affected by the blow, for the movement imparted to the anvil will be slow as compared with that of the hammer.

Let the two inelastic masses be $m_{,}$ and $m_{,,}$, moving with the respective velocities $v_{,}$ and $v_{,,}$, and together moving after impact with the velocity V; the respective momenta of the masses before impact were $m_{,}v_{,}$ and $m_{,,}v_{,,}$; that of the conjoined mass after impact is $(m_{,} + m_{,,})$ V. Hence

$$m_{,}v_{,} + m_{,,}v_{,,} = (m_{,} + m_{,,})\,V.$$

$$V = \frac{m_{,}v_{,} + m_{,,}v_{,,}}{m_{,} + m_{,,}}. \quad (1.)$$

It was found experimentally by Newton that, in such a case, the momentum lost by one body was equal to that gained by the other. To express this algebraically, if M represent the momentum gained by one and lost by the other

$$m_{,}V - m_{,}v_{,} = M. \quad (2.)$$

$$m_{,,}v_{,,} - m_{,,}V = M. \quad (3.)$$

From either of these, with the aid of equation (1) we find

$$\mathrm{M} = \frac{m_{\prime} m_{\prime\prime} (v_{\prime\prime} - v_{\prime})}{m_{\prime} + m_{\prime\prime}}.$$

Apparent loss of Energy.—In this case the kinetic energy after impact $\left(i.e., \ \frac{1}{2} \text{ mass} \times \mathrm{V}^2 = \frac{1}{2}(m_{\prime} + m_{\prime\prime}) \left(\frac{m_{\prime} v_{\prime} + m_{\prime\prime} v_{\prime\prime}}{m_{\prime} + m_{\prime\prime}}\right)^2 = \frac{1}{2} \frac{(m_{\prime} v_{\prime} + m_{\prime\prime} v_{\prime\prime})^2}{m_{\prime} + m_{\prime\prime}}\right)$ is less than the sum of the kinetic energies before impact (which were $\frac{1}{2} m_{\prime} v_{\prime}^2$ and $\frac{1}{2} m_{\prime\prime} v_{\prime\prime}^2$ respectively). This is not true if $v_{\prime}$ be equal to $v_{\prime\prime}$, but in that case the two bodies would be travelling in the same direction with equal speed, and the one could not overtake and strike the other. The energy which has apparently disappeared has assumed the form of Heat.

Impact of Elastic Bodies.—We may here anticipate a statement of the nature of Elasticity so far as to say that a perfectly elastic body, possessed of a certain amount of kinetic energy, and striking a perfectly rigid body, will rebound, and will possess as much kinetic energy after the impact as before it; for it leaves the rigid body with a velocity equal to that with which it had approached it.* The mass and the velocity being numerically unchanged, the momentum is numerically equal after impact to that before it; but as it is no longer mv but $m \times (-v) = -mv$, it has become negative, and has therefore altered by an amount equal to $2mv$. If the body be imperfectly elastic, so that the velocity is not completely regained, it is found experimentally that it returns with a certain fraction, λ, of its original velocity (this fraction, λ, being called the **coefficient of restitution**), and the change of velocity is not $2v$ but $(1 + \lambda)\, v$; and its momentum has become negative and $= -\lambda . mv$, so that it has changed by the amount $(1 + \lambda)\, mv$.

If two masses $m_{\prime}$ and $m_{\prime\prime}$, moving with velocities $v_{\prime}$ and $v_{\prime\prime}$, and formed of such material that the coefficient of restitution between them is λ, strike one another, they will, after impact, travel with velocities $\mathrm{V}_{\prime}$ and $\mathrm{V}_{\prime\prime}$. The momentum gained by the one is equal to that lost by the other; but it is not equal, as it is in the case of inelastic bodies where $\lambda = 0$, simply to $\frac{m_{\prime} m_{\prime\prime} (v_{\prime\prime} - v_{\prime})}{m_{\prime} + m_{\prime\prime}}$, but to $(1 + \lambda) \times$ that quantity. This equality of momenta is expressed by the equations—

(1) Gained by $m_{\prime}$; $m_{\prime} \mathrm{V}_{\prime} - m_{\prime} v_{\prime} = (1 + \lambda) \frac{m_{\prime} m_{\prime\prime} (v_{\prime\prime} - v_{\prime})}{m_{\prime} + m_{\prime\prime}}$.

(2) Lost by $m_{\prime\prime}$; $m_{\prime\prime} v_{\prime\prime} - m_{\prime\prime} \mathrm{V}_{\prime\prime} = (1 + \lambda) \frac{m_{\prime} m_{\prime\prime} (v_{\prime\prime} - v_{\prime})}{m_{\prime} + m_{\prime\prime}}$.

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* Such is the elementary theory. There is, however, no perfectly elastic body, and even if there were, a part of its energy would necessarily be spent, upon impact, in setting up vibrations in it, and the speed of rebound could never come up to the theoretical limit.

Whence

$$V_{,} = v_{,} + (1 + \lambda)\frac{m_{,,}(v_{,,} - v_{,})}{m_{,} + m_{,,}}.$$

$$V_{,,} = v_{,,} - (1 + \lambda)\frac{m_{,}(v_{,,} - v_{,})}{m_{,} + m_{,,}}.$$

Take, as a particular instance, the case in which the elasticity is perfect or the restitution complete (*i.e.*, $\lambda = 1$); and the balls which strike one another are of equal weight, so that $m_{,} = m_{,,}$; then $V_{,} = v_{,,}$ and $V_{,,} = v_{,}$; *i.e.* :—

Two equal and perfectly elastic balls striking one another directly in the line joining their centres exchange their velocities, and that whether they meet or overtake one another.

Oblique Impact.—If a ball strike a rigid surface obliquely, its motion relative to that surface may be resolved into two components: one parallel to it, which is not affected by the impact; the other at right angles to it, which, after impact, will be wholly or in part restored in the reverse direction. Reference to Fig. 60 will show that if $\lambda = 1$, the angle of reflexion will be equal to the angle of incidence; while if λ be less than unity, the angle of reflexion will be proportionately less acute, to an extent easily determined by construction.

If the oblique impact be between two balls, the investigation is based on similar principles. Take a line joining their centres of figure; in a direction at right angles to this line the motion is unaffected by the impact, and the component in this direction will be the same after impact as before it; in the line joining the centres of figure, the velocities $V_{,}$ and $V_{,,}$ may be found as in the preceding discussion. By compounding the component velocities after impact, the resultant velocities and their direction may be found. In practice, as the two balls are passing one another in contact, friction between their surfaces causes a relative delay of one aspect of each, and causes rotation of the balls; the energy necessary for this is taken from that theoretically available for the direct translational movements of the balls as wholes.

Energy in impact of Elastic Bodies.—In the case of perfectly elastic bodies, the energy after impact is equal to that before it; $\frac{1}{2}m_{,}V_{,}^2 + \frac{1}{2}m_{,,}V_{,,}^2 = \frac{1}{2}m_{,}v_{,}^2 + \frac{1}{2}m_{,,}v_{,,}^2$. If λ be less than 1, there is apparent loss of Energy, which has assumed some other form than that of motion of the mass. If a horse with loose traces rush forward and jolt a car, the energy which disappears is wasted in the form of heat, or deleteriously spent in disintegration of the materials of the car, or in bruising the animal.

"Centrifugal Force," so-called.—It has been already shown (p. 77) that when a body describes a curved path, there is an acceleration towards the centre $= v^2/r$, where v is the tangential velocity in the curve, and r the instantaneous radius of curvature.

If the path be a circle of radius R, the acceleration is constantly $=v^2/\mathrm{R}$. The component force drawing the body from the tangential path is therefore one which produces an acceleration towards the centre $=v^2/\mathrm{R}$; and it is itself equal to mv^2/R.

Suppose a stone of mass m to be whirled round like a slingstone by a string, but in a perfectly circular path. This circular path may be supposed to be made up of very numerous short straight lines or elements (p. 57), each of which is tangential. The actual velocity along any one of these tangential elements during any one instant may be hypothetically resolved during the next instant into two components; one along the circle $= v$; one away from it, and corresponding to an outward acceleration $a = v^2/\mathrm{R}$. Of these two components the former freely manifests itself as velocity v along every successive element or, practically, as a continuous velocity v in the circular path; the latter, the outward component, never manifests itself, for it is, at every instant, counteracted by tension in the string. This tension is a stress set up in the string by the action of its molecular forces, when the whirling ball tends to pull the outer end of the string outwards; and numerically it is, across every complete cross-section of the string, a total tension equal to that which would have been established by the application of mv^2/R units of force.

If this string snapped or were suddenly cut, this tension would cease; there would then be nothing to hinder the actual tangential velocity at the instant of snapping from persisting during the next and succeeding instants; the motion of the stone would therefore be continuous motion in a straight line, the tangent to the curved path at the point where the stone had happened to be at the instant of snapping, and the stone thus liberated and flying off at a tangent would then obey Newton's First Law of Motion.

The stone flies off at a tangent, and not straight from the centre; there is therefore no counteraction on the part of the string of any tendency on the part of the stone to fly off in some direction straight away from the centre; there is therefore in the old sense of the term no so-called "centrifugal force" counteracted by the tension of the string. The tension of the string is, however, equivalent to a force $= mv^2/\mathrm{R}$, acting upon the stone, directed inwards along the string, and the actual inward acceleration of which, v^2/R, balances at every instant the outward acceleration, the centrifugal acceleration of the whirling stone, acting upon the string.

Any string will snap if force be applied to it beyond a certain limit. If a string be just so strong that N grammes of matter may be suspended on it without its snapping, it can survive the application of force equal to 981N dynes. If this string be used to whirl a slingstone of mass m, it will snap unless the velocity v be such that mv^2/R is less than 981N—that is, v must be less than $\sqrt{981\mathrm{NR}/m}$. If the velocity exceed this limit, the string will snap. As the velocity increases, its centrifugal component increases, and requires a greater force or reaction to be exerted in a direction towards the centre in order to bend the path into the same curve in a shorter time. In the same way, if a fly stand on the rim of a rotating wheel, the adhesion between the foot of the fly and the rim of the wheel necessary in order to enable the fly to retain its footing may become so great that the fly cannot hold on and is hurled off at a tangent.

When a grindstone or flywheel is rotated too rapidly, the molecular forces of cohesion cannot keep the particles together against their tendency to fly off at a tangent.

If the earth rotated on its axis seventeen times as fast as it does, the attraction of gravitation, the effect of which is even now masked to some extent by the rotation of the earth, would only just be able, at the equator, to keep bodies from flying off its surface at a tangent.

The greater the velocity of a railway train the greater is its tendency to fly off the track as it is rounding curves.

If a drop of oil be suspended in a mixture of spirit and water, so that it is free to assume any form, and if a motion of rotation be communicated to it, the globular drop assumes the form of an oblate spheroid, and bulges at its equator; for particles at its original equator have, when set in motion, a greater velocity than those nearer its poles. For the same reason the earth itself has assumed the form of an oblate spheroid.

In the trundling of a wet mop, when the drops fly off because they do not adhere firmly enough to enable them to retain their position—in the rotation of a steam governor, the balls of which fly asunder as the speed of the engine increases, thereby actuating an appropriate train of mechanism which to a greater or less extent shuts off the steam—we find examples of this phenomenon. If a man were placed on a revolving table, with his feet towards the centre, the blood in his body would be urged towards his head; and this has actually been proposed as treatment in bloodlessness of the brain.

When a circular cylindrical vessel containing water is rotated on its axis, the water is heaped up towards the sides of the vessel. If the speed exceed a certain limit, the water will be hurled over the sides of the vessel, and if the supply of water and the rotation be continuous, an engine may expend its energy in thus continuously lifting water against gravity. This principle is applied in Siemens's governor for machinery; when the engine goes too fast it begins to spend energy in producing this current of water. The form of the surface of water thus produced is always parabolic.

When light and heavy particles in mixture are whirled, the heavier fly outwards; thus milk, if rotated, separates into heavier milk externally and lighter cream internally.

If a particle move in a circle whose radius is r with angular velocity ω, the actual Space traversed by it in time t is $r\omega t$; its Velocity per second is $r\omega$; the Force necessary to impart this velocity in one second is $mr\omega$; the Energy of its movement is $\frac{1}{2}mr^2\omega^2$.

Moment of Inertia.—If a mass M and a point in space, internal or external to the mass, be considered in relation to each other, the name Moment of Inertia is given, and the symbol I is usually assigned to the numerical quantity which is obtained by summing up in appropriate units the products of the mass $\overline{m}$ of each particle of the mass into the square of its corresponding distance $\overline{d}$. This operation generally requires the aid of the Integral Calculus, but the resultant sum, $\Sigma\overline{m}\overline{d}^2 = \mathrm{I}$, is a numerical quantity, and is always positive.

Radius of Gyration.—Suppose a uniform disc of radius 1 ft. to rotate round its centre under the action of a given force, it rotates with less angular velocity than it would have assumed if the same force had been applied to the same matter gathered nearer the centre; for $f = mr\omega$, and if the mean value of r be greater, the value of ω must be less if f, the force applied, be constant. Such a disc rotates, on the other hand, with greater angular velocity than that which it would have assumed if the matter had been gathered near the circumference. Between these two limits there must be a distance from the centre such that if the whole mass had been concentrated there, the angular velocity acquired under the influence of a given force would have been the same as that actually assumed by the disc. This distance is the Radius of Gyration with reference to that point. The criterion of the radius of gyration is, that if the whole mass M were placed at the distance k (k = rad. of gyr.) from the point of suspension, it would have with reference to it the same Moment of Inertia as that actually possessed by the physical mass in question. Hence $\mathrm{M}k^2 = \mathrm{I} = \Sigma\overline{m}\overline{d}^2$.

Radii of Gyration and Moments of Inertia in particular cases.—(1.) A uniform rod of length l rotates with the same angular velocity as if its mass were accumulated at a distance $l/\sqrt{3}$ from the point of suspension. The rad. of gyr., $k = l/\sqrt{3}$; the mom. of inertia, $\mathrm{I} = \mathrm{M}l^2/3$.

(2.) A uniform rod of length, l, poised on its centre; $k = l/2\sqrt{3}$; $\mathrm{I} = \mathrm{M}l^2/12$.

(3.) A rectangular lamina of sides a and b, poised at its centre; rotation round an axis at right angles to the lamina; $k = \sqrt{(a^2 + b^2)/2\sqrt{3}}$; $I = M(a^2 + b^2)/12$.

(4.) A circular disc of radius r, and of uniform thickness; rotation round an axis at right angles to the disc and passing through its centre; $k = r/\sqrt{2}$; $I = \frac{1}{2}Mr^2$.

(5.) A solid cylinder rotating round its axis; same as (4).

(6.) A solid ring cut out of a uniform disc of any thickness; inner and outer radii $r_{,}$ and $r_{,,}$; rotation round an axis passing through the centre of the ring and at right angles to the plane of the ring; $k = \sqrt{(r_{,}^2 + r_{,,}^2)/2}$; $I = M(r_{,}^2 + r_{,,}^2)/2$.

(7.) Solid ring, whose cross-section is a circle of radius a; distance between the centre of the ring and the centre of this circle $= b$; rotation round an axis passing through the centre of the ring and at right angles to its plane; $k = \sqrt{b^2 + \frac{3}{4}a^2}$; $I = M(b^2 + \frac{3}{4}a^2)$.

(8.) Spherical shell of radius r; rotation round any diameter; $k = r\sqrt{2/3}$; $I = \frac{2}{3}Mr^2$.

(9.) Solid sphere of radius r; rotation round any diameter; $k = r\sqrt{2/5}$; $I = \frac{2}{5}Mr^2$.

(10.) Let MK^2 be the moment of inertia round an axis passing through the centre of gravity of a mass of any form; round any other axis, parallel to the former and at a distance h from it, the moment of inertia Mk^2 is $M(K^2 + h^2)$[*] where M is the whole mass.

Energy of a rotating body.—The energy of a particle in rotational movement is $\frac{1}{2}mr^2\omega^2$; that of a system of particles each at its own distance $\bar{r}$ and with its own mass $\bar{m}$ must be $\Sigma(\frac{1}{2}\overline{mr}^2\omega^2) = \frac{1}{2}\omega^2\Sigma(\overline{mr^2})$; ω, the angular velocity, being the same in all particles of a rotating body. But $\Sigma\overline{mr}^2 = I$, the moment of inertia: therefore the energy of a rotating body is $\frac{1}{2}\omega^2 I$, or, where k is the radius of gyration, $= \frac{1}{2}\omega^2 Mk^2$. The energy of the ring of example (6) above is therefore $M(r_{,}^2 + r_{,,}^2)\,\omega^2/4$.

A flywheel in motion possesses a large amount of kinetic energy; and if an obstacle be placed in the way of the engine, the engine cannot be stopped by it unless the flywheel can be arrested also: this would involve the sudden exercise of a very great force; hence an engine with a heavy flywheel rapidly rotating can overcome a very great resistance, and in this way, for ordinary resistances, it is prevented from manifesting any very great irregularity of motion.

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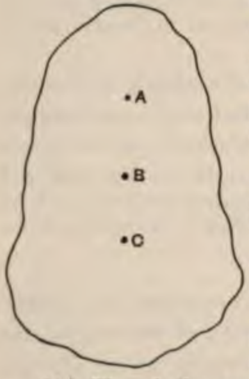
[*] Draw a triangle ABC; A represents the centre of gravity of the object spun, through which the central axis of rotation passes, perpendicular to the paper; B is the position of the other axis parallel to the former; C *any* point whatsoever in the mass rotated. Draw a line CD from C at right angles to AB or to AB produced. Then $BC^2 = AC^2 + AB^2 \pm 2\,AB.AD$. AB is the distance h between the two axes; BC the distance of the particle C from the new axis, AC its distance from the centre-of-gravity axis. If the particle at C have mass m, $m.BC^2 = m.AC^2 + m.h^2 \pm 2m.h.AD$. Now sum up for all such particles as C, and we have $\Sigma(m.BC^2) = \Sigma(m.AC^2) + \Sigma(m.h^2) \pm 2h\,\Sigma(m.AD)$. The last term disappears, for all round the centre of gravity AD has as many positive as negative values; $\Sigma(m.BC^2)$ is Mk^2, the moment of inertia round B; $\Sigma(m.AC^2)$ is MK^2, the moment of inertia round A; $\Sigma(m.h^2)$ is Mh^2. Whence $Mk^2 = M(K^2 + h^2)$.

If a flywheel whose energy is $\frac{1}{2}\omega^2 Mk^2$ were called upon to expend n units of energy in overcoming a certain resistance, the energy in it after doing so would be $(\frac{1}{2}\omega^2 Mk^2 - n)$, and a new angular velocity $\omega_{,}$ would be assumed, such that $\frac{1}{2}\omega_{,}^2 Mk^2 = (\frac{1}{2}\omega^2 Mk^2 - n)$. The amount of kinetic energy in a flywheel thus fluctuates. If a very large flywheel have a heavy rim, and if the spokes be relatively thin, the radius of gyration is practically the distance between the centre of the wheel and the middle of the thickness of the rim; and the energy is, approximately, $\{\frac{1}{2}\omega^2 M . (\text{mean radius})^2\}$.

Suspended Body.—Suppose a heavy body of mass M to be suspended at a point, and then, that point of suspension retaining its fixed position, to swing down so far that its centre of figure sinks through a vertical height h, acquiring an angular velocity ω. The kinetic energy acquired by the body considered as rotating round the point of suspension is $\frac{1}{2}\omega^2 I$; the potential energy lost by descent of the mass M through height h is Mgh. These are equal. Hence $\frac{1}{2}\omega^2 I = Mgh$; or $\omega^2 = 2Mgh/I$. But $I = Mk^2$ where k is the radius of gyration; whence $\omega^2 = 2gh/k^2$.

Centre of Oscillation.—In Fig. 79 let A be the point of suspension of a body, B its centre of figure or of mass (centre of gravity), k the length of the radius of gyration of the mass with reference to the point of suspension A: then there is in the same straight line with A and B, and on the opposite side of B from A, a point C called the *Centre of Oscillation*, which has the following properties :—

Fig. 79.



(1.) The body may be swung upon A or upon C indifferently, and in either case it will oscillate pendulum-wise with equal rapidity.

(2.) The body thus suspended at A or at C will oscillate at the same rate as an ideal simple pendulum of the length AC. (Proved at p. 199.)

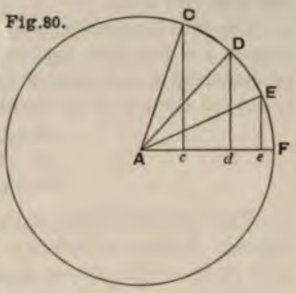
(3.) This body will, if struck at C, oscillate round A without producing any pressure on the supporting axis at A.

(4.) Though the support at A were withdrawn—as, for instance, if the body float submerged in water—yet if the point C were struck, the point A would remain at rest, and all the part of the body lying above A would move in a direction opposite to that in which C is struck. For every point C at which a body may be struck, or for every **centre of percussion,** there is a corresponding point A on the other side of the centre of figure through which passes an **axis of spontaneous rotation,** round which the body rotates. If the lower part of any object be suddenly pulled forwards, the upper part will move backwards. This property is found applied in the jaw of echinoidei; the upper end of each of the five jaws is suddenly tilted outwards, and the lower, the tooth-bearing ends, are tilted together.

(5.) The distance AC is equal to k^2/AB when the body is suspended at A, k being the radius of gyration in this case; or to $k_{,}^2/CB$ when suspended at C, $k_{,}$ being the radius of gyration in this case. The radii of gyration are so related that $k^2/AB = k_{,}^2/CB$. (See p. 199.)

Force causing rotation constant in direction.—If a body be caused to rotate by a force whose direction is the same or nearly so throughout the movement, the effect of the force varies greatly. In Fig. 80 let AC, AD, AE, AF, be successive positions of a rod rotating round A, and acted upon by a force applied at the extremity remote from A, and always parallel to the lines C*c*, D*d*, E*e*. In the position AF the effect produced by the force is a maximum, for two reasons: (1) that the component effective in producing rotation is there the greatest; and (2) that the force is applied with the greatest "leverage," or so as to have the greatest possible "moment." In this way the forearm moves with the greatest swiftness at the middle of flexion.

Fig. 80.



Accelerated Motion.—The discussion of the accelerated motion of a particle moving with constant acceleration—*i.e.*, under the continuous influence of a constant force—has already led us to the formulæ—

$$v = \mathrm{V} \pm at, \quad \text{(i.)}$$
$$s = \mathrm{V}t \pm \tfrac{1}{2}at^2, \quad \text{(ii.)}$$
$$v^2 = \mathrm{V}^2 \pm 2as, \quad \text{(iii.)}$$

where V represents the velocity of a particle at the beginning, and v that at the end of time t, s the space traversed during that time, and a the acceleration (positive or negative, as the case may be) per unit of time. The most familiar examples of this kind of movement are those in which bodies exposed to the constant influence of gravity fall with constantly-increasing speed.

Problems.

1. A train travelling at 50 miles an hour comes into collision with a fixed obstacle and is abruptly stopped: the passengers receive a blow. What height must one fall in order to receive a similar blow?—*Ans.* 50 miles an hour $= 73 \cdot \dot{3}$ feet per second. A body falling from rest ($\mathrm{V} = 0$) acquires a speed of $73 \cdot \dot{3}$ per sec. in falling 83·51 feet, for

$$v^2 = \mathrm{V}^2 \pm 2as. \quad \text{(iii.)};$$
$$(73 \cdot \dot{3})^2 = 0 + (2 \times 32 \cdot 2 \times s); \therefore s = 83 \cdot 51.$$

A blow in a collision at 50 miles an hour is equivalent to a blow received in consequence of a fall of 83·51 feet; for a body which has fallen 83·51 feet is in consequence travelling at 50 miles an hour at the instant.

2. A ball weighing 5 ounces is hurled upwards. It is supposed that

while it is in the hand it is swung through 4 feet: the thrower during this swing continuously exerts an accelerating pressure on it. This pressure must be equivalent to the effort which would be put forth in raising some definite weight in the same position of the body. What is this weight if the ball rise 100 feet?

The ball leaves the hand with an unknown velocity V; it rises through a space $s = 100$ feet against a force (gravity), the negative acceleration (a) produced by which is 32·2 feet per second; it comes for an instant to rest ($v = 0$) at the top of its course.

$$v^2 = V^2 \pm 2as. \quad \text{(iii.)}$$
$$0 = V^2 - (2 \times 32{\cdot}2 \times 100).$$
$$V^2 = 6440. \quad V = 80{\cdot}25.$$

The question thus becomes—Under the influence of a force f acting through 4 feet, a velocity 80·25 feet per second is imparted to a mass $m = \frac{5}{16}$ lb. Find f; or, since $f = ma$ and $m = \frac{5}{16}$, find a.

$v^2 = V^2 \pm 2as$ (iii.); $V = 0$; a is positive.

$\therefore$ $v^2 = 2as$; but $v = 80{\cdot}25$, the velocity with which the ball leaves the hand.

$6440 = 2a \times 4 = 8a. \quad a = 805$ feet.

$f = ma = 226\frac{9}{16}$ British units of force.

But wt. of 1 lb.-mass = 32·2 Brit. units of force.

$$f = \text{wt. of } \frac{226\frac{9}{16}}{32{\cdot}2} \text{ lb.-masses} = 7\tfrac{13}{16} \text{ lbs.}$$

The effort then is the same as that put forth in upholding a weight of 7 lbs. 13 oz.

3. A shot is fired vertically from a gun whose barrel is 30 inches long: it rises half a mile. Compare the acceleration of a body falling under the influence of gravity with that under which the bullet acquires such velocity in the space of 30 inches.

First find the velocity of the shot as it leaves the gun. In its course in the air (friction being entirely neglected) it commences with the unknown velocity V, traverses space $s = 2640$ feet against gravity which produces acceleration $a = -32{\cdot}2$, and comes to rest ($v = 0$) for an instant.

$$v^2 = V^2 \pm 2as. \quad \text{(iii.)}$$
$$0 = V^2 - (2 \times 32{\cdot}2 \times 2640). \quad V = \sqrt{170016} = 412{\cdot}3.$$

In the gun-barrel a velocity v, 412·3 feet per second, is acquired within a space $s = 2\frac{1}{2}$ feet. We want to know the acceleration. Here $v = 412{\cdot}3$; $V = 0$; $s = 2{\cdot}5$; a is unknown, but is positive.

$$v^2 = V^2 \pm 2as. \quad \text{(iii.)}$$
$$(412{\cdot}3)^2 = 2a \times 2\tfrac{1}{2}.$$
$$170016 = 5a. \quad a = 34003{\cdot}2 \text{ feet per second.}$$

If the barrel were long enough to expose the projectile to the influence of such an accelerating force for a whole second, this velocity would be acquired, and the barrel would be 17001·6 feet long.

But gravity produces an acceleration of 32·2 feet per second. Hence the acceleration due to the gunpowder is greater than that of gravity in the ratio of $\frac{34003{\cdot}2}{32{\cdot}2}$, or 1056 : 1. The *force* exerted by the powder $= ma = 34003{\cdot}2m$, and in the case of an ounce-bullet would be equal to 2125·2 British units of force.

4. With what "force" will a 10-lb. mass falling 100 feet strike at the end of its course? As it stands, this question is devoid of sense, for it does not specify the time during which the momentum is changed on impact. If the body struck were rigid, and the falling mass perfectly elastic, it would rise on rebounding to an equal height of 100 feet. During the impact it must have come to rest. Let us arbitrarily assume it to have taken $\frac{1}{2000}$ sec. to come to rest, and an equal period—$\frac{1}{2000}$ sec.—to gain its upward initial velocity: this upward initial velocity is 80·25 feet per second, for

$$v^2 = \mathrm{V}^2 - 2as, \text{ where } a = 32{\cdot}2.$$
$$0 = \mathrm{V}^2 - (2 \times 32{\cdot}2 \times 100).$$
$$\mathrm{V} = 80{\cdot}25.$$

The question thus becomes—What is the mean pressure between the body which has fallen and that on which it falls, if a speed of 80·25 feet per second can be arrested or developed by it in $\frac{1}{2000}$ sec.? The answer is—Since $v = at$: $v = 80{\cdot}25$; $t = \frac{1}{2000}$; $a = 160{,}500$; and $f = ma = 1{,}605{,}000$ British units of force $=$ the wt. of $\dfrac{1{,}605{,}000}{32{\cdot}2} = 4984{\cdot}47$ lbs.

5. A ball weighing 10 lbs. falls from a height of 100 feet on a rigid floor. It is flattened to the extent of $\frac{1}{30}$ inch, measured in the direction of its motion: it recovers its form and rebounds. What is the time taken to bring the ball to rest, and what is the mean total pressure between the ball and the floor on which it falls? Here a velocity of 80·24954 ft. per sec. is arrested in the space of $\frac{1}{30}$ inch: what is the retarding acceleration a? what is the corresponding pressure f? what is the time t?

$$v^2 = \mathrm{V}^2 - 2as. \quad \text{(iii.)}$$
$$0 = (80{\cdot}24954)^2 - (2a \times \tfrac{1}{30} \text{ inch.})$$
$$= 6440 - (2a \times \tfrac{1}{360} \text{ foot.})$$
$$a = \tfrac{1}{2}(6440 \times 360) = 1{,}159{,}200 \text{ feet per second.}$$

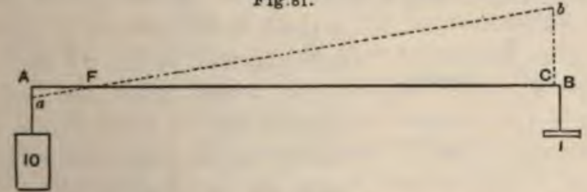
Again, $f = ma = 1{,}159{,}200 \times 10 = 11{,}592{,}000$ British units of force
$= \text{the weight of } (11{,}592{,}000 \div 32{\cdot}2) = 360{,}000$ lbs. $=$ *mean* pressure.

Lastly, $v = at$. (i.)

$$80{\cdot}24954 = 1{,}159{,}200t. \quad t = \tfrac{1}{14444} \text{ sec.}$$

The Principle of Moments.—In Fig. 81 a linear body is poised at the point F; at A suppose a force equal to 10 units,

Fig. 81.



at B a force equal to 1 unit. The former, acting alone, would turn the bar round F through an angle θ, and the work done at A by the force f is equal to $fs = 10$ units $\times$ A$a = 10$ AF $\sin\theta$. The latter force applied at B would turn the bar round F through an angle, say ϕ, and would in producing rotation do work $= 1 \times$

FB sin ϕ. The work done by the force applied at A during any small displacement is in the opposite sense to that done by that applied at B. Together they may balance one another, and produce equilibrium. If the bar be rigid, $\phi = \theta$. If the one force raise as much as the other depresses every point of the bar, there is equilibrium. As regards rotation round F, the forces are of opposite sign; they are accordingly $+10$ at A, tending to produce **positive** rotation in the direction opposed to that of the hands of a watch, and -1 at B, tending to produce a **negative** rotation. Hence, if there be equilibrium, the work done by force $= +10$ acting at A, and that done by force -1 acting at B are together $= 0$.

$$(10 \times \mathrm{AF} \sin \theta) + (-1 \times \mathrm{BF} \sin \theta) = 0. \quad (1.)$$
$$(10 \times \mathrm{AF}) + (-1 \times \mathrm{BF}) = 0;$$

or, generally, if the forces be P and Q,—

$$(\mathrm{P} \times \mathrm{AF}) + (\mathrm{Q} \times \mathrm{BF}) = 0;$$
$$\text{or } \mathrm{P} : \mathrm{Q} :: \mathrm{BF} : \mathrm{AF}.$$

The forces which balance one another are inversely proportional to their distances from the fixed point F.

Thus a smaller force acting at a greater distance can balance a greater force acting at a less distance. The Importance of the greater force with reference to the point F is exactly the same as that of the smaller force, which has the advantage of greater distance, or greater "leverage" or "purchase."

This Importance of a force not passing through a point is called the **Moment** of that force round that point. It is equal to the amount of the force × the shortest distance from the point *to the line of application* of the force. The shortest distance from a point to a line is well known to be a line drawn from the point to the line in question, at right angles to the latter. In Fig. 81 the moments of the forces round the point F are respectively $10 \times \mathrm{AF}$ and $-1 \times \mathrm{BF}$. In Fig. 82 the forces acting at A and at B are not parallel; their lines of application are AE and BE; the distances of these lines from F are FC and FD at right angles to AE and BE: the moments of the respective forces round F are Force A × distance CF, and Force B × distance FD; and if the forces

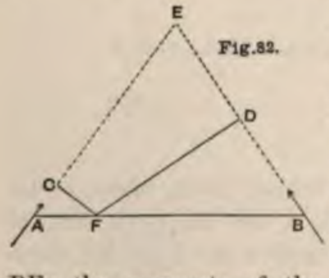


Fig.82.

are to produce equal and contrary rotational effects round F, so that there may be rest and statical equilibrium, their moments must be equal and of opposite sign, so that their sum = 0. This is the *Principle of Moments.*

If in Fig. 81 the forces at A and B acted at the ends of an immovable rod, there would be a reaction = 11 units spread over the whole extent of the rod, but more intense near the point A: if the rod were held fast only at one fixed point F, all the reaction (= 11 units) is concentrated at that point, if it be such a point that there is no tendency to rotation round it—*i.e.*, if the moments round the point of resistance = 0; if these be not = 0, the pressure on it is still 11 lbs., but the energy is partly spent in producing rotation round that point. If the reaction pass through the point round which the moments = 0, there is neither translation nor rotation, and hence the three forces are in equilibrium: these are, 10 units at A, 1 unit at B, parallel to the former, and 11 units at F, parallel but in the opposite direction.

Thus Fig. 83 is established as indicating the conditions of equilibrium of two parallel forces, P and Q; a third, R, equal to their sum, must act in the opposite direction at F, a point round which their moments vanish or are together equal to zero. If the two conditions be satisfied (1) that P + Q = −R, and (2) that the moments of P and Q round F be equal and opposite, there will be statical equilibrium: if the former be not satisfied there will be translation; if the latter there will be rotation; if both be violated there will be both translation and rotation.

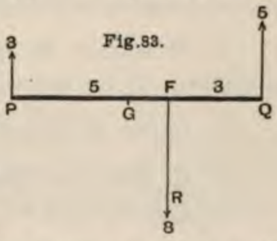


Fig.83.

Now let the point of application of the force Q be shifted to the right: the force P must increase in order that its moment may remain equal to that of Q. If Q be transferred to an indefinite distance, the force P would have to become indefinitely great in order to balance it. Two unequal forces, tending to produce rotation, may be balanced by a single force: P and R are balanced by Q. In this case P and Q have opposite and equal moments round F; R has no moment round F, its own point of action. There is equilibrium here between P, Q, and R; their moments round F are together = 0. So are their moments round any other point, as may be easily proved. In general, whatever point is considered, if there is to be no rotation round that point, the sum of all the moments of all the forces acting round that point, each taken with its proper sign, must be equal to 0.

Couples.—Two forces not directly opposed, and concurring in producing rotation, may sometimes, as in some of the Examples of Couples below, be called a Couple, whether these forces be equal and parallel or not. In another sense the word Couple is restricted to two equal and parallel forces causing rotation of a symmetrical body round its centre of mass, when that centre is situated midway between these forces and in the straight line joining their points of application. The standard definition of a Couple is, however, a generalised one, is independent of any fixed point of rotation, and is based upon the following considerations. Two forces always have a resultant and may be balanced by a third except in one case, viz., that in which the two forces are equal and opposed in their direction, but not opposed in the same straight line. This pair of equal forces constitutes a **Couple,** strictly so-called; and the Standard Definition of a Couple is—two equal and parallel forces opposed in direction but not in the same straight line.

A Couple, as thus defined, has the following properties:—

1. It cannot be balanced by any one force at any finite distance. If P become equal to R in Fig. 83, Q vanishes.

2. It can be balanced by an opposed couple.

3. It produces rotation round any point which may happen to be fixed, whether within the same plane or not.

If, on the other hand, two unequal forces (P and R of Fig. 83, in the absence of Q) tend to produce rotation, there is always at least one point such that, if this point be held fixed, there can be no rotation.

4. It produces no pressure upon the point fixed, wherever that point may be situated.

If, on the other hand, two unequal forces, opposed in direction but not opposed in the same straight line, act upon a body, there is a tendency to translation. If the handles of a copying-press be equally acted upon by both hands, there will be rotation simply; if the hands act unequally, the copying-press, with the table on which it is fixed, may be pulled or pushed over.

5. The algebraic sum of the moments round all points in space is the same.

These special properties have earned for this pair of equal forces the specific name of Couple; and an example of a Couple, as thus defined, is furnished by every case in which Reaction, though equal and opposite to Action, is not in the same straight line with it: and that whether the reaction be due to a support, to friction, or to the inertia of the body acted upon.

If a Couple be applied to a body of which no particle is held fixed, there will be rotation round a point which lies between the points of application of the two forces, and the position of which depends upon the form of the body. If the body be symmetrical and the forces symmetrically applied with reference to its centre of mass, there will be rotation round the centre of mass whether that centre be held fixed or not: and there will be no translation.

This is the case which, as in Magnetism, is of importance in this volume: the generalised definition is of importance in the study of the moving parts of machinery.

Moment of a Couple.—The sum of the moments round all points in space is the same. Take the midpoint: the whole distance between the forces is l: the moment of each force round the midpoint is $F\frac{1}{2}l$, where F is either force: both forces concur in producing rotation: the joint moment is $Fl = M$, the Moment of the Couple, the product of either of the equal forces into the distance between them.

Examples of Couples.—The action of the two hands on the handles of a copying-press is that of a couple: one pulls, the other pushes.

Examples abound in the muscular and osseous system.* Such are—the elbow joint, where the triceps pulls the olecranon process backwards, and the reaction of the articular surface of the humerus against the sigmoid cavity of the ulna constitutes the other member of the couple; the jaw in lateral chew-

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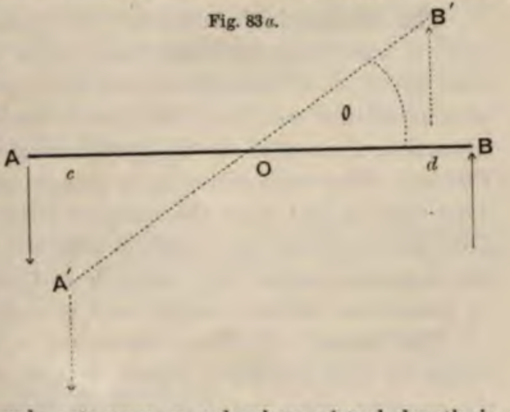
* Numerous examples may be found discussed in Hermann Meyer: *Die Statik u. Mechanik des menschl. Knochengerüstes*; Leipzig, Engelmann.

ing, where the external pterygoid muscle may pull one side of the jaw forward while the result of the action of the hinder fibres of the opposite temporal muscle, together with the corresponding muscles below the jaw, is to pull the opposite side of the jaw backwards; the weight of the head when a person stands in a very erect position is equivalent to a force acting along a line passing through a point a little behind the occipital condyles, and this, together with the reaction between the atlas and the occipital condyles, forms a couple which is equilibrated by the muscles of the front of the neck; the same weight of the head when it bends forward a little passes along a line a little in front of the condyles, and it forms with the reaction of the atlas a couple, which is balanced by the contraction of the muscles of the back of the neck: when these contractions slacken, as when a person is falling asleep, the head is rotated by the couple on a transverse axis, and it drops forward or backward according to the position in which it happens to be at the time when muscular contraction ceases to balance its weight.

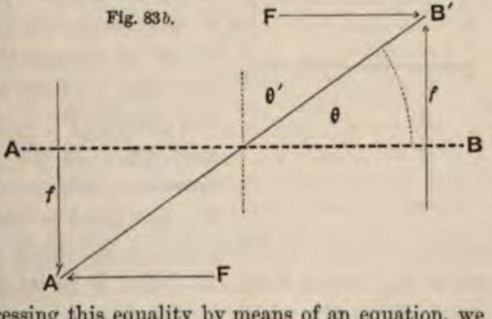
Equilibrium of Couples.—Let a couple consisting of two equal forces act always in one and the same direction, pulling the particle A (Fig. 83*a*) and pushing the particle B, and let A and B be so connected as to form a system capable of rotation round the point O midway between them. When AB is at right angles to the couple, the Moment of the Couple is equal to twice the product of either force into the arm OA or OB; it is therefore equal to either force $f \times$ the length AB. Let the system rotate into the position A′B′ making an angle θ with its previous direction; the couple acts upon a rod whose virtual length is reduced, by way of projection, to *cd* or AB cos θ. The moment of the couple is now f.AB cos θ, and when θ is 90° the moment of the couple is reduced to zero, and there is no further effect.

Fig. 83*a*.



Now let two similar couples act upon the same system AB, and let their directions be at right angles to one another and their actions opposed. There will be equilibrium when F (Fig. 83*b*) pushes B so as to diminish θ just as much as f pushes it so as to increase θ. At that moment, and in that position of AB, the effective moments of the two couples are equal. The one is f.AB cos θ; the other is F.AB cos θ'. Expressing this equality by means of an equation, we have—f.AB cos θ = F.AB cos θ' = F.AB sin θ. Hence $f \cos \theta = \mathrm{F} \sin \theta$ or

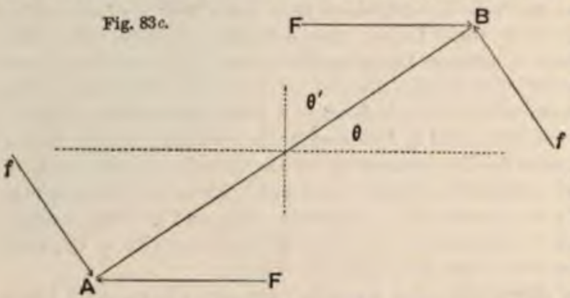
Fig. 83*b*.



$f : \mathrm{F} :: \tan\theta : 1$, where θ is the deflection from a position parallel to F, F. This proposition is applied in the construction of the Tangent Galvanometer.

Again, let the one couple *ff* have a direction always at right angles to the direction of AB, while the other, FF, has any direction whatsoever not at right angles to AB. AB is deflected through an angle θ from a position parallel to F, F. The f couple is $f.\mathrm{AB}$; the F couple is $\mathrm{F.AB}\sin\theta$ as before. These couples being in equilibrium, we have $f.\,\mathrm{AB} = \mathrm{F.AB}\sin\theta$ or $f : \mathrm{F} :: \sin\theta : 1$. This proposition is applied in the Sine Galvanometer.

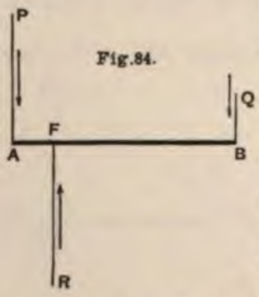
Fig. 83c.



The Mechanical Powers.—The principle of moments or—what is essentially the same thing—the principle that the work done by or on a machine = 0, or that, on the whole, there is no accumulation of work in a machine, is the key to the explanation of the action of many of the Simple Machines or so-called Mechanical Powers. The work done by a simple machine is equal to that done upon it, and upon the machine itself there is no work done. This is, of course, not strictly accurate; but simple machines are supposed, in the first instance and for the sake of theory, to be themselves without weight, and to work without friction.

The Lever.—This is a bar of any substance, rigid enough to retain its form under the forces applied to it. We consider it at the moment when the forces or pressures applied to it are all in equilibrium, so that there is no movement. If the point A be pressed down with force P, if the fixed point or Fulcrum be at F, and if the point B be pressed down with force Q, then round F the moments $(\mathrm{P.AF}) + (-\mathrm{Q.BF}) = 0$. If AF be shorter than BF, Q is numerically less than P; then the smaller Q can balance the greater P; a practical mechanical advantage. As an example of this take a crowbar: the man's strength is exerted at B; the fixed point is at F, and the weight of the body to be lifted acts downwards at A. Suppose the lever to be 42 inches long, the point F to be 2 inches from A, and the man's strength, which is competent to raise 56 lbs., to be exerted at B: then $\mathrm{AF} = 2$, $\mathrm{BF} = 40$, and $\mathrm{Q} : \mathrm{P} :: 2 : 40$; whence the man

Fig. 84.



can, by exerting at B a force of 56 lbs., keep a mass, weighing 10 cwt. and resting on A, from moving downwards; an effort a little greater will lift it.

On the other hand, a force equal to the weight of 1120 lbs. applied at P can only balance a weight of 56 lbs. at Q: here there is great mechanical disadvantage.

An example of this occurs in the case of forceps, with blades relatively long: the pressure which they can exert at the tip is relatively small, for each blade of the forceps is a lever supported at the hinge. Extremely long scissors do not cut so well at the point as near the joint.

But what is gained or lost in force is lost or gained in range of movement; for the work done ($=Fs$) by the one arm is always the same as that done upon the other.

Whatever the special arrangement of the two forces and the reaction, the principle is always the same, that the lever is studied at the instant of equilibrium, when round the fulcrum the sum of the moments $= 0$; then an excess either of the Force applied or of the Resistance, beyond their proportions at that instant, will cause rotation round the fulcrum.

There is a popular division of levers into three classes, which it is well to explain; Fig. 84 illustrates them all.

Class I.—The fixed point is at F in Fig. 84. A crowbar, a handspike, a pair of forceps, scissors, or shears, a poker, a common balance,—all these have the fulcrum or fixed point between the point of application of the force and that of the resistance.

Class II.—If the fulcrum be at A in Fig. 84, and the force be applied at B, the resistance overcome at F is numerically greater than the force applied at B, as is found by taking moments round A. $AF \times R = (AB \times Q)$. Examples of this are furnished by nut-crackers, where the resisting nut is nearer the hinge than the hand is; by the oar of a boat, in which the force is applied at the handle while the tip of the oar is approximately at rest, and the resistance of the boat is overcome between these; by a claw hammer used for extracting nails, where the fulcrum is at the end of the claw, the force is applied through the handle, and the resisting head of the nail is between these points; by a wheelbarrow, in which the fulcrum is at the axle of the wheel, the raising force is applied at the handle, and the resistance to be overcome is the weight of the substance in the barrow between the handle and the wheel.

Class III.—This is the same as the second class, except that the Force and the Resistance have changed places. As example of this we find that in a pair of tongs for sugar or for coal, in which the fixed point is at the hinge or the flexible end, the resistance is encountered near the end, and the force is encountered between these points. The pressure that can be applied by such an arrangement is comparatively feeble, while to overcome any given resistance the force applied must be proportionately very great. This is seen in opening a gate by pressing on it near the hinges; a considerable force has to be exerted. Such an arrangement, in which force is sacrificed in order to gain amplitude of movement, is of ordinary occurrence in the muscular system. The biceps is inserted into the radius at a point about one-sixth of the distance between the axis of rotation of the elbow joint and the centre of the palm of the hand. In order to raise a pound-mass in the hand, that muscle, if it acted alone, would have to exert a force which would directly lift 6 lbs.; but, on the other hand, the forearm has a range and rapidity of movement which it would not have had had the muscles been inserted in the position of greatest mechanical advantage, not to mention the inconvenience of having muscles extending from prominence to prominence of the skeleton like the rigging of a ship. The pectoral muscle of a bird, the deltoid muscle of man, his glutei muscles, actuate conspicuous examples of osseous levers of the third order.

Problems.—1. Two porters bear a burden, 56 lbs. in weight, by means of a bar of such length that the distance between shoulder and shoulder is 70 inches. The weight is suspended from a point 40 inches from the shoulder of one of the porters. What share of the burden is borne by the shoulder of each respectively?—*Ans.* This is a case of Fig. 84, in which the weight of the burden corresponds to R, and the upward shoulder-reactions to P and Q. There is no tendency to rotation round F, which is relatively fixed; hence the reactions at A and B must be such that their moments round F are equal. Hence the two equations, $P+Q=56$ and $40\ P=30\ Q$, give $P=24$, $Q=32$. The porter nearer to the burden carries 32 lbs., the one further from it carries 24 lbs.

2. A nut-cracker 6 inches long has a nut in it an inch from the hinge. The hand exerts a force equal to the weight of 4 lbs.: what is the total stress on the hinge?—*Ans.* The nut, as long as it does not yield, affords a fixed point: the total stress on the hinge = the weight of 24 lbs.

The Wheel and Axle.—The lever, when it has done work and raised a burden against resistance, moves into a position where the leverage and the effective component are both diminished, as seen in Fig. 80, if the force retain, or nearly retain, its original direction. If by any means matter could be so arranged that a lever would, when it had moved out of its position of greatest advantage, be replaced in the most favourable position by another lever, to which the burden and the force applied will be shifted, the apparatus thus constructed would in some respects be more useful than a simple lever.

This criterion is satisfied as regards levers of the first order by the Wheel and Axle. This consists of a large wheel or cylinder and a small one, both on the same axis, and capable of rotating together on that axis.

Each wheel may, if solid, be regarded as consisting of an infinite number of spokes. One of these spokes in the larger wheel, and one running in the opposite direction from the centre in the smaller wheel, together make up, when they are for an instant at right angles to the lines of application of the force and the resistance, a lever in the most favourable position. As soon as this has left the position of greatest advantage by reason of rotation of the system, its place is at once taken by another.

The weight of a large mass hung on the smaller wheel and that of a smaller mass hung on the larger wheel will balance one another, if their moments round the axis of rotation be equal. The weights may be replaced by a force applied at the circumference of the larger wheel, and a larger resistance balancing this at the margin of the smaller wheel. This is the principle of the **capstan** and the **winch**—the former used on ships for raising the anchor, the latter in use for drawing buckets up wells. In the former the spokes of the larger wheel are few, while the smaller takes the form of a cylinder or drum; in the latter the smaller takes the same cylindrical form, while the larger consists virtually of only one spoke, the handle, which is turned through all successive positions in a circle.

The wheel and axle is a statical instrument as long as its moments round the axis are together $= 0$; but when one of the moments is numerically greater than the other, there is rotation.

Wheelwork.—If a force be applied to the first wheel of a chain of wheelwork, so that it acquires an angular velocity, ω, and if the last wheel of the chain have, in consequence of this, an angular velocity $\omega_{,}$, the force which the last wheel can exert is, as compared with that which the first wheel alone might exert when running with angular velocity ω, as $\omega : \omega_{,}$. The principle holds good, whatever the nature or complication of the mechanism which intervenes between the first and the last wheel. In a crane or in a lathe arranged for metal-cutting, we see the wheelwork so devised that the last axis moves very slowly, and with a correspondingly great power of overcoming resistance.

The Inclined Plane.—The mechanical advantage of this machine depends on the principle of the resolution of a force into its components.

When a body is pushed up an inclined plane by a force or push just sufficient, and no more, to prevent it from moving down the plane in obedience to gravity, there is equilibrium between three forces—viz., this Force, acting along the slope of the incline, the Weight of the body acting vertically, and the Reaction between the body and the surface of the plane, acting at right angles to the latter. These three forces can be represented by the sides of a right-angled triangle, in which the

Hypotenuse : Height : Base
as Weight of body : Push up the plane : Reaction.

If the push be applied horizontally, the three forces in equilibrium—which are the Weight of the body downwards, the

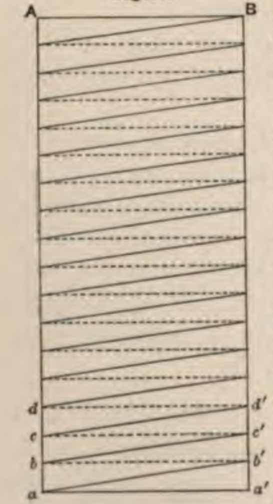
Reaction at right angles to the surface of the plane, the Push up the plane applied horizontally—will have the relation of the sides of a right-angled triangle, in which

Hypotenuse : Height : Base
as Reaction : Horizontal Force : Weight of body.

Velocity of fall down a frictionless inclined plane.—If a body fall down an inclined plane, the potential energy lost by it in virtue of its vertical descent h is mgh; there is none gained or lost in virtue of horizontal motion, in which there is no work done by or against gravity. The kinetic energy acquired is $\frac{1}{2}mv^2$. These must be equal; hence $v = \sqrt{2gh}$, the same speed as would have been acquired by a vertical fall. In the latter case, however, the direction of motion would have been directly downwards; in the former it is in the direction of the plane. The reaction of the plane has not modified the speed of the fall; it has modified its direction. The speed at which the body is moving down the plane after effecting a vertical descent h is thus the *same velocity* as that which it would have acquired if it had fallen vertically through that height h. But it has travelled through a greater space in order to attain this speed. The acceleration down the plane is therefore smaller; and a body falling down a smooth slope of 1 in 20 would take a *greater time* to reach the bottom than it would take to fall vertically through an equivalent height, in the ratio of 20 : 1.

If the body travelled down a succession of inclined planes, or down a curve, the same ultimate velocity would be acquired: the reaction of the curve alters the direction but not the speed. If it *rolled* down the plane or curve, a part of its energy would be rotational; it would acquire a correspondingly smaller velocity of fall.

Fig. 85.



The Screw.—In Fig. 85, across the rectangular parallelogram Aaa'B are drawn equidistant lines aa', bb', cc', dd', etc., at right angles to Aa and Ba'. The lines ab', bc', cd', etc., are drawn as there shown. If the surface Aaa'B be wrapped round a cylinder whose circumference is equal to AB, the line Aa will coincide with Ba', and the lines ab', bc', cd', etc., will form a continuous spiral line $abcd$ round the cylinder, and will trace out the form of the thread of a screw whose *pitch* is ab, the distance between the equidistant lines ab', bc', etc. Hence the thread of a screw is seen to correspond to a narrow inclined plane wrapped round a cylinder.

A screw is usually used as a mechanical power for the sake of moving a body through a small space with great force, as in the copying-press: in this the mechanical advantage is due to two causes :—(1) in accordance with the principle of moments, the resistance borne by the thread of the screw is at a less distance from the axis of rotation than are the points of application of the pull and push of the two hands to the handle; and (2) the resistance directly encountered by the point of the screw has merely a component effective along the thread of the screw, this component varying with its pitch. In fact, the screw is an inclined plane in which the force is applied horizontally, the resistance vertically, and the reaction at right angles to the thread; and we have seen that in this case the

Force : Resistance overcome : : Height of incl. plane : its Base.

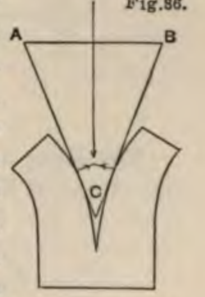
Hence, the less the pitch of the screw—the greater the number of turns to the inch—as well as the greater the leverage of the handles, the greater the mechanical advantage that can be derived from its use.

Problem.

What is the mechanical advantage which can be obtained in a copying-press of the following construction :—Effective radius of the arms 12 inches—screw $1\frac{1}{2}$ inches thick—pitch $\frac{1}{8}$ inch ?—*Ans.* The hands move through 1 inch, while the point of the screw descends $\frac{1}{128}$ inch. $\therefore F^1 = 128F$.

The Wedge.—A wedge, as seen in Fig. 86, is practically a double inclined plane movable between resistances. During a blow there are at work (1) the driving force acting downwards through the centre of AB, (2) a reaction at right angles to AC, and (3) one at right angles to BC; these latter being through the point of contact, or, if there be contact over the whole of AC and BC, through the centre of these lines. These must cross in a point if the equilibrium, which subsists the instant before the wedge commences to move, be considered; and they must be represented by the sides of a triangle. Round the point at which they meet the moments = 0.

Fig. 86.



Pulleys.—These well-known objects are wheels, solid or spoked, mounted in a framework or block, which is either movable or fixed to a beam or other solid attachment. The simplest use of a pulley is to change the direction of application of a force applied to a cord. The total tension of the cord on one side of a pulley would, if there were no friction, be equal to that on the other side of it, while the motion of the cord on the one side of the pulley is in any case equal to that on the other side, whatever be the size of the pulley, and whatever be the amount of the flexure to which the cord is subjected. A single pulley thus produces no mechanical advantage if it simply serve this purpose; but if this pulley be itself movable against a resistance—if, for instance, a heavy mass be suspended from it, while the other end of the cord is attached, say, to the roof—a movement of the suspended mass through one inch would correspond to the pulling in of two inches of cord, and the hand exerting the force would move through a space twice as great as that traversed by the pulley. Thus, by the intervention of a cord, one end of which is fixed, and of a single pulley round which the cord is bent, a resistance 2 may be overcome by a force just greater than 1. This principle of reduplication of a string round a pulley is taken advantage of, and practically turned to use in combinations of pulleys, in any of which the mechanical advantage is the numerical ratio of the amount of string pulled out to the corresponding movement of the body pulled upon.

The Bell-crank.—If in Fig. 87 the rigid body ABC, which can rotate round A, have a force applied to it at C, its tendency to rotate round A may cause motion at B against a resistance R. The principle of moments shows us that, whatever the ornamental shape of the crank, the relation of the resistance R overcome to the force P exerted depends on the relative lengths of the effective arms AB and AC.

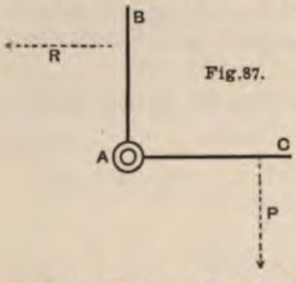


Fig. 87.

The Knee.—In Fig. 88 if two bars be jointed at O, and their ends A and B be confined to a given straight line CD, a movement of the hinge O athwart the direction of the line CD corresponds, especially when AO and OB are nearly in the same straight line, to a relative motion of A and B, which is proportionately very small. Hence A and B are thrust asunder with a force greater than that which acts upon the hinge and presses it into its central position. This contrivance is used in some copying-presses, hand printing-presses, and railway-ticket endorsing machines. It is seen in the human knee: when the leg is straightened out a vigorous thrust upwards and forwards is given to the body, and a corresponding one downwards and backwards to the earth on which the foot presses.

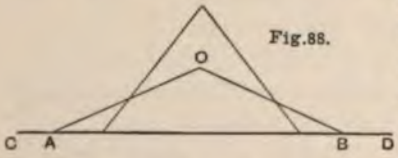


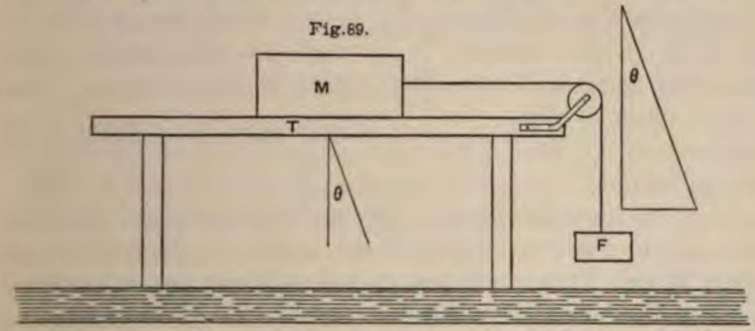
Fig. 88.

A wire stretched between two points, and loaded by a weight or by the pressure of the wind, is a *knee* whose action is reversed. It tends to pull together the two supports to which it is fixed ; and if there were any movement of these supports, it would be small in comparison with the corresponding movement of the centre of the wire. Thus the force acting upon the supports and resisted by them is greater than that acting upon the wire itself.

FRICTION.

Friction between Solids.—If a mass M be supported on a table T, and the weight of a mass F be employed to pull it towards the edge, the body M will not commence moving unless the mass of F bear a certain proportion to that of M. This proportion, a fraction less than unity, is the **coefficient of statical friction**, is usually represented by the symbol μ, and has to be experimentally found.

Fig.89.



$$F : M :: \mu : 1, \text{ or } F = \mu M.$$

Experiment has shown that the coefficient of friction depends upon (1) the nature of the substances of which M and T consist, (2) the smoothness or roughness of their surfaces, (3) the presence or absence of lubricating substances—oil, soap, blacklead—between them.

The coefficient of friction is the same between the same two substances, whatever the mass of M—that is to say, the mass F must bear to the mass M always the same ratio; thus the Friction itself—the resistance to sliding between M and T, overcome by the weight of F—varies directly with the weight of the mass to be moved, and therefore varies directly with the pressure between the given surfaces. Again, the mass M may be distributed in any way, and the contact between M and T may be by a surface large or small. If the area of contact be diminished, the pressure on each unit of area will be increased, and therefore the friction on each unit of area will be proportionately greater; but the number of units of area over which this resistance is exerted is correspondingly smaller, so that the mass F, the weight of which is just competent to pull the mass M towards the edge, remains the same. Hence the total friction is independent of the area of contact between two given masses; but the friction per unit-area of contact varies directly as the pressure per unit-area.

Limiting Angle.—In Fig. 89 we may consider the equilibrium subsisting at the instant before the mass M begins to slide on T. The mass M is at rest under (1) its own weight acting vertically, (2) the force F acting horizontally, and (3) the reaction R between M and T. This is inclined at an angle θ to the vertical. The horizontal component is to the vertical as $\tan\theta$ is to 1. But $\frac{\text{horiz.-compon.}}{\text{vert.-compon.}} = \frac{F}{M} = \mu$; $\therefore\ \mu = \tan\ \theta =$ the coefficient of statical friction.

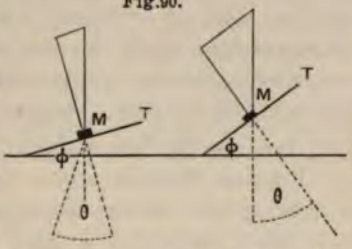
Any force applied to M, or any set of forces whose resultant acts on it in a line making with the normal to the surface between M and T an angle less than θ, will not produce sliding. If there were no friction, any force applied to M in a direction differing in the least degree from the normal would have a component which would produce sliding; but friction makes it necessary that a force should be wide of the normal to the surfaces of contact by something more than the Limiting Angle θ before sliding can be caused by it.

If a flat piece of wood be placed on a table, and pressed against the table by a stick held at right angles to it, it will not slip: the stick may be inclined somewhat, and still it will not slip: when the stick is inclined more than a certain amount the piece of wood commences to slip on the table.

If M be pressed against T by a force P acting in a line which makes any angle ϕ with the normal, we resolve it into two components: one at right angles to the surface, $= P\cos\phi$, produces pressure between these surfaces; the other, $P\sin\phi$, is the component which tends to produce sliding. If $\frac{P\sin\phi}{P\cos\phi} = \frac{\text{sliding component}}{\text{pressure}}$ be less than or equal to μ, there will be no sliding; if it be greater than μ (*i.e.*, if ϕ be greater than θ), sliding will occur. If there be no sliding, the component which tends to produce it sets up a condition of stress between the particles of the two bodies, and thus a reaction is set up by molecular forces, equal and opposite in that direction. The amount of this reaction depends on the molecular conditions of the substances, and is only to be determined by experiment.

Angle of Repose.—Suppose a mass to be placed on a table, and the table to be tilted up until the mass is just about to begin to slide. At that moment there is equilibrium. To what angle can the table be tilted up? Let μ be the coefficient of friction between the mass and the table. The one will then slip on the other if a force be communicated between them in the direction of a line making, with the normal to the surfaces of contact, an angle greater than θ. In Fig. 90 are shown two positions of the table T bearing the mass M. In both the dotted lines indicate the limiting angle θ; in both the weight W of the mass M acts vertically downwards. The equilibrium, then, is between (1) the pressure produced between the mass and the table, at right angles to the latter, $= \text{W} \cos \phi$; (2) the sliding component, $= \text{W} \sin \phi$; and (3) the reaction of the table, equal and opposite to W, the weight of the body. In the first case of Fig. 90 the reaction between the body and the table falls within the limiting angle, and there is no sliding: the sliding component is less than $\mu \times$ the pressure. In the second case the limit is reached; the sliding component is just equal to $\mu \times$ the pressure, and is just able to balance the friction. If the table were tilted up any farther, sliding would occur. But in the second case it is easy to show that the angle ϕ, to which the table has been tilted, is itself equal to θ, the limiting angle. Hence the angle θ is also called the Angle of Repose. Upon the coefficient of mutual friction depend in this way the angles at which heaps of sand, of grain, and the like, will adjust themselves when poured out and allowed to find their own position.

Fig. 90.



This angle θ has, then, three properties: (1) the Coefficient of Statical Friction, $\mu = \tan \theta$; (2) θ is the Angle of Repose; (3) it is the Limiting Angle.

Kinetical Friction.—After slipping has commenced, the acceleration produced by a given force is less than it would have been if there had been no friction. The force is apparently diminished in a certain proportion: it has lost a fraction k of its amount; the velocity produced is diminished in that proportion; energy is wasted, but not destroyed, in the form of heat. The "coefficient of kinetical friction," k, is, however, not so great as μ, the coefficient of statical friction.

Influence of Duration of Contact.—When two bodies have been in contact for a long time, the particles of each develop such relations to one another that μ, the coefficient of statical friction, increases with the duration of contact; it is more difficult to make a body slide on another with which it has been long in contact than on one on which it has freshly been placed. When one surface slides on another, the particles seem to have no time to assume such relations, and the coefficient of kinetical friction is comparatively small; at very great velocities it is even somewhat smaller than at ordinary velocities; but when the velocity of sliding is very small, the condition approximates to one of relative rest, the coefficient of kinetical friction approximates to that of statical friction, and a larger proportion of energy disappears at very low speeds than at high.

Friction, then, is not a force: it is a resistance or reaction; it corresponds to the absorption of energy by its transformation into a molecular kinetic or potential form: in the former case the energy absorbed may perhaps assume the form of the energy of electrical condition, but ultimately it takes that of heat; in the latter case it corresponds to a stress between the particles of the bodies, to pull which asunder requires a certain amount of force. But Friction acts and may enter into calculations as if it were a Force, never coming into action unless force be applied, always tending to prevent slipping, and its total amount is either proportional to the total pressure between the bodies in contact, if there be rest, or else to the velocity of motion, if there be relative motion.

So for friction between solids; but **Friction of Solids against Liquids** depends directly upon the extent of surface exposed; it further increases with the velocity, for it is proportional to v^2. $F \propto (\text{area} \times v^2)$. It is found that well-oiled bearings present this fluid friction; perhaps the friction of sharp skates against smooth ice may be found to be in the same category, the ice being melted as the skate runs.

At ordinary working velocities of axles in their bearings the coefficient of kinetical friction is, between metal and metal, approximately constant. If castor oil be used as a lubricant, the coefficient of friction is, at low speeds, very small; but it increases rapidly as the speed rises. If water or thin petroleum oil be used as a lubricant, the friction at speeds beyond a certain limit is very small; but at speeds whose average is below that limit there is alternate "biting" and slipping. Hence for axles at low speeds, thick oils; for high speeds, thin oils.

Belting.—There is a very interesting and familiar case in which friction serves as a means for the transmission of energy—that is, transmission by machine-belting. A rotating wheel has a belt tightly drawn over it, as also over a second wheel, not too near. The belt must be tight, so that there may be more pressure between the leather and the iron. If the wheel be very small or the motion be very rapid, the mutual pressure between the leather and the iron may be lessened by the inertia of the belt, which tends to pass the wheel and to be carried on. The friction is proportional to the pressure between the wheel and the belt, for the relation of the wheel to the belt is practically one of rest, though the surfaces in contact are changed from instant to instant. There being no slipping, the friction is statical, and is proportional to the pressure. Though the belt and the wheel do not move relatively to one another, they move relatively to surrounding objects, and the belt is set in motion. If the second wheel be free to rotate on an axis, portion after portion of its rim tends to remain at rest relatively to the leather, and the second wheel is set in motion round its axis. The tension of the belt, that is, the Total Tension, is greater nearer the driving power than it is on the other side of the wheel driven. This is because energy has been taken up in preventing relative motion of the belt and the driven wheel, or—another mode of expressing the same thing—in producing absolute motion of the latter. This difference of tension is equivalent to a force directly applied to the rim of the wheel. Thus the kinetic energy of the driving wheel is in part imparted to the leather belt and the rotating wheel; these come to form a part of the same system with the driving wheel; the latter cannot rotate so fast when it is driving a second wheel as it can when not doing so, the same energy being supplied to it; and thus energy is transmitted.

Such is the theory of belting when there is no slipping; but in practice there is always some slipping. The part of the belt in front of the pulley is under greater tension than the part behind; it is therefore more stretched out and assumes a greater length; and this involves slipping, which causes a loss of energy spent in deforming the belt, and ultimately transformed into heat in the belt; a loss which in the case of leather belting is appreciable, but which in the case of indiarubber belting is very considerable. (Osborne Reynolds.)

The efficiency of belting is greatly increased when the rim of the wheel is lined with leather, hair side outwards, the hair side of the leather belt being inwards.

Resistance to Traction.—When a heavy body is moved along a frictionless level path, no work is done by or against gravity. The force required to set the body in motion is proportional to its mass and to the velocity imparted in a unit of time. $F = ma$. But kinetic friction would apparently diminish the force by a fraction k. Thus it is equivalent to a retarding force of kF or kma. Thus the friction is proportional to the mass m, and therefore to the weight of a vehicle. It is said, then, that the friction overcome by the engine of a railway train is equal to a resistance of so many pounds to each ton of the train's weight. In this way, a train of 100 tons may be pulled along a level road by an engine which overcomes a resistance of 50 lbs. per ton, and thus the work to be done by the engine is the same as it would have been had the load been 5000 lbs. drawn vertically upwards at the velocity with which the train is travelling.

The resistances are said to be so many pounds per ton, nothing being said of the velocity. This is because, within wide limits, the Coefficient, k, of kinetical friction is practically independent of the velocity; though, when the speed becomes very small, the waste of energy occasioned by friction becomes proportionately large, the converse holding good at high speeds.

When a horse draws a cart up a hill the slope of which is 1 (vertical rise) in 10 (road-length), the principle of the inclined plane shows us that the animal has to put forth an effort corresponding to lifting a weight of $\frac{1}{10}$ the mass moved; but further, it has to overcome the frictional resistances, which are equivalent to an extra weight.

When a man walks his knee is straightened, and his body is projected forwards and upwards at each step. The impulse may be resolved into two components: one upward, which may raise the centre of gravity of the body about an inch or an inch-and-a-quarter; one forward, which has to overcome the intermittent resistances introduced by the stoppage occurring at the end of each step, when the foot of the opposite side strikes the ground. This is an intermittent frictional resistance.

If there were no friction between the wheels of a railway train and the rails of the railroad, there would be slipping but no progress. Friction between the wheels and the rails—friction proportional to the weight of the vehicles—has the effect of preventing slipping; but (as in the case of belting) this corresponds to the maintenance of a state of rolling adhesion, under which each wheel is turned round and rolls upon the rail.

The mechanical powers, when friction is taken into account, give rise to several problems; but the physical principle underlying the whole subject is the same, that friction acts in the same way as a force opposed to sliding, and that it is proportional to the pressure or to the actual velocity of sliding, as the case may be. As an example let us take this question: A copying-press is pressed hard down on the copying-book; the hands are removed; the book remains under pressure;—why does the screw not come up? The reaction of the book has a component up the line of the thread of the screw; this would tend to send up the screw, but it is counterbalanced by friction, acting as a resistance in the opposite direction down the thread. It may be left to the reader to show (1) that the better the screw is oiled the less able will it be to retain its hold; and (2) that a screw of too large a pitch (one the turns of whose thread are too far apart) may fail to hold the book down. The upward pressure is resolved into two components, of which the one along the thread of the screw must not be greater than $\mu \times$ the component at right angles to the thread; if it be greater than this the screw will slip upwards in its nut.

Problem.

If an unsymmetrical wedge have a force applied at the centre of one side, and its reactions at the centres of its other sides (the ends being neglected), and if there be friction: prove that there is a point round which the sum of the moments $= 0$; that that point is not the centre of figure; and that the moments round the centre of figure will indicate that there is a rotation of the whole wedge in such a sense that the centre of figure is carried forward under the action of the driving force farther than is that point round which the wedge rotates.

Friction on a raindrop.—A raindrop falling *in vacuo* through a height h feet would acquire a velocity $v = \sqrt{2gh} = 8{\cdot}249\ \sqrt{h}$ feet per second. Its starting point might easily be so distant that a blow from a raindrop travelling under these circumstances would be fatal to any living being struck by it. But at every instant of its course it is subject to kinetic friction tending to reduce its velocity at the instant; at the same time it is subject to the accelerating force of gravity: and thus there must be a certain velocity at which the retardation of friction and the acceleration due to gravity will balance one another, and the drop, if it once attained this speed, would retain it, and fall with a constant velocity. This happens in the case of the raindrop, and also in the case of a stone or granule falling in deep water.

Friction of a rope round a post.—This is familiar in the example of a rope passed round a post on a quay in order to hold fast a ship. If any little part or element of the rope be considered, it will be seen that the friction is proportional to the pressure of that part of the rope on the post, and to a certain extent it tends to prevent slipping; in this it partly counteracts the tension of the rope; the total tension communicated to the end of the element of the rope further from the applied force is less in consequence of this than it would have been if there had been no friction. If we trace out in this way, along the rope, the gradual diminution of tension, we find that the tension, after a complete turn of the rope round the post, dwindles down to a constant fraction $1/c$ of the original tension. Between a flexible rope and wood this constant fraction is about $\frac{1}{9}$; hence a force of 1 lb. could prevent a force of 9 lbs. from pulling a flexible rope round a post round which it had been passed so as to form a complete turn. After two turns the tension becomes $\frac{1}{81}$; after ten turns it becomes $1/9^{10} = 1/3486,784401$. Hence a man exerting a force of 1 lb. at the end of a rope wound ten times round a post would be able to resist a pull of about one-and-a-half million tons. Of course this is not attained in practice, because no ropes are thoroughly flexible, and none are strong enough to stand such stresses; but a perfectly flexible rope would diminish tension in this manner without reference to the diameter of the post round which it is wrapt.

Rolling Friction.—When a ball is set to roll on ice it goes farther than it can on a wooden floor; farther on that than on a carpet; farther on a carpet than on grass. The rotation is, however, at length stopped. To produce a rotation a couple, or something equivalent to a couple, is needed; to stop rotation a single force is competent. The greater the moment of the force opposed to the rotation, the sooner will the rotation be stopped. The rotation of the ball is stopped by the resistance of friction. This is equal to a small force acting at the surface of the ball, and bearing a constant ratio to the pressure produced by its weight. This ratio is very small. The resistance, then, to a wheel rolling along the ground is much less than the resistance to the same object pressing against a brake. It is very much easier to move the trunk of a tree by setting it on logs which roll on the ground and under the trunk, than it is to drag it along the ground. There is less friction at a well-oiled hinge or well-lubricated joint than there would be in any other contrivance used for transferring a given mass from one position to another. If a wheel, instead of having its axle supported in bearings, have it supported on a couple of pairs of Friction-wheels which are free to rotate, the axle as it turns does not rub against a fixed bearing, but the friction-wheels yield and rotate, so that the rotating axle is supported by surfaces which travel at the same rate with it, and the friction is accordingly very small.

The friction is affected by the relative softness of the surfaces in contact. (Osborne Reynolds). An iron wheel rolling upon an indiarubber plane will raise up before it a little mound of indiarubber; and if it stop, this little mound will recover its form and drive the wheel backwards, thus making it oscillate. The friction of iron upon indiarubber is thus ten times as great as the friction of iron upon iron. Conversely an indiarubber tire is deformed in the same way against a hard surface. This tendency to thrusting forward the upper layer of both, but especially the softer substance, results, in the case of iron railway rails, in the wear of the rail by scaling off of successive laminæ of iron.

A similar result may influence most cases of ordinary friction, as in the spreading of putty with the thumb, to take an extreme case.

Friction-Dynamometers.—Friction may be utilised as a means of measurement of Rate of Doing Work. Suppose a cord passed round a revolving pulley; the two ends of the cord pass away from the pulley, both vertically or otherwise in line with one another; the lower end is stretched by a weight mg dynes; the upper end pulls upon a fixed spring and imparts to it a strain which indicates a total tension of N dynes. The weight mg is so great and tightens the string so much that the whole of the energy of the pulley is spent in overcoming friction, and the pulley stops at once when the driving power is withdrawn. The string wraps round the circumference of the pulley, *i.e.*, $2\pi r$ cm.; the velocity of that circumference in passing any point of the string is, if the pulley rotate n times per second, $v = n \,.\, 2\pi r$ cm. per second; the force overcome, f, is the difference between mg and N, *i.e.* $(mg - \mathrm{N})$ dynes; the product fv is therefore equal to $n \,.\, 2\pi r.\ (mg - \mathrm{N})$. This product fv measures in ergs the work done per second by the revolving pulley, the Rate of Doing Work of the pulley (p. 41). Instruments of this class may be graduated so as to indicate, by the amount of distortion of a spring, the working value of a steam-engine in horse-powers: the whole power of the engine is turned on to the dynamometer for a brief period, and the scale-reading of the spring observed, as well as the speed of rotation of the pulley.

Activity in Belting.—The transmission of energy by belting is subject to the law that Activity $= \mathrm{F}v$; v being the velocity at which the belt runs, and F the tension (*i.e.* total tension) along the belt; this tension being, essentially, the difference of tension between the outgoing and the incoming parts of the belt. Let, for instance, the speed of the belt be 400 feet a minute or, say, 200 cm. per second; and let the effective total tension on the belt be equal to the weight of 100 kilogrammes or to 98,100,000 dynes; then the Activity, or rate of transmission of energy $= \mathrm{F}v =$ Tension × Velocity $= 98{,}100{,}000 \times 200 = 19620{,}000000$ ergs-per-second, or about $2\frac{1}{2}$ horse-power.

If the velocity be very great, the tension may be small. Thus let the velocity be 6000 feet per minute or, say, 3000 cm. per second, a speed which has been attained in practice; and let the desired activity of transmission be 100 horse-power, or 745,948,005,000 ergs per second; we have Activity $=$ $\mathrm{F}v$, or $745{,}948{,}005{,}000 = 3000\ \mathrm{F}$; whence $\mathrm{F} = 248{,}649{,}335$ dynes, and the tension F is therefore equal to the weight of $248{,}649{,}335/981 = 253{,}465$ grammes or 253·465 kilogrammes. Such a tension would (since steel has a "breaking weight" of 33 tons per square inch) be barely able to snap a steel wire of $\frac{1}{4}$ cm. diameter; whence a slender steel wire-rope may, at very high speeds, be safely used to transmit large amounts of energy; a conclusion which experience has confirmed.

CHAPTER VII.

ATTRACTION AND POTENTIAL.

WHEN one body in contact with others forms with them a system, a Conservative System, which will be put in a condition of stress when the bodies are removed from contact with one another, these bodies are said to be Attracted towards one another; and if they be fixed in such a position that the stress of the system is permanent, the condition is one of statical equilibrium. When a spring is drawn out and fixed by a catch there is equilibrium between the recoil of the spring and the molecular forces within the catch, which resist its deformation; when a heavy stone is placed on a wooden table, if the table be strong enough to support the stone, there will be equilibrium between the weight of the stone and the resistance to crushing offered by the wooden support. If the spring be released it will fly back: if the supporting table be removed the stone will fall. In the former case there is a visible medium, the spring, the elasticity of which comes into play; in the latter case there is no such elastic medium visible.

There is no direct analogy between the two cases. In the former, the greater the displacement the greater the stress; in the latter, the greater the mutual distance the less the mutual attraction.

Let us consider attraction, which, whatever may be its cause, obeys the particular law (the so-called "Law of Inverse Squares") that a mass M and a mass m are attracted by each other with a force which depends on each mass directly, and on the square of the distance between them inversely. Then $f \propto \mathrm{M}m/d^2$; that is, $f = k.\mathrm{M}m/d^2$.

In a system of particles of this kind we must further assume —and experience warrants us in so doing—that every particle is connected with every other particle by an independent attraction; then the total attraction of one set of particles for another set of particles has to be found by a process of summation. To effect this summation the aid of the Integral Calculus has in general to be called in; the process is, however, of this kind:—The mass and the distance of each particle from every other being taken into account, the attraction between each particle and every other particle is to be separately found, and the whole attractions are then to be summed up. In simpler cases, mutually attracting masses may be considered as acting at their centres of figure; then the mean distance between two such masses is the distance between their centres of figure, and each mass may be supposed to be concentrated at its centre of figure.

Attraction in particular cases.—(1.) A hollow spherical shell, whose thickness is infinitesimal, attracts an external particle as if all its mass were gathered at its centre. Its area is $4\pi r^2$; the amount of mass per unit of surface (its "surface-density") $= \sigma$; its mass M is therefore $4\pi r^2\sigma$. It acts on mass m placed at a mean distance a from the centre of the shell as if the whole mass $4\pi r^2\sigma$ were at that centre, and the attraction $f = k.4\pi r^2\sigma.m/a^2$. If the attracted particle be of unit-mass, $m = 1$, and the attraction of the shell on a unit-particle is $k.4\pi r^2\sigma/a^2$.

If the external particle be *just outside* the shell, so that its distance a from the centre is practically equal to r the radius of the shell, $a = r$ and $f = k.4\pi\sigma$.

(2.) A solid sphere and an external particle will in the same way act on one another as if all the mass of the sphere were gathered at the centre. The attraction between a sphere whose radius is R and whose amount of mass per unit of volume ("volume-density") is ρ, and a unit particle at a distance a from the centre of the sphere, is $k.(\frac{4}{3}\pi R^3)\,\rho/a^2$ (for the volume of a sphere of radius R is $\frac{4}{3}\pi R^3$): if the particle be just on the surface of the sphere the attraction is $k.(\frac{4}{3}\pi R^3)\,\rho/R^2 = k.\frac{4}{3}\pi R\rho$.

(3.) An attracting spherical shell of any thickness, if this thickness be uniform, has no action whatsoever on a heavy particle contained within it. For every area of the shell on one side of the particle which may attract it in one direction, there is another on the other side attracting it in an opposite direction; and the one exactly balances the other, for what advantage the one area may have in size the other exactly makes up for in proximity. Thus it is not possible to find any area of the sphere, the attracting effect of which on the particle within the sphere is not exactly counterbalanced by the opposed attracting effect of an opposite area. The particle, attracted equally in every direction, remains at rest.

No other law than that of the inverse square of the distance will give this entire absence of effect within a hollow spherical shell of uniform thickness, as will easily be found on trial.

If the shell have any other form than the spherical, it must, in order to retain this absence of interior effect, have a thickness which is other than a uniform one. For example: an ellipsoidal shell whose inner ellipsoidal surface is concentric and confocal with the exterior ellipsoidal surface has a thickness which at any point is proportional to the shortest distance between the centre and the tangent to the ellipsoid touching the point in question (see Fig. 197); and such a shell has, under the law of inverse squares, no action at any point within it. Such a shell is thickest at the extremities of its major axis.

If a point be just outside such a shell, at a point where the surface-density is σ, the attraction just outside $= k\,.\,4\pi\sigma$; just inside, it $= 0$; and thus the particle, if it pass through the shell, passes from a field where the attraction on a unit particle is $k\,.\,4\pi\sigma$ to another where the attraction differs from its former value by $k\,.\,4\pi\sigma$.

(4.) Between a particle and an arc of a circle of radius r, at the centre of which the particle stands, $f = k.(r\sigma/r^2) \times 2 \sin \frac{1}{2}$ angle subtended $= k.(\sigma/r).2 \sin \frac{1}{2}\theta$; just as if a mass equal to that of a chord of the arc, with the same density as the arc, were concentrated at the midpoint of the arc.

(5.) Between a semicircle and a particle at its centre ; the angle subtended is 180° ; the force is $k.(\sigma/r) \times 2 \sin 90^\circ = k.2\sigma/r$.

(6.) Between a definite line AB and a unit-particle at D opposite the centre C of the line AB. Draw AD and BD. With centre D and radius DC draw a circular arc limited by the lines AD and BD. The line AB and this circular arc exert the same attraction on a unit-particle at D : and the attraction of the circular arc we know from (4.).

(7.) Between an indefinite line and a unit-particle at a distance r from it ; the angle subtended is 180° ; the force is, as in (5.), equal to $k\,.\,2\sigma/r$.

(8.) Between a hemispherical shell and a particle at its centre, the attraction is $f = k\,.\,2\pi\sigma$, and is independent of the radius.

(9.) Between an indefinite plane and a body at a finite distance x from it, the attraction is the same as in case 8, and $f = k\,.\,2\pi\sigma$. This is independent of the distance x. If the particle pass through the infinite plane it passes to a region where the attraction is $-k\,.\,2\pi\sigma$, because it acts in the opposite direction, and therefore differs from its former amount by $k\,.\,4\pi\sigma$.

Since the force acting is independent of the distance x we may make $x = 0$. The unit-mass acted upon is now a unit-mass of the substance of the plane itself, and it is acted upon by a force $k\,.\,2\pi\sigma$. The matter distributed over a sq. cm. is σ, and this is acted upon by a force $(k.2\pi\sigma).\sigma = k.2\pi\sigma^2$. The force acting on the substance of the plane itself is therefore $k\,.\,2\pi\sigma^2$ per sq. cm., and is at right angles to the surface.

Convention as to Attraction and Repulsion.—A force of this kind is conventionally said to be Positive when its effect is to separate (or to increase the distance between) the bodies by whose relative motion it is manifested. Thus a repulsive force is positive ; an attractive, which diminishes the distance between two masses, is negative. This convention is opposed to the ordinary use of speech.

Potential Energy in case of Repulsion.—If, as the phrase goes, two bodies repel one another, and if one or both of them be free to move, their mutual separation may be carried on to an infinite distance. As long as it is still possible under any specified circumstances for the bodies to become still farther separated by their mutual repulsion, there is still some potential energy in that system which consists of the two masses (together with the intervening medium, if there be any such) ; the mutually repelling bodies must therefore be separated to an infinite distance from one another before their repulsion can cease to act, before the Potential Energy of the system becomes reduced to zero. When, on the other hand, the bodies which repel one another are in contact, the Potential Energy of the system is as great as it can possibly be.

Work done by Repulsion. — If two masses, m and $m_{,}$, situated at a distance r from one another and repelling one another with a force $= k.mm_{,}/r^2$, be allowed to separate through a little distance δr, the system has exchanged a configuration in which the mutual force was $k.mm_{,}/r^2$ for one in which it has been diminished to $k.mm_{,}/\overline{(r + \delta r)^2}$; and the work done by the repulsion is the product of the mean force into the space δr. If δr be taken small enough this product becomes, with an indefinitely close approximation to accuracy, equal to $(k.mm_{,}/r^2) \times \delta r$. If the bodies increase their distance, making the distance $(r + \delta r)$ grow to $(r + 2\delta r)$, the work done in this stage is $\left(k.mm_{,}/\overline{(r + \delta r)^2}\right) \times \delta r$. Summing up by means of the Integral Calculus the work done by the repelling force in separating the bodies, stage by stage, from a mutual distance r to a distance R, we find that it is $k.mm_{,}\left(\frac{1}{r} - \frac{1}{\mathrm{R}}\right)$.

This proposition may be otherwise presented in the form of a positive statement. The work done is the product of the Space traversed, $(\mathrm{R} - r)$, into the Mean Force; but the mean force in question is not the arithmetical but the geometrical mean between the extreme values; that is, these extreme values being $k.mm_{,}/r^2$ and $k.mm_{,}/\mathrm{R}^2$, the mean force is the square root of their product, or $k.mm_{,}/\mathrm{R}r$; the latter being multiplied by the space traversed, $\mathrm{R} - r$, gives the product $k.mm'(\mathrm{R} - r)/\mathrm{R}r$ or $k.mm_{,}\left(\frac{1}{r} - \frac{1}{\mathrm{R}}\right)$ as the work done.

Hence the work done by a repulsion (which at any distance r is equal to $k.mm_{,}/r^2$) in separating two masses from a distance r to a distance $2r$, is $k.mm_{,}.(1/r - 1/2r) = k.mm_{,}/2r$; from distance r to an infinite distance ∞, the work done is equal to $k.mm_{,}(1/r - 1/\infty) = k.mm_{,}(1/r - 0) = k.mm_{,}/r$. Hence if a certain amount of work be done by the repulsion in doubling the distance between two mutually repelling bodies, the repulsion would do exactly twice as many units of work, and no more, in separating the two bodies to an infinite distance from one another.

The Potential Energy unexhausted at any given distance.—To remove a mass $m_{,}$ from a point at a distance R from a fixed repelling mass m to an infinite distance would involve expenditure (by the repulsion) of work $= kmm_{,}\left(\frac{1}{R} - \frac{1}{\infty}\right) = \frac{kmm_{,}}{R}$: this work not having been done when the distance between the bodies is finite, the potential energy of the system is so far unexhausted. At an infinite distance, $R = \infty$, and the unexhausted potential energy is $\frac{kmm_{,}}{\infty} = 0$.

The unexhausted Potential Energy of two bodies of masses m and $m_{,}$, repelling one another and situated at a distance R, is $k.mm_{,}/R$; this is called the Mutual Potential of the two masses.

If one of them be a unit, the mutual potential is $k.m/R$. This is numerically equal to the Potential as defined in the next paragraph.

Direction of Movement.—At an infinite distance, where the potential energy attributed to a body there placed would be zero, there would be no force impelling to any further separation. At any place where the potential energy has a positive value, it will tend to exhaust itself, and a body there placed will, if free to do so, move away towards some place where it would have less potential energy. But the Potential Energy which a Unit-mass would have if placed at a particular Point in Space,—the work which would have to be done by the repelling force in removing the unit-mass from that point to an infinite distance, or done against repulsion in conveying the unit-mass from an infinite distance to that point,—may be stated as an attribute of that Point in Space and may be called its **Potential.** This may be numerically high or low. Then, under a repelling force, a body tends to move from a place of high potential to a more distant place of low potential, and if the body be free to move in that sense, the force will do work; while if the body be moved from a place where the potential is low to one where it is high, the movement is effected against repulsion or resistance, and work is done against the repelling force.

The Direction of the Force is opposed to the direction in which the potential increases most rapidly; and its amount at any point is (in any given direction) equal to the mean **decrease** of potential per unit distance traversed (in that direction): and the product Fs, the Work Done, is numerically equal to the whole **diminution** of potential in the whole distance traversed.

Potential a condition at a point in space.—We must distinguish between the *Potential Energy* which a *mass* may be said to have in virtue of its position at a certain point, and of its consequent relation to neighbouring masses; and the *Potential* of that point in space. The condition at that point of space is such that *if* a unit-mass were placed there, the forces acting on it would do V units of work in conveying it to an infinite distance, or would, on the other hand, have V units of work done against them if the unit-mass were forced against them from an infinite distance to that point: and this is a property, numerically expressible by the numerical value of V, but independent of the actual presence or absence of any unit-mass at that point.

The potential at a point situated at a distance r from a mass Q is $V = k.Q/r$.

Work done against Attraction.—If a body be at a given distance r from an attracting mass, the action between the two bodies is a force tending to approximate them: work is done by the attracting force in doing this: but "the work done *by* the attracting force in separating the bodies to an infinite distance" is a negative quantity, for work ($= V$ units) would have to be done *against* the attraction in producing this movement; and the **potential** at a distance r from the attracting mass is $-V$, a **negative** quantity.

Potential in the special case of Gravitation.—A pound-mass standing on the surface of the earth would (if the earth were a sphere of radius 4000 miles) require the expenditure of 21,120,000 foot-pounds of work, and no more, to remove it to an infinite distance, this force being exerted *against* the gravitation; and therefore any point on the surface of the earth, thus assumed to be spherical, would be at a negative potential of $-21{,}120{,}000\ g$, while the potential of any point at an indefinitely great distance would be zero. By a special exception, however, the Potential of a point at the surface of the earth is considered to be zero, and a body lying on the earth has no potential energy; while a pound-mass removed to an indefinite distance could have no more than 21,120,000 foot-pounds of potential energy stored up in it; and the gravitation-potential of a point at an infinite distance is $+ (21{,}120{,}000 \times 32.2)$ foot-poundals.

Absolute Zero of Potential.—A point is at zero potential when a body placed there would have no potential energy. This is the condition of a point at an infinite distance from all repelling masses.

Fields of Space in opposite conditions.—If there be two bodies, the one attracting, the other repelling: a unit-mass brought near the former will on the whole be attracted, as a small magnet is attracted by the nearer pole of a large magnet more than it is repelled by the farther pole; if brought near the other mass, it will on the whole be repelled. The space in the neighbourhood of the attracting mass will be a field of space in which the potential is negative; round the repelling body there will be a field of force of positive potential.

Continuity of Potential through Zero value.—A particle passing from a region of positive potential into one of negative potential must pass through a point where the potential is zero; for if it were possible for it to do otherwise there would be physical discontinuity. As it thus moves, the positive potential energy of the body is gradually exhausted, becomes zero, and then becomes a negative quantity.

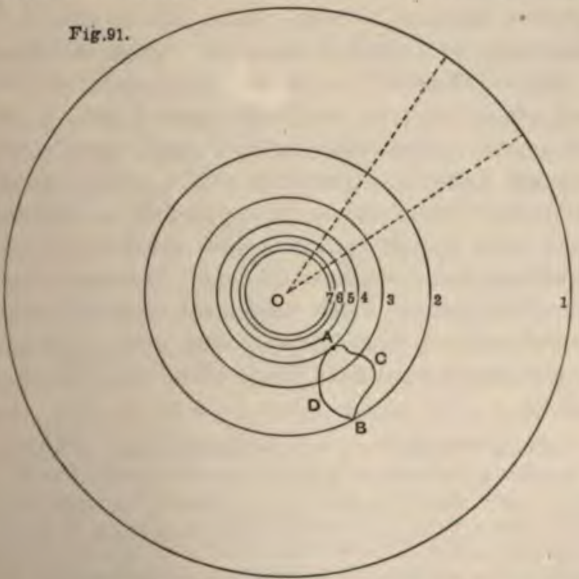
Arbitrary Zero of Potential.—We may arbitrarily assume any point or surface in the neighbourhood of attracting or repelling masses as one whose $V = 0$; then those places which have a greater potential are said to be localities of positive potential, and those at which the potential is less are said to be localities of negative potential. This is convenient, for absolute zero we know no more than we know absolute rest.

Analogy of Sea-level.—Let us assume that the surface of the earth is the sea-level taken at high-water mark. This is an arbitrary assumption, for low-water mark might have just as well been chosen. If a body be placed at a certain height above sea-level, gravitation may do a certain amount of work in bringing it down to that level, for the mass placed at that height has a certain amount of potential energy: at a less height it has less potential energy; at the sea-level it has none; if placed below the sea-level, its potential energy is, on this assumption, a negative quantity. Hence the gravitation-potential above sea-level is of opposite sign to that below it.

Obviously it would be possible, instead of saying that a point is so many feet above or below sea-level, to say that a pound-mass there placed would have V units, + or −, of potential energy if there placed, and thus to define the distance between that point and sea-level by its gravitation-potential.

Equipotential Surfaces.—In Fig. 91, O is a repelling particle.

Fig.91.



All points at equal distances from it are at the same potential.

If these be joined they form a sphere. The potential at every point of the surface of one of these imaginary spheres is the same, and may be represented by V_1. This sphere is an *equipotential surface* for potential V_1. Within this, and concentric with it, lies another sphere, the potential at every point of which is V_2. Within this lie successive shells or imaginary spherical surfaces, over each of which the potential is equal. If these surfaces be chosen such that their potentials have a common difference—that is, that $V_2 - V_1 = V_3 - V_2 = V_4 - V_3$, etc.—and if these differences each represent one unit of work, a set of equipotential surfaces thus obtained is called a "*System of Equipotential Surfaces.*"

Motion parallel to Equipotential Surfaces does not involve work done either by or against the attracting or repelling force.

Motion across Equipotential Surfaces, from one surface to another, implies movement from a place where the potential has one value to a spot where it has another. A unit-mass moving away from the second to the first surface in Fig. 91, loses potential energy $= V_2 - V_1$: on a mass m the repelling force would do work $= m \times (V_2 - V_1)$. A mass m moved up from equipotential surface No. 10 to surface No. 15 in a system of such surfaces, whatever be their form, would have work $= 5m$ units done upon it against the repelling force.

The **work done** would be the same *whatever be the points* of the respective surfaces between which the motion is effected. Any transference of a particle from one equipotential surface to another may be effected by a vertical translation from the one to the other, which involves work, compounded with a translation along the second equipotential surface, which involves none.

The **work done** by a transference of a particle from a point A on one equipotential surface to a point B on another is also always the same, by *whatever path* the transference be effected, provided always that there be no friction. The most complex path may be resolved into so much movement at right angles to the equipotential surfaces, which implies work done by or against the forces, and so much parallel to them, which consumes or liberates no energy.

This may also be proved by a *reductio ad absurdum*. If in Fig. 91 there were two possible paths between A and B, one of which, ACB, corresponded to a units of work done by a unit-mass of matter traversing it, while the other, ADB, corresponded to a greater amount, b units, of work ; then it would be possible to cause a body to fall from A to B down the path ADB, corresponding to the greater work, and by falling to pull directly or indirectly a mass equal to its own up the easier path BCA : it would itself acquire kinetic energy corresponding to energy $= b - a$; the body thus pulled up along BCA might in its turn fall down the path ADB, and raise along the path BCA the mass which had previously traversed the path ADB, again with gain of energy equal to $b - a$. Thus the circuit might be kept up with continuous gain of energy, and this contrivance might be utilised as a perpetual motor; but this is an impossibility; therefore there is an equal expenditure or liberation of energy, so far as the attracting or repelling forces are concerned, in effecting a transference along every possible path between any two given points in space.

Analogy of Surfaces of equal level. Obviously the same propositions apply if we read the word level for potential.

If A and B be two equal particles, A attracting and B repelling external particles, the space surrounding A will be a region of negative potential, while the potential of the neighbourhood of B is positive. Over a plane symmetrically situated with respect to A and B the potential will be zero, while the attraction will carry bodies across the equipotential surfaces to the point A.

Distance between Concentric Equipotential-Surfaces.—In a system of concentric spherical equipotential-surfaces, the distance between every pair of these surfaces is proportional to the square of their mean distance (*i.e.*, of the geometrical mean) from the centre of the single attracting or repelling mass.

Two concentric spherical equipotential surfaces whose potentials are V and (V + 1), and whose respective radii are R and R^1; we wish to find the value of $R - R^1$.

$$V = k\frac{m}{R}, \text{ and } (V + 1) = k\,\frac{m}{R^1}\text{; whence } R - R^1 = km\,\frac{1}{V\,(V + 1)} = \frac{RR^1}{km}.$$

Thus, if the equipotential surfaces be those surrounding the earth, over which the potential due to **Gravitation** is constant, and if the distances between the surfaces be such that transfer of a pound-mass from any one surface to the next one represents a foot-pound of work done: then, at the distance of one earth's-radius from the centre of the earth—that is to say, on the surface of the earth—the distance between two equipotential surfaces is one foot; twice as far from the centre—that is, 4000 miles (nearly) from the surface of the earth—the distance is 4 feet, and the same amount of work which would raise a pound-mass through one foot near the surface of the earth would, at a height of 4000 miles, raise it 4 feet; and similarly, at a height of 8000 miles, it would raise it 9 feet, and so on. Thus at a very great distance exceedingly long paths would be traversed by a pound-mass as the result of doing a single foot-pound of work on it.

It follows that if the equipotential surfaces be chosen *at equal* *distances* from one another, the amount of work corresponding to the removal of a mass from one surface to the next is in the inverse ratio of the square of the mean distance of the two surfaces from the attracting mass.

Two concentric spherical equipotential surfaces whose potentials are V and V_1, and whose radii are R and $R + 1$; $V = km/R$; $V_1 = km/(R + 1)$; $\therefore V_1 - V = km/\overline{R\,(R + 1)}$.

A pound-mass at a distance of 240,000 miles (= 60 radii nearly) from the earth's centre will be attracted by the earth with a force which bears to the attraction at the earth's surface the proportion of $(1/60)^2 : (1)^2 = 1 : 3600$. Hence, to move a pound-mass through one foot—that is, from an equipotential surface by any path to any point on an equipotential surface one foot distant from it—at a distance of 240,000 miles, or, roughly, at the distance of the moon, would involve the expenditure of approximately $\frac{1}{3600}$ foot-pound of work.

Free movement always at right angles to Equipotential Surfaces.—Whatever be the form of the equipotential surface, it always happens that a body placed on such a surface and free to move, will tend to move, under the influence of the attracting or repelling forces, in a direction at right angles to that surface. This is because the forces of attraction or repulsion can have no component tending to produce motion in any direction along a surface of equal potential, or parallel to it.

Lines of Force.—Thus, if the equipotential surfaces be concentric spheres, as those of Fig. 91, a body repelled from O will travel along radial lines such as are exemplified by the dotted lines in that figure. When the equipotential surfaces have a more complex form, the lines along which a body tends to travel are more complex, as is shown in Figs. 241 and 242. These lines, always at right angles to the equipotential surfaces which they cross, are called Lines of Force.

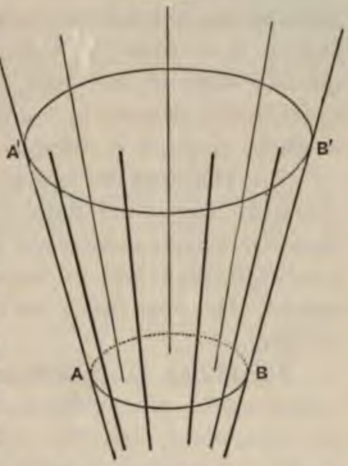
Space in the neighbourhood of an attracting or repelling body may be conceived to be pervaded by a system of Lines of Force, along which bodies will move if free to do so. The work done on a particle thus set in motion by any attraction or repulsion is the product of the mean force by the space traversed; the latter must be measured along the line of force which is the body's actual path.

Lines of force are analogous to lines of steepest fall in topography; water poured out will at any spot run in the direction of steepest fall; and a body acted upon in a field of force will tend to fall away from a spot of higher potential to one of lower potential, following the direction of most rapid potential-fall, the line of force.

Tubes of Force.—Suppose AB to be a portion of an equipotential surface: lines of force pass through the equipotential surface: some of these lines graze the edge of the area AB; these cut off an area A′B′ from another given equipotential surface. The space comprised between these equipotential areas and the marginal lines of force is called a Tube of Force. This space may be supposed to be filled with a bundle of lines of force, extending from AB to A′B′.

Tubes of force have this property, that so far as the area A′B′ cut by them from one equipotential surface is greater than the area AB cut off from another, so does the intensity of the attracting force over any unit of area diminish; so that if s and s' be the respective areas of AB and A′B′, and F, F′ the respective forces per unit of area acting across these equipotential areas, the product, force × area, is constant, and $Fs = F's'$. Thus the force at the level A′B′ is less than that at AB in inverse proportion to the relative magnitude of the area A′B′ cut off by a tube of force.

Fig. 92.



Tubes of Force drawn in such fashion as each to contain one line of force are called Unit Tubes of Force.

Number of Lines of Force.—The forces at any two points may be compared by stating the relative numbers of the lines of force which pass through units of area of those equipotential surfaces which pass respectively through each of the points compared; the fewer these lines the less the local intensity of the force. Thus in Fig. 91 the lines of force which cross the outer spheres are less numerous per unit of area of the sphere than those which cross the inner spheres, and the force there is correspondingly less.

Systems of Surfaces and Lines.—The space in the neighbourhood of an attracting or repelling mass or system of masses may thus be mapped out by a system of equipotential surfaces and lines of force, and such a region of space is called a **Field of Force.** The system of surfaces and lines may be so constructed that (1) the work done in passing from one equipotential surface to the next is always the same, one unit of work; and (2) the lines of force are drawn in just such numbers that at a place where the force is equal to unity, one line of force passes through the corresponding equipotential surface in each unit of area of that surface. Then the following advantages are secured:—

(1.) The potential at any point in the field of space surrounding the attracting or repelling mass or masses is found by determining on which imaginary equipotential surface that point stands.

(2.) If unit-length of a line of force cross n equipotential surfaces, the mean force along that line along the course of that part of it is equal to n units; for the difference of potential of the two ends of that part of the line of force $= n$; it is also equal to Fs, because it represents numerically a certain amount of work; but $s = 1$; whence $n = F$.

(3.) The force at any point of the field corresponds to the extent to which the lines of force are crowded together; and thence it may be determined by the number of lines of force which pass through a unit of area of the corresponding equipotential surface, that area being so chosen as to comprise the point in question.

Variations in Difference of Potential.—Any movement of a body across the surfaces of equal potential, if these surfaces be not equidistant, alters the relative difference of potential between its two extremities, because a body approaching a repelling or attracting mass meets and cuts more equipotential surfaces than it quits, as may be seen from Fig. 91; and, *vice versâ*, a receding body meets fewer surfaces than it quits. In the former case the movement tends to cause an increase in the difference between the potentials of the extremities of the body moved in the non-uniform field of force; in the latter it tends to diminish it.

An increase in the central attraction or repulsion has the same effect as an approach; a diminution the same as a recession.

Theorem.—If a closed surface be drawn round a system of attracting or repelling masses, the number of lines or unit tubes of force traversing the surface is numerically equal to $k.4\pi Q$, where Q is the algebraical sum of the whole matter within the closed surface, and when the law of force is that $f = k.mm_{\prime}/\text{distance}^2$.

Take first a single particle q at the centre of a spherical surface. The force per unit of area is $k.q/r^2$; the whole surface is $4\pi r^2$; the force over the whole surface, *i.e.*, the number of lines of force crossing the surface, is $k.q/r^2 \times 4\pi r^2 = k.4\pi q$.

Take next any particle q at any point within a closed surface of any form. Any small area δs is taken, which subtends at the particle a solid angle ω, and the normal to which is inclined at an angle ϵ to a line drawn from the particle to the centre of δs. The area δs is equal to $r^2\omega/\cos\epsilon$, where r is its mean distance from the particle; the normal force per unit of area is $k.q.\cos\epsilon/r^2$; the product $(r^2\omega/\cos\epsilon)\,k.(q\cos\epsilon/r^2) = k.q\omega$ represents the number of lines of force passing through the element of surface, a number which is seen to be independent of the obliquity or the distance of the element of surface considered. When the surface completely surrounds the particle, the solid angle subtended by the surface is $\omega = 4\pi$, and the force due to this particle and acting through the whole surface is $k.4\pi q$. So for every particle, wherever situated, within the closed surface, and if the sum of the q's be Q, the number of lines of force traversing the whole surface is $k.4\pi Q$.

If some of the q's be attracting, some repelling, they must be affected with their proper signs, and Q is their algebraic sum.

If the surface be one which is repeatedly indented so as to be repeatedly traversed by lines of force, the exits must be more numerous by one than the entrances; the exits and entrances in any region of the surface subtending the solid angle ω must compensate one another, with the exception of the last exit; this alone contributes to the aggregate number of lines of force finally issuing from the surface. The closed surface may thus be of any degree of complexity, without affecting the numerical value of the whole system of lines, as enunciated by the above theorem.

Isodynamic Surfaces and "Lines of Slope."—If all those points in a field of Force be connected, at which the force is equal, we have a set of **isodynamic** surfaces. These may coincide with equipotential surfaces, as in the case of gravitation; but they may have a totally different lie. For example, the field of electromagnetic force surrounding a long straight wire, bearing a current of electricity, is permeated by equipotential surfaces, plane and radiating from the straight wire; the lines of force, cutting these at right angles, are concentric circles round the wire; the isodynamic surfaces are concentric cylindrical surfaces surrounding the wire. At right angles to the isodynamic surfaces we may imagine lines, the so-called "**Lines of Slope,**" which trend in the directions in which the intensity of the force in the field falls away most rapidly. In the case of gravitation these trend in the same directions as the lines of force; in the case of the straight current they radiate from the wire at right angles to it, and are therefore at right angles to the lines of force.

In a uniform field of force there are no such lines or surfaces, for the whole region is isodynamic.

CHAPTER VIII.

GRAVITATION AND THE PENDULUM.

Law of Gravitation.—Every particle of matter in the Universe is attracted by every other particle with a force varying directly as the mass of each particle, and inversely as the square of the distance between them.

We have already seen that the Weight of a body is a synonym for the Force with which it is attracted by the earth. The law just enunciated indicates that the weight of a double mass is twice that of a single mass, and so on. This seems a truism; but it is an experimental result, not a truism, that the weight of a mass of lead is equal to that of an equal mass of wood. This might have been otherwise. The mass of a given piece of wood is known to be equal to that of a certain piece of lead by the experimental fact that equal forces acting on each for equal times produce equal velocities: $F = ma$; these velocities being those of short horizontal trajectories, which are independent of gravitation. Now a piece of iron and a piece of cork whose masses are thus found to be equal, will, if placed in the neighbourhood of a magnet, be found to be by no means affected by equal accelerations towards the magnet; yet they are both equally attracted by the earth, have both the same weight in the balance, and, if caused to fall through a vacuum (the friction of the air being thus removed), are found to fall with concurrently equal velocities.

It is remarked that horizontal trajectories are independent of gravitation. A cannon ball at the moon, if it had weighed 60 lbs. on the earth, would weigh only 10 lbs. there: and so, as has been said, it might be dropped on the toes of the observer there without serious consequences; but it would be found exactly as difficult there as here to heave the ball horizontally, for it would still contain 60 pounds mass.

Again, a heavy and a light mass of any substance fall at the same rate through a vacuum. It was long believed that the heaviest bodies fall fastest; but Galileo experimentally disproved this. The attraction of the earth for a large mass is greater than for a small one, but the mass to be moved increases in the same proportion as the attraction; and thus the acceleration produced is the same in all cases, and is independent of the amount as well as of the substance of the falling mass.

Cavendish's Experiment.—This was a direct measurement of the attraction of masses for one another. Light balls were poised on a rod and their position carefully noted: large balls of lead were carefully brought near them: the light balls were attracted by the heavy masses, and their displacement measured. Great experimental precautions were necessary, such as the observation of the position of the balls with a telescope placed at a distance, the avoidance of draughts of air and of vibrations, etc.; the result showed that if lead balls had been employed as large as the earth, the attraction of such balls would have been greater than the actual attraction of the earth in the ratio of 11·35 to 5·67: but lead is 11·35 times as heavy as water; hence the earth as a whole is 5·67 times as heavy as an equal bulk of water, or the **density of the earth** is 5·67.

If two masses be respectively M and m, and their distance d, the attraction between these masses $\propto \mathrm{M}m/d^2$ or $f = k.\mathrm{M}m/d^2$, where k is a constant. The earth being approximately spherical attracts falling bodies of mass m as if its own mass, M, were gathered together at the centre, about 637,000,000 cm. from the surface. Its mass M is 6140,000000,000000,000000,000000 grammes. A mass = 1 gramme is attracted by the earth with a force

$$f = 981 \text{ dynes: hence } f = 981 = k\frac{\mathrm{M}m}{d^2} = k\frac{(614 \times 10^{25}) \times 1}{(637{,}000{,}000)^2}.$$

$$k = 981 \times \frac{(637{,}000{,}000)^2}{614 \times 10^{25}} = \frac{1}{15{,}430{,}000}.$$

The astronomical unit of mass is 15,430000 grammes and the corresponding unit of force 15,430000 dynes. With such units the constant $k = 1$.

Accelerated Motion under Gravity.—A body free to fall *in vacuo* would be subject to constant acceleration of about 981 cm. or 32·2 feet—that is, of g units of length—per second, and its movement would be described by the three formulæ of page 69; the + sign being used when the attraction of gravity acts in the same sense as the original velocity V; the − sign when it acts in an opposite sense.

When bodies fall through the air there is **friction** between the air and the falling body. This is found to vary as the radius of the sphere if the falling body be an exceedingly small sphere; and generally it increases with, but is not proportionate to, the surface exposed. Thus a feather, which presents much surface, falls more slowly than a similar feather rolled into a ball.

Path of a Projectile.—We have already seen that combination of a uniform rectilinear movement with a uniformly accelerated movement, not in the same direction, results in movement in a parabolic path. This is the theoretical course of a bullet flying *in vacuo;* but the actual course of a shell or bullet in the air differs widely from this on account of friction, its path being at first somewhat straight and ending with a somewhat sudden fall.

If a shot were fired horizontally *in vacuo* at such a rate V (about 26,077 feet per second), that g, the acceleration downwards ($=32{\cdot}2$ ft. per second) would be V^2/r, r being the distance of the earth's centre from the bullet's path, the shot would never fall to the ground, but would travel round the earth at the level of the gun's mouth.

Any object travelling past the earth with a velocity V such that at the distance r the acceleration g, due to gravity, is equal to V^2/r, will travel round the earth and not cease to do so; one travelling at a greater velocity will pass the earth, being deflected by it from its path; one travelling at a less velocity will fall in towards the earth until r diminishes so far that V^2 becomes equal to rg. The surface of the earth may be reached before this limit is attained; if not, the body would, *in vacuo*, travel round the earth at a distance r from the centre such that $r=V^2/g$.

Universal Gravitation.—The fact of terrestrial gravitation and many of its laws were well known before Newton's time; he stated the law of gravitation as a universal one: "Gravitatem in corpora universa fieri," etc.—*Principia*, Bk. III. Prop. vii. and Corol. 2.

The moon makes a revolution round the earth in about 2,360,000 seconds in an orbit whose mean radius is 59·964 times the earth's equatorial radius. The formula $g=V^2/r$ shows that this corresponds to an actual fall of the moon towards the earth of $1/(112{\cdot}48)$ foot per second: this, compounded with the tangential velocity at every instant, keeps the moon in its orbit. This acceleration, due to the attraction of the earth, is $1/(59{\cdot}964)^2$ of 32·2 ft.; thus the moon is under the influence of terrestrial attraction which obeys the law that $f \propto d^{-2}$. Newton made similar deductions from other astronomical phenomena, particularly those of the satellites of Jupiter, and ultimately asserted the universality of the law of gravitation.

The moon attracts the water of the sea, and thus produces a lunar tide on the side of the earth nearest it; it also pulls the earth away from the water on the farther side, and produces a lunar tide on the farther side. The sun produces similar tides $\frac{2}{5}$ as great. These two sets of tides may concur or may partly counteract one another.

Variations of the acceleration of gravity on the earth's surface.—At the equator the acceleration of gravity is, at the sea-level, 978·1028 cm. per second: at the pole it would be 983·1084, if the law of variation in accessible regions of the earth's surface be obeyed there. This law is, that at any place whose latitude is λ, the local acceleration of gravity is, in cm., $g = (980{\cdot}6056 - 2{\cdot}5028 \cos 2\lambda - {\cdot}000003h)$, where h is, in cm., the height of the observing station above the sea-level. This diminution of gravity—equal masses weighing less, and therefore distorting spring-balances less, in regions nearer the equator—is due to two concurrent causes: (1) That the mean **equatorial radius is greater** than the polar; the polar radius is 635,639,000 cm.; the longest equatorial radius (from lat. 14°· 23′ E. to lat. 165°· 37′ W.) is 637,839,000 cm.; the shortest equatorial radius, at right angles to the former, is 637,792,000 cm.* (2) The **rotation of the earth.** If the earth came to rest, the acceleration of gravity would be increased by $g/289$ or 3·3908 cm. per second, and the weight of bodies would be increased in the ratio of 289 to 290. If the earth rotated $17(=\sqrt{289})$ times as fast as it does, loose objects would, at the equator, fly off its surface at a tangent.

The acceleration due to gravity is in Paris 980·94, at Greenwich 981·17, at Manchester 981·30, at Edinburgh 981·54 cm. per second.

The velocity of rotation at the equator is 456,510 cm. per second: whence $V^2/r = 3{\cdot}3908$.

Local Variations.—In the neighbourhood of a high range of mountains a plumb-line inclines towards the mountains. The ebb and flow of water in the Firth of Forth affects the apparent latitude of Edinburgh by about $\frac{1}{14400}$ degree, for when the water is at high tide, plumb-lines are inclined towards it, and the mercury used as a means of producing perfectly level mirrors is, in the vessels containing it, heaped up towards the mass of sea-water.

At sea the effect of gravity is less than it is on land, because the mass of water under the spring-balance is lighter than a corresponding amount of rock would have been. The depth of the sea may be determined by a graduated instrument of the nature of a spring-balance, sufficiently sensitive to take account of these variations.

Measurement of the Local Force of Gravity.—The force of gravity must, like all other forces, be measured by its acceleration. This may be done directly by **Attwood's machine,** already described. Observation of a single fall cannot, however, give accurate results, and the value of g is best determined by the oscillations of a **pendulum.**

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* Col. A. R. Clarke's values are 635,638,756 cm., 637,837,929 cm., and 637,791,478; the longest equatorial axis being from 8° 15′ W. to 171° 45′ E. of Greenwich (*Phil. Maga.*, 1878).

There are at the basis of this determination four main facts: (1) that a pendulum of a given length will oscillate through small arcs in equal times, of whatever substance it be made—this last experimental result being due to Newton; (2) that the relation between l the length of a simple pendulum, T its time of complete to-and-fro oscillation, and g the local acceleration of gravitation, is given by the formula $g = 4\pi^2 l/T^2$ presently to be proved; (3) that the length l of a simple pendulum may be very accurately observed, for in practice it is equal to the distance between two points on a solid rod, called a compound pendulum; and (4) that the time of one oscillation may be very accurately observed by counting the number of oscillations in a sufficiently long period of time. Hence g can be found to any nicety.

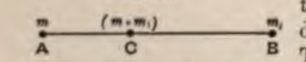
Centre of Gravity.—The earth is approximately spherical, and bodies on the surface have all their particles drawn approximately towards its centre. But the centre is so distant that, within the limits of ordinary terrestrial objects, the gravitation forces acting on the several particles of a body are nearly parallel to one another, and their resultant acts on the Centre of Figure. This centre is called the Centre of Gravity of a body attracted by the earth.

The centre of gravity of any plane figure may be found by cutting that figure out in cardboard, and suspending the card first from any one point and then from any other. A line drawn vertically downwards from the first point of suspension when the body is suspended from it, and another line drawn in the same way from the second point of suspension, will cross one another at the centre of gravity.

Whatever be the form or the arrangement of matter in a body, if it be suspended from any point arbitrarily chosen, the centre of gravity is in a line vertically drawn through the point of suspension—vertically here meaning at right angles to the free horizontal surface of liquid at that place. If the centre of gravity be found by two suspensions, the vertical lines drawn from any other points of suspension will all pass through the same centre.

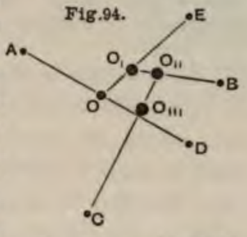
Centre of Gravity of Two Masses.—In Fig. 93 the two bodies, A and B, whose masses are m and $m_{,}$, will have their centre of gravity at a point C, which is determined by the equation $m \times AC = m_{,} \times BC$. The whole mass $m + m_{,}$ may, as regards other bodies, be considered as if it were aggregated at the point C.

Fig. 93.



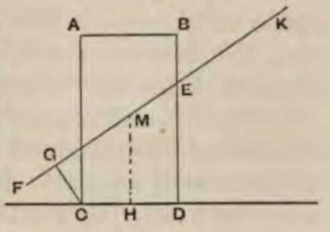
Centre of Gravity of a System of Masses.—This is found by taking account of each *seriatim*. In Fig. 94 let the bodies be A, B, C, D, E, whose respective masses are $m_{,}$, $m_{,,}$, $m_{,,,}$, $m_{,,,,}$ and m_{v}. First the centre of gravity of any two, say A and D, is found at O; A and D are supposed to be replaced by a mass $m_{,}+m_{,,,,}$ at O. Next the centre of gravity between another of the masses, say E, and the imaginary mass $m_{,}+m_{,,,,}$ at O is found to be at $O_{,}$; at this point $O_{,}$, there is supposed to be placed a body whose mass is $m_{,}+m_{,,,,}+m_{v}$. In the same way, the centre between this imaginary mass and another, say $m_{,,}$ at B, may be found at $O_{,,}$. Finally the centre between this and the mass $m_{,,,}$ is found at $O_{,,,}$, and this is the last operation, for, as regards external masses, the system, ABCDE, acts as if it were a mass $(m_{,}+m_{,,}+m_{,,,}+m_{,,,,}+m_{v})$ concentrated at the point $O_{,,,}$; this point is therefore the centre of gravity of the system. The same point will be found whatever the order in which the masses are considered.

Fig. 94.



Overturning a body.—Let ABCD be a block of material supported on a base CD. How great a force applied at E, in the direction EF, is necessary to overturn the block? The question is really one of moments round C, for if the force along EF prevail over the weight of the block, it will do so by turning it over the point C. From that point C, CG is the shortest line drawn to meet the line EF, and CH is the shortest distance to the line MH, along which the force of gravity may be considered to act. At the instant when overturning is just going to commence, the moments round C must be equal, and CG × force along EF = CH × wt. of body. Therefore, if the force along EF be greater than weight of body × CH/CG, the body will be overturned. The greater CH is, the greater must be the force exerted along EF in order to overturn the body; the smaller CH is, the less need that force be. When CH = 0—*i.e.*, when the centre of gravity M is vertically above C—any force, however small, will upset the block ABCD; while, if H be on the other side of C, the block cannot stand unless propped up. In this way a body resting on a wide base is less easily upset than one standing on a narrow; one in which a vertical line drawn from the centre of gravity falls outside the base of support cannot stand unsupported; while one in which the centre of gravity stands over the very edge of the base of support is upset by the least disturbance.

Fig. 95.



A microscope, then, ought for the sake of steadiness to have a wide base; and since a tripod stand is the most steady form of support, for reasons already stated (see Spherometer), instruments of this class should be supported on broad tripod stands. An old man using a staff widens his basis of support by virtually converting his two legs and the staff into a broad tripod stand.

It amounts to the same thing whether the base of an object be relatively broad or its centre of gravity be relatively low. If the centre of gravity be relatively high or the base relatively narrow,—as in the case of young animals learning to walk, children learning to walk, persons learning to move on skates, or on stilts, or on a narrow rail, or rope, or wire, or a bicycle, or a person standing on one foot or on his heels,—a relatively small displacement of the object will readily cause the centre of gravity to be placed vertically over a point beyond the base of support; then the object, if it be not propped up, or if the centre of gravity be not brought over the base of support, or the base of support not brought up under the centre of gravity, will topple over.

If, on the other hand, the base be relatively wide, or the centre of gravity be relatively low, as in the case of a lampstand loaded at its base, the task of upsetting such an object is greater, since the centre of gravity is in such a case less easily induced to pass to a position vertically beyond the base.

Curious positions may be assumed by objects when they are so balanced that the centre of gravity is vertically over some point in the basis of support. A man's centre of gravity is at a point about the front of his last lumbar vertebra. If he carry a burden, then, in order to bring the centre of gravity of the conjoined mass of his body and the burden borne by him into a position vertically over some part of the narrow basis of support furnished by his feet (heels, and balls of great toes, and lines joining these), he must stoop; if the burden be towards the front of the body, as in the case of obese persons, the gait becomes very erect.

When a body is suspended from any point in its own substance and set a-swinging, its centre of gravity ultimately finds its way into the lowest position possible.

Work done in overturning a body.—If there be rotation round the point C of Fig. 95, so far that M, the centre of gravity of the body, comes to be immediately over that point and overturning is effected, the centre of gravity is raised through a certain height h. The weight of the body, mg, × that height, h, is the work which must be done before overturning can be accomplished.

Angle of Overturning.—If the force applied at E in Fig. 95 be applied in a direction too nearly parallel to BD, its moment may be too small (its arm CG being too short) to produce overturning. At a certain definite angle, BEK, there will be equilibrium, the arm CG being of exactly such length as will make the moment of EF equal to that of the weight of the body. If, then, this angle BEK be less than θ, the angle of repose, the body will overturn before sliding; if BEK be greater than θ, the body will slide before overturning.

Equilibrium, Stable, Unstable, and Neutral.—If the centre of gravity be so situated in a body that work has to be done in disturbing it—that is to say, if the centre of gravity be already at the lowest possible position—the equilibrium is stable. If a ball lie in a bowl, work must be done in order to effect any displacement of it, for no displacement can be effected without raising the centre of gravity of the ball, and thus imparting potential energy to it. When the ball is let go, it rolls back and oscillates in the bowl until it comes to rest. The same thing is seen in a swing, a cradle, a rocking-horse, a pendulum, or a ship well ballasted, which are all in Stable Equilibrium; in the last case the oscillations somewhat resemble those of a pendulum whose point of suspension and whose length both vary.

If a body have its centre of gravity placed above its point of support, so that any displacement lowers its centre of gravity, then the body already has potential energy, which it is disposed to convert into kinetic by the fall of its centre of gravity to the lowest possible point. Hence in bodies thus in Unstable Equilibrium, a very slight disturbance may cause a very great displacement, disproportionate to the disturbance, but depending on the potential energy stored in the system. In this case are boats in which people stand, high chairs in which children are seated, cars which are heavily loaded atop, deck-loaded ships, and, in short, everything which is "topheavy."

When no work is done upon or by an object, as far as the attracting forces are concerned, when it is displaced, the Equilibrium is Neutral. A uniform sphere may be displaced and assume a new position without either raising or depressing its centre of gravity.

A sphere floating in water is in neutral equilibrium; a plank floating in the usual way is in stable, while a plank floating with its edges vertical is in unstable equilibrium.

Simple Pendulum.—This is an ideal. It is a heavy particle suspended by a weightless cord. An approximation to a simple pendulum is obtained by suspending a small bullet by a very thin wire. The length of this pendulum is the distance between the point of suspension and the centre of the bullet.

If in Fig. 96 a simple pendulum of length $l = AC$ be reprerented as displaced from the vertical position through the angle θ, m being the mass of the bob, and mg consequently its weight, when the bob is at C the force of gravity may be resolved into two components: one $= mg \sin \theta$ in the direction of the tangent at C, and tending to bring the bob towards the middle line with acceleration $= g \sin \theta$; and a radial component $= mg \cos \theta$, which renders the cord tense. The displacement of the bob, the distance between B and C measured along the arc, is equal to $l\theta$. Now, as long as θ is small, some 2° or 3° at most, $\sin \theta$ and the angle θ are nearly equal, and the tangential acceleration, which is equal to $g \sin \theta$, bears to the displacement $l\theta$ an almost constant ratio, for $g \sin \theta . / l\theta =$ (approximately) g/l. But we know that when a body after displacement is subject to a force tending to bring it back, which produces an acceleration proportional to the displacement, the result is a S.H.M.; and thus a pendulum very slightly displaced oscillates in S.H.M.

Fig.96.

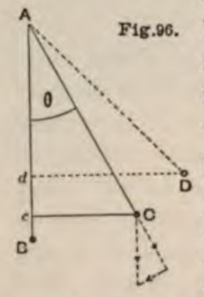
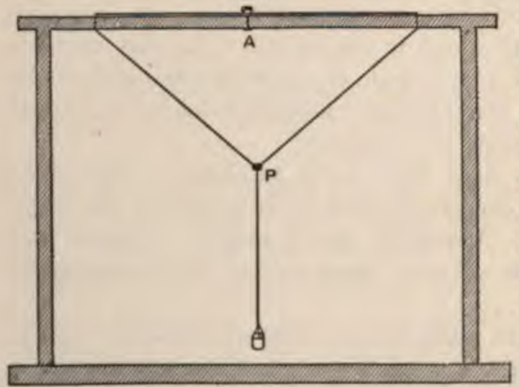


Fig.97.



Simple Harmonic Motions experimentally performed.—A simple pendulum approximately describes a S.H.M. A pendulum whose bob consists of a flask containing coloured fluid or ink, which pours from a narrow orifice as the flask oscillates, will record the path of the pendulum. A quantity of sand may be used instead of ink. The apparatus shown in Fig. 97 (Blackburn's pendulum) may be used for the description of the compounded H.M.'s exemplified in Figs. 35-40. If the bob, whose path is recorded by sand, be displaced in a line making an angle of, say, 45° with the line of the cross-bar, there will be two simultaneous and independent oscillations set up when it is liberated: one from the cross-bar and at right angles to it; one from the point P and at right angles to the other oscillation. By adjusting the relative lengths of the effective long and short pendulum, by means of a peg A, round which a certain quantity of cord is wound, or by shifting a ring at P, an indefinite variety of such figures may be produced.

The **Time** taken by a simple pendulum to effect one complete oscillation—one "*swing-swang*"—depends on the square root of its length l, and varies inversely as the square root of g, the local acceleration of gravity. It is equal to $2\pi\sqrt{l/g}$. Thus a clock will go slower at the equator than in polar regions; a clock will go slower when its pendulum has been lengthened by heat: a clock with a ten-inch pendulum will tick twice as often as one with a forty-inch pendulum.

In S.H.M. the angular velocity ω in the circle of reference is equal to $\sqrt{\dfrac{\text{acceleration at any point of the S.H.M.}}{\text{displacement at that point.}}}$

That is, $\omega = \sqrt{g \sin\theta./l\theta} = \sqrt{g/l}.$

But $\omega =$ (the angular path traversed in time T ÷ the time T); and if the time be so chosen that in it the body describing the S.H.M. would perform exactly one revolution in the circle of reference—that is, if T be the period of one complete oscillation back and fore, or *swing-swang* of the pendulum,— $\omega = 2\pi/\mathrm{T}$; whence $\mathrm{T} = 2\pi\sqrt{l/g}$, and $g = 4\pi^2 l/\mathrm{T}^2$.

The value of T may be otherwise written. The moment of inertia $\mathrm{I} = ml^2$; the weight $w = mg$; whence $\mathrm{T} = 2\pi\sqrt{l/g} = 2\pi\sqrt{\mathrm{I}/wl}$.

We see from the equation $g = (4\pi^2/\mathrm{T}^2)l$ that of those pendulums which oscillate at equal rates in different places, the lengths are proportional to the local intensities of gravity.

The leg acts partly as a pendulum, and in natural locomotion a person with short legs has a tendency to take shorter and quicker steps than a person with longer limbs.

Isochronous Oscillations of a Simple Pendulum.—The equation $\mathrm{T} = 2\pi\sqrt{l/g}$ shows that as long as $(g \,.\, \sin\theta./l\theta)$ may be considered to be equal to g/l,—that is, as long as the angle θ is not too wide,—the period of oscillation does not depend on the amplitude of oscillation, for θ does not enter into that equation; and thus within certain limits a pendulum swings in equal periods through comparatively large or comparatively small arcs.

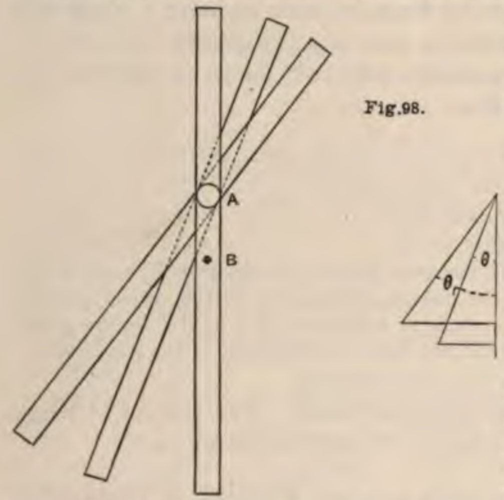
Length of the Ideal Simple Second-Pendulum.—The seconds pendulum performs one "complete oscillation" in two seconds. The equation $g = 4\pi^2 l/\mathrm{T}^2$, when $\mathrm{T} = 2$ sec., gives

$l = g/\pi^2 = 3{\cdot}261608\dot{3}$ feet or 39·1393 inches at the latitude of London, at sea-level, and when the barometer and thermometer are at standard heights (30 inches of mercury, 60° F.)

Work done in moving a Simple Pendulum.—In Fig. 96 suppose the bob to be displaced from C to D; its angular displacement θ is increased to $\theta_,$. When at C its vertical height above B is Bc—that is, $AB - Ac$, or $l - l\cos\theta$. When at D its vertical height above B is $l - l\cos\theta_,$; and its vertical height above C is $cd = Ac - Ad = l\cos\theta - l\cos\theta_,$. When the bob, whose weight is mg, is raised from C to D, the work done against gravity is $mg \times l\,(\cos\theta - \cos\theta_,)$.

Compound Pendulum.—Let a body of any shape be poised on a point or axis of suspension, as from the point A, or an axis passing through the point A in Fig. 79 or Fig. 98; let the radius of gyration of the body with respect to this point A be k; and h the distance of the centre of gravity B below A. If two positions be taken, as in Fig. 98, respectively corresponding to displacements through angles θ and $\theta_,$, the vertical distances between the centre of gravity and the point of suspension are respectively $h\cos\theta$ and $h\cos\theta_,$. The work done by the body during a movement from the higher position to the lower is $mg \times (h\cos\theta - h\cos\theta_,) = mgh\,(\cos\theta - \cos\theta_,) =$ (by page 151) $\frac{1}{2}I\omega^2 = \frac{1}{2}mk^2\omega^2$.

Fig.98.



$$\therefore\ \omega^2 = 2gh\,(\cos\theta - \cos\theta_,)/k^2. \qquad (1.)$$

The work done by the bob of a simple pendulum during a similar displacement is $mgl\,(\cos\theta - \cos\theta_,)$. In this fall it acquires angular velocity $\omega_,$, and kinetic energy $= \frac{1}{2}I\omega_,^2 = \frac{1}{2}ml^2 \,.\, \omega_,^2$. The kinetic energy acquired is equal to the potential energy lost, whence—

$$\tfrac{1}{2}ml^2 \,.\, \omega_,^2 = mgl\,(\cos\theta - \cos\theta_,).$$
$$\omega_,^2 = 2g\,(\cos\theta - \cos\theta_,)/l. \qquad (2.)$$

If the simple pendulum were of such a length as to oscillate at the same rate as the compound one—that is, if ω, the angular velocity of the one $= \omega_,$, that of the other, we find from (1) and (2) that—

$$2gh\,(\cos\theta - \cos\theta_,)/k^2 = 2g\,(\cos\theta - \cos\theta_,)/l.$$
$$\therefore\ l = k^2/h.$$

Hence the compound pendulum oscillates at the same rate as a theoretical simple pendulum of length k^2/h; that is, its period is $T = 2\pi\sqrt{k^2/hg}$.

This length, k^2/h, the length of the equivalent simple pendulum, is the distance between the centre of suspension and the centre of oscillation. Let

MK^2 be the moment of inertia of the compound pendulum round its centre of gravity. Then round the centre of suspension the moment of inertia is $M(K^2+h^2)$ and the length l of the equivalent simple pendulum is $K^2+h^2\,.\,/h$. The distance $h_{,}$ between the centre of oscillation and the centre of gravity is $h_{,}=l-h=(K^2+h^2\,.\,/h)-h=K^2/h$.

Now suspend the body from the centre of oscillation. The moment round this point is $M(K^2+h_{,}^2)=M(K^2+(K^2/h)^2)=$ say, $Mk_{,}^2$. The length $l_{,}$ of the equivalent simple pendulum is $k_{,}^2/h_{,}=\{K^2+(K^2/h)^2\}\div(K^2/h)=(K^2+h^2)/h$. But this is also the value of l. Whence $l=l_{,}$ and the centres of suspension and of oscillation, the distance between which is the length of the equivalent simple pendulum, are interchangeable. Therefore—

If a body of any form be suspended at a certain point and be found to oscillate at a certain rate; if another point in the body be found after trial at which the body being suspended will oscillate at the same rate,—then the distance between these interchangeable points of suspension is the true length of the ideal simple pendulum oscillating at the observed rate.

Problem.—Prove that a cylindrical rod will swing at the same rate, whether it be suspended from its extremity or from a point one-third of the length from the extremity.

Ballistic Pendulum.—Suppose a heavy mass M to be suspended from a point. Into this heavy mass let a bullet of mass m be horizontally fired, striking with velocity v, and let the bullet sink into it so as to form a conjoint mass $M+m$, whose centre of mass is at a distance h below the point of suspension. The energy of the striking bullet is $mv^2/2$; this energy is wholly imparted to the conjoint mass before that mass has had time to become appreciably displaced. In virtue of this energy imparted the whole is displaced so far that the suspending cord comes to make an angle θ with its original position, work being thus done against gravity, equal to $(M+m)gh(1-\cos\theta)$ or $(M+m)gh\,.\,2\sin^2\dfrac{\theta}{2}$; the whole then falls back and thereafter oscillates in ordinary pendulum fashion. The energy imparted and the work done against gravity are equal; whence

$$mv^2/2=(M+m)gh\,.\,2\sin^2(\theta/2),$$

or

$$\sin(\theta/2)=\sqrt{\frac{m}{M+m}}\cdot\sqrt{\frac{1}{gh}}\cdot\tfrac{1}{2}\cdot v,$$

$$\textit{i.e.},\ \sin(\theta/2)\propto v.$$

The throw is such that the sine of half the angle of deflection is proportional to the velocity of the impinging bullet, and therefore to the square root of its energy.

If ω be the angular velocity imparted to the swinging mass, the energy imparted may also be written as $\frac{1}{2}I\omega^2$; whence

$$I\omega^2=2(M+m)gh\,.\,2\sin^2\overline{\theta/2}$$

and

$$\omega=2\sqrt{(M+m)gh/I}\,.\,\sin\overline{\theta/2}.$$

Stability of a Ballistic Pendulum.—The amounts of energy which must be imparted in order to jerk such a pendulum through 15°, 30°, 45°, 60°, 75°, 90°, 105°, 120°, 135°, 150°, 165°, or 180°, are the products of $(M+m)gh$ into 0·0341, 0·134, 0·2929, 0·5, 0·7412, 1·0000, 1·2588, 1·5, 1·7071, 1·866, 1·9659, or 2·0000 respectively. From these figures we see that the Stability of a pendulum in any position—the amount of energy which must be imparted to it in order to throw it *farther* through one degree—is greatest when the throw is already about 90°. Thereafter it diminishes; and when the energy imparted exceeds $2(M+m)gh$, or when the velocity of the impinging bullet exceeds a certain limit ($v = 2\sqrt{(M+m)gh/m}$), the pendulum is thrown right over, and describes a somersault. Problems of this nature are of great importance in connection with the stability of ships.

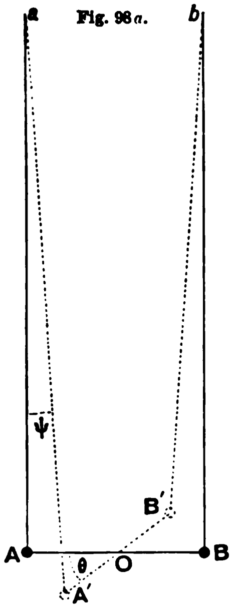


Fig. 98a.

Bifilar Suspension.—If two masses, A, B, at the extremities of a weightless rod AB be suspended by the parallel cords *a*A and *b*B, and if these be displaced so that round O, the central point of AB, there is rotation through an angle θ, there is necessarily a lifting up of both A and B, and there will be components of gravitation tending to stretch each suspending string, and components tending to restore A and B to their original positions. We need only consider the cord *a*A. Its lower extremity is swung horizontally through an angle θ with respect to O; the whole cord is deflected from its vertical position through an angle ψ such that $aA \,.\, \tan\psi =$ the chord of the arc of $\theta = OA \,.\, 2 \sin \overline{\theta/2}$. The component of gravitation tending to restore A′ to A, acting towards A is equal to $mg \tan\psi$. Its moment round O is $(mg \tan\psi) \,.\, (OA \,.\, \cos \overline{\theta/2})$. The whole moment of the couple is $2mg \tan\psi \,.\, OA \,.\, \cos\overline{\theta/2} = 2mg\,(OA^2/aA)\; 2 \sin\overline{\theta/2} \,.\, \cos\overline{\theta/2} = 2mg\,(OA^2/aA) \sin\theta$. The moment of the restoring force is thus proportional to the sine of the angle of deflection, and the oscillations of such a system are approximately simple harmonic.

CHAPTER IX.

MATTER.

THE essential nature of matter—its substratum—is unknown to us; we only know matter by those of its properties which we perceive by our senses. These properties are subject to our direct observation and to our study, and from them we may infer as to the constitution of matter much which we cannot directly perceive.

THE PROPERTIES OF MATTER.

Some of these properties of matter are general, so that if they were other than they actually are, the nature of our universe would be totally different; and thus, in relation to the matter of the existent universe, these properties are with sufficient appropriateness said to be **essential.** For instance, all experience leads us to say that matter must necessarily exist in definite or measurable Quantity; and, since quantity of matter is expressed briefly by the word **Mass,** we say that every body must have a definite mass, for it is to us, with our range of ideas, impossible to conceive of a definite body having a physical existence, but consisting of an indefinite quantity of matter. There are many bodies of which we do not definitely know the mass, but every body must have some definite mass, great or small. If the mass of a body be great, the body is said to be **massive**; if its mass be small, it is usually said to be **light,** though that adjective is properly antithetical to *heavy*, a perfectly distinct idea. A massive gate is difficult to move, not because it is heavy, for gravity does not affect the horizontal swing of a gate on its hinges, except indeed by affecting the friction at the hinges; it is difficult to move because its mass m is great; and since $f = ma$, to produce a given acceleration a, if the mass m be large, the force applied must be great. It would, in theory, be equally considerable were the gate and its hinges removed to a region where the effect of gravity vanished, and the gate had therefore not even a feather's weight.

As regards Quality of matter, experience shows us that every substance with which we are acquainted is made up of one or other or more or fewer of about seventy different kinds of matter. These kinds of matter are called **elements.** They are considered to be distinct kinds of matter, and are called separate elements simply as a confession of our relative experimental impotence, and of our complete failure up to this time to break up any one of them into simpler substances, or to build any one up by any synthetic process. A piece of brass may be by analytic processes resolved into its component copper and zinc, and when copper and zinc are fused together in proper proportions, brass of a similar quality may be made; but no one has broken up either zinc or copper into any simpler components, neither have these metals been made by causing any simpler substances to combine. These metals are, then, Elements; and the substances which they form by entering into combination with other elements, as well as the circumstances under which these combinations are effected, form the subject-matter of the science of Chemistry. The description of any given substance—chlorine, nitrogen, calcium—as an element is thus seen to be entirely provisional. The experience of 1807 may possibly be repeated when least expected. Before that date lime, soda, and potash were enumerated in the list of elements, though, from their strong likeness to metallic oxides, it was vehemently suspected that they were really not elements at all, but oxides. When Sir Humphry Davy brought the galvanic battery of the Royal Institution to bear upon masses of these substances he resolved them into oxygen and into metals never seen till then; and thus the list of elements suffered a profound modification. Now evidence of a speculative character, on the one hand, based (Mendelejeff and Newlands) upon the remarkable relations existing between the chemical properties of the elements and their atomic weights, and also (Gladstone) upon the relations between these chemical properties and the extent to which such of the elements as are transparent or form transparent compounds possess, either when pure or in combination, the power of refracting a beam of incident light; and evidence, on the other hand, of a directly observational character, based (Lockyer) upon the results of spectrum analysis as applied to the stellar bodies, results which seem to show that many elements are decomposed by intense heat into simpler elements; this mass of evidence lends cumulative support to a belief, which is rapidly gaining ground, that all the elements differ from one another only in their intimate structure, and have a common basis which may not impossibly be hydrogen; or in other words, that all the elements are structural modifications of one form of Matter. Thus even the alchemist's dream of the transmutation of metals cannot now be treated with such unmitigated contempt as it received thirty or even ten years ago, though it may continue to be a dream to the realisation of which no approach is possible, on account of the necessary limitations of our experimental appliances.

In reference to Space: every mass of matter must at any instant occupy a definite **volume** of space: it must have some **Extension** in tridimensional space—it must have dimensions expressible in terms of length, breadth, and thickness. As a natural consequence every mass of matter must have some definite **form,** whether that form be imposed on it by surrounding matter or not—whether, like a solid rock, it have a form of its own, or, like water in a basin, it have a form which depends partly on the form of the vessel containing it, or, like gas confined in a gasometer, its form as well as its volume depend on that of the vessel in which it is enclosed.

Among the properties of matter which are said to be essential we usually find mentioned that known as **impenetrability.** This means that two masses of matter cannot occupy the same space. In view of the peculiar phenomena attending the solution of substances in water—a very large quantity of different salts being soluble in water without materially increasing its bulk—we cannot state this absolutely. But matter is believed to be composed of minute masses called Molecules, and of these it is held to be true that two cannot coincide in position. But these are not in contact even in solids, and so a body is always free to shrink in size—as, for instance, when it is cooled down or compressed—because the distance between its molecules is capable of diminution; and thus a quantity of water, which is not a continuous substance, may receive between its own molecules a number of molecules of other substances, and so form a solution, without entirely sacrificing that freedom of movement past one another which its own molecules possess—that is, without entirely losing its fluidity. The **impenetrability** of matter is, then, a property of **molecules,** not necessarily of masses.

If a certain bulk of metallic potassium contain 45 atoms or half-molecules of potassium, an equal bulk of caustic potash will contain 70 atoms of potassium and 140 of hydrogen and oxygen.

In respect of Time: Lapse of time brings about no change either in the quantity of matter or in its quality—that is, matter is **indestructible** both in regard to its total quantity and to the quantity of each element. Such is the ordinary belief; the former statement is in accord with the universal experience of Chemistry; but he would be bold who, from the experience of mankind on the surface of the earth, should venture to deny that in the interior of this planet there may even now, as the earth is cooling, be an increase taking place in the quantity of the more complex at the expense of the more simple elements; not to speak of the positive probability which spectrum analysis lends to a belief that this kind of action is actually going on in the fixed stars. Be that as it may, within our experimental range of knowledge there is no transmutation of elements and no destruction or creation of matter. Matter changes its forms and its combinations incessantly, but it can always be traced up by chemical analysis. A closed glass tube containing oxygen and powdered charcoal weighs exactly the same after the charcoal has been induced to burn in the oxygen, and thus to disappear, as it did before that action; the gaseous carbonic acid producéd is, though invisible, equal in total weight, and therefore in mass, to the sum of the carbon and oxygen which composed it.

The general properties of matter.—By a distinction which is somewhat arbitrary the preceding properties are said to be essential, while those of inertia, weight, divisibility, and porosity are said to be general, because found to be possessed by all matter. The statement that **inertia** is a general property of matter simply means that Newton's first law of motion is a universal result of experiment.

All bodies possess **weight** at the earth's surface and within experimental or observational limits. A mass placed on the earth's surface is attracted by the earth, by the sun, the moon, the planets of the solar system, and in a less degree—the attraction being so small that we have no direct evidence of its existence—by the distant fixed stars; the resultant differs so very little from the direct attraction of the earth that the latter alone may be considered as pulling the mass downwards towards its centre; but this is only a first approximation, for a more careful discussion of the intensity and direction of the resultant force helps us to explain the phenomena of the Tides.

All masses are **divisible**; the only question which here emerges is that as to indefinite divisibility. Is a given mass—

say of chalk—divisible to infinity, or would we after division effected with sufficient frequency obtain a small mass of chalk which, if further divided, would be no longer chalk, but might perhaps be broken up into lime and carbonic acid? The facts of chemical equivalence, as ascertained by the balance, seem to be susceptible of no natural explanation other than that matter is made up of such ultimate particles, and hence matter is concluded not to be infinitely divisible. We shall recur to this subject.

Nobert engraved parallel lines on glass at a mutual distance of 1/40,000 cm., half the wave-length for blue light. No microscope made can show these as distinct lines.

All matter is porous or possesses **porosity**. Hydrogen gas leaks through white-hot iron under pressure; cold water can be pressed through iron, as may be seen in Bramah's hydraulic press, or through lead, as in Francis Bacon's famous experiment, in which he took a shell of lead filled with water and compressed it; the water oozed through the lead and stood in drops and beads on the surface of the shell.

Contingent properties of matter.—Some of the properties of matter are contingent, and depend on the particular kind of matter considered and on the surrounding circumstances. As examples, we may take the facility with which a body is heated, the rate at which heat can run along it, the ease or difficulty with which light can pass through it, and so on.

The quantity of matter per unit of space is defined as the **density** of the mass filling that space. Thus a gramme of water occupies a cubic centimetre, and according to the C.G.S. or centimetre-gramme-second system of measurement, the density of water is (1 gramme/1 c.c.) = 1. More accurately (Kupffer), 1 cub. cm. water at 3°·9 C. weighs 1·000,013 standard grammes, and its density is 1·000,013. In the same way the density of lead is 11·5, because 1 c.c. weighs 11·5 grammes.

In general, if m be the mass contained in volume V and ρ the density, $m/V = \rho$, or $m = V\rho$.

Every kind of matter, simple or compound, has a special density of its own; thus a given bulk of lead contains 11·5 times as much mass as the same bulk of water. Water is taken as the standard of density; its specific density is said to be = 1, though sometimes, in estimating the density of fluids, it is, in order to avoid decimal fractions, reckoned as 1000. In the same way, the specific density of lead is 11·5, and those of all substances may be experimentally found and recorded in a **table of specific densities,** or, as it is more commonly called, a table of specific gravities. These are experimentally found by taking advantage of the fact that weight is proportional to mass; $w = mg$. The piece of lead which will occupy a given space not only contains $11\frac{1}{2}$ times as much mass, but also *weighs* $11\frac{1}{2}$ times as much as the quantity of water which would fill the same space. Thus the density of a body, as compared with that of water, presents a ratio—its *specific density*—which is numerically identical with the ratio—its *specific gravity*—of its weight to that of an equal bulk of water.

If m and $m_{,}$ be the masses of equal volumes of the body and of water, V these equal volumes;

$$\text{Sp. density} = \frac{\text{density of the body}}{\text{density of water}} = \frac{m/\text{V}}{m_{,}/\text{V}} = \frac{m}{m_{,}}.$$

$$\text{Sp. gravity} = \frac{\text{weight of the body}}{\text{weight of equal bulk of water}} = \frac{mg}{m_{,}g} = \frac{m}{m_{,}}.$$

Hence sp. density and sp. gravity are numerically identical ratios.

Sketch of the Experimental Methods of finding the Specific Density of Bodies.

(a) **Solids.**—1. Weigh the body in air (properly *in vacuo*); measure its bulk by dipping it (suspended by a thin string) into water contained in vessel A, and observing the rise of level in that vessel; take it out: out of a known quantity of water in vessel B pour enough water into vessel A to produce an equal rise of level in the vessel A; find the weight of the water that has been poured out of vessel B. Then the weight of the body ÷ the wt. of the equal bulk of water poured out of B = the sp. density of the body. The practical objection to this method is, that the body when taken from vessel A removes some of the water.

2. Weigh the body in air; put it in a vessel—a "specific-gravity flask"—marked distinctively at a certain level; fill with water up to the marked level; weigh. Empty the vessel and fill with water alone up to the mark; weigh.

Then weight of vessel, body and water up to level, $= \text{V} + \text{B} + w$.
" " vessel and water alone up to level $= \text{V} + \text{W}$.
$\therefore$ wt. of water which replaces the body is $\text{W} - w$,
and wt. of body is B.

$$\therefore \text{Sp. gr.} = \frac{\text{wt. of body}}{\text{wt. of equal bulk of water}} = \frac{\text{B}}{\text{W} - w}.$$

3. Take advantage of the following proposition in Hydrostatics:—A body suspended in a liquid is buoyed up by that liquid to such an extent as to diminish in apparent weight by an amount equal to the weight of the bulk of the liquid which it may be considered as displacing. If a body of exactly the same density as water be suspended in water, it will neither sink nor rise; its apparent weight will = 0; its sp. gr. = (wt. in air ÷ loss in water)

$= 1/1 = 1$. If it be more dense it will sink, but slowly, for while its mass is unaltered, the force acting on that mass is apparently diminished. If it be less dense than water, it will rise; its weight will appear to be less than nothing, negative.

A cork of volume v and density ·8 will have a mass $v \times {\cdot}8 = {\cdot}8v$. An equal bulk of water would have a mass $v \times 1 = v$. The weight of this mass of water would be mass $\times\, g = vg$; the weight of the mass of cork is similarly ${\cdot}8vg$. The apparent weight of the mass of cork will be ${\cdot}8vg - vg = -{\cdot}2vg$; *i.e.*, its downward acceleration will be negative, or the cork will move upwards. This upward accelerating force, $= {\cdot}2vg$, acting on a mass ${\cdot}8v$, will produce an acceleration (since $a = f/m$) $= {\cdot}2vg/{\cdot}8v = \frac{1}{4}g$, and but for friction in the water the cork would rise with an upward acceleration of 8·05 feet per second.

Therefore weigh a body in air; suspend it by a fine thread from the pan of a balance, so that it just sinks wholly into water. It will appear to weigh less. Then the sp. gr. = (weight in air ÷ apparent loss of weight in water), or, accurately, (weight *in vacuo* ÷ apparent loss of weight in water). If the body be lighter than water, attach to it a piece of heavy substance—say, lead—of known weight and known density (lead = 11·5). Then the weight of the lead is $11{\cdot}5\ Vg$; that of the light substance is $\rho\, V_{,}g$; together they weigh $11{\cdot}5\ Vg + \rho V_{,}g$. In water they weigh $10{\cdot}5\ Vg + (\rho - 1)\, V_{,}g$. The loss of the lead alone must be Vg, or 1/11·5th of its weight; whence the loss of the light body in water is easily determined by difference; and its density, the fraction (weight in air ÷ loss in water), found.

4. If the solid be soluble or be otherwise acted on by water, some other fluid is made use of, such as naphtha or turpentine, the specific density of which is known. If the specific density of the solid with reference to the liquid, found by any of the above mentioned methods, be s/l, and the sp. d. of the liquid with reference to water be l/w, then the product of these specific densities $= s/l \times l/w = s/w$ is the sp. d. of the solid as compared with water.

(b) **Of Liquids.**—1. Fill a weighed vessel up to a certain mark with the liquid: the whole weighs so much: by difference find the weight of the liquid alone. Empty the vessel and repeat the process with water; the water which fills the vessel up to the same mark weighs so much. Then the sp. gr. of the liquid = (wt. of given bulk of liquid ÷ wt. of same bulk of water).

2. A body immersed in water appears to lose $1/x$th of its weight; immersed in the fluid to be tested it appears to lose $1/y$th of its weight. This body is x times (x being a whole number or a fraction) as dense as water, y times as dense as the fluid. The density of the fluid is to that of water as the apparent loss $1/y$ in the fluid is to the apparent loss $1/x$ in the water—that is, as $x : y$; and the sp. d. of the fluid is x/y.

3. By the use of Hydrometers, Alcoholometers, and the like. The principle of these instruments is the following:—A body which floats in water without being wholly submerged is in equilibrium under the action of two forces—(1) the weight of the whole body; (2) the buoyancy of the water, equal to the weight of the part of the water displaced. Thus ice floats in water with $\frac{918}{1000}$ of its bulk submerged. If the volume of a mass of ice be v and its specific density ρ, the weight of the mass is $v\rho g$, and the weight of the mass of water equal in volume to the submerged part of the mass of ice is ${\cdot}918v \times g$. These are equal; $v\rho g = {\cdot}918vg$; whence $\rho = {\cdot}918$, the specific density of ice as compared with water = 1. Thus the sp. d. of a body floating in water = (part immersed ÷ whole volume). This method might be used to determine the sp. d. of solids were it not for practical difficulties of

measurement, which, for the sake of explanation, we here suppose overcome. If a mass of ice be placed in chloroform, it will float with ·6125 of its mass submerged; the sp. gr. of ice in reference to chloroform is ·6125. The problem comes to be this: Water is $\frac{10000}{9180}$ times as heavy as ice, chloroform is $\frac{10000}{6125}$ times as heavy as ice—What are the relative densities of water and chloroform? Chloroform is heavier than water in the ratio of $\frac{10000}{6125} : \frac{10000}{9180}$, or 9180 : 6125, or 1·497 : 1; that is, its sp. d. is 1·497. We see, then, that the comparative sp. d. of liquids could be ascertained by finding to what depth bodies floating in them will sink, if we could perform the necessary measurements with the requisite accuracy. But instruments which have been graduated by the instrument-maker, who observes and marks on them with more or less care the depth to which they sink in various liquids of known specific gravity, and marks the corresponding position by proper figures, are in common use for promptly ascertaining the density of various liquids. They are usually made with large bulbs, generally loaded with mercury or lead-shot, in order that they may float vertically, and they have a narrow stem which is graduated. They are caused to float in the liquid whose density is to be found: the graduated stem stands more or less out of the liquid; the figure upon it which corresponds most nearly with the general level of the surface of the liquid is read off and recorded. It is convenient in effecting such readings to arrange a piece of black paper to serve as a background, and to place the liquid to be tested in a glass vessel.

Rousseau's Densimeter bears at its summit a little cavity. In this a cubic centimetre of the fluid is placed: according to the depth to which this makes the instrument sink in water is the density of the fluid determined, according to the graduation performed beforehand by the instrument-maker.

Fahrenheit's Areometer consists of a similar instrument provided at its summit with a little platform or pan. It is placed in water at 3°·9 C., and loaded by small additional weights placed in the pan, until the areometer sinks in the water just so far that the level of the water coincides with a certain mark on the instrument. Then the sum of the known weight of the areometer + the additional weights put on $= W =$ the weight of the bulk of water displaced. It is removed, dried, and placed in the liquid to be tested, and again loaded till it stands *at the same level* as before. The weight of the instrument + the weights now added $= W_{,} =$ the weight of an equal bulk of the liquid to be tested. Then $W_{,}/W$, the ratio of these weights, is the specific density of the liquid in question. If, for example, the instrument weigh 800 grs. and float in water at the proper level when loaded with 200 grains; loaded with 80 grains it floats at the same level in an aqueous solution of ammonia: the total weights are 1000 and 880; the density of ammonia solution is ·880.

Nicholson's Areometer, which is sometimes used for determining the density of solids, is a modification of these instruments. It bears two platforms—one at the summit, out of the water, and a lower one in the water. The body whose density is to be found is placed on the upper pan along with weights just sufficient to sink the areometer to a certain mark. The body is transferred to the lower pan, beneath the level of the water. More weights must be placed in the upper pan to cause the areometer to sink to the same level; these weights are equal to the apparent loss of weight suffered by the body when placed in the water on the lower pan. Then (weight of body ÷ the additional weights) = density of the solid.

4. By Specific-gravity Bulbs. Bulbs are sold which are known to float without rising or sinking in liquids of the sp. gr. marked in numbers upon them. A number of them are thrown into the liquid; those which bear too high a number sink, those which are too light rise; the one exactly corresponding, if there be one, is at rest anywhere in the fluid.

(c) **Of Gases.**—The density of a gas is found by an application of the same principles as those employed in determining that of a liquid. A copper or glass vessel, as light as is consistent with adequate bulk and strength, has the contained air extracted from it by means of a good air-pump; it is then weighed empty; it is very slowly filled with the gas and left for some time in communication with a reservoir of it, its stopcock is closed, and the vessel thus filled with the gas is again weighed: the weight of the volume of gas which fills the vessel is thus ascertained. It is again emptied by means of the air-pump, and then air is allowed to enter it. After standing for some time in order to acquire the temperature of the room, it is again closed and weighed: thus the weight of that volume of air which fills the vessel is found. Then the weight of the gas ÷ wt. of equal vol. of air = density of the gas in reference to air as a standard.

It is convenient for chemical purposes to take the rarest gas,—that is, the least dense gas, Hydrogen,—as a standard of density, and then we say that the specific density or sp. gr. of Hydrogen is 1, that of air 14·47, that of Oxygen 16, and so on.

A cubic centimetre of Hydrogen weighs ·0000895682 grammes, a cubic cm. of air weighs (at Paris) ·0012932 grammes at the freezing point of water and at the barometric pressure of 760 mm. of mercury. A cub. cm. of the lightest liquid known —liquefied marsh-gas (which liquefies at $-73^\circ{\cdot}5$ C., if the pressure be raised to 56·8 atmospheres)—weighs, according to Wroblewski, 0·37 gramme; the same volume of water weighs 1 gramme at $3^\circ{\cdot}9$ C., or more accurately it weighs 1·000013 standard platinum grammes; the same volume of lithium, the least dense solid metal, weighs ·5936 grammes, and of hammered platinum weighs 21·25 grammes. Thus we see that, bulk for bulk, solid platinum is nearly 240,000 times as dense as gaseous hydrogen.

For the estimation of Vapour-density, see p. 359.

THE STATES OF MATTER.

We know that in popular language there are said to be three states of matter, the solid, the liquid, and the gaseous. Closer observation shows us that these merge into one another, and that, in addition to these, matter exists in conditions more recondite; thus we shall have to pass in review the following conditions:—(1) Rigid solid, (2) soft solid, (3) viscous liquid, (4) mobile liquid, (5) vapour, (6) critical state, (7) gas, (8) radiant matter. These may be all classified under the two heads of Solid and Fluid, which are not, however, separated from each other by any distinct line of demarcation.

A **perfect solid** is an ideal body which, when brought into a condition of stress, if it do become deformed, becomes deformed to a definite extent, and then retains its newly-acquired bulk or shape for an indefinite period of time, so long, that is, as the same condition of stress is continuously kept up in it; thus the ratio $\frac{\text{deformation}}{\text{deforming force}}$ or $\frac{\text{strain}}{\text{stress}} = \textit{const.}$ A steel spring becomes stretched by the weight of a mass hung upon it: it is stretched to a definite extent, and it then retains the same length for any length of time, or rather it appears to do so when we do not use sufficiently accurate measurements; if it did so perfectly, steel would be a perfect solid. A **fluid,** on the other hand, is a substance which, if continuously acted upon by deforming force, continuously yields with continuously increasing deformation; and thus the ratio $\frac{\text{deformation}}{\text{deforming force}}$ is not constant, but increases with the lapse of time.

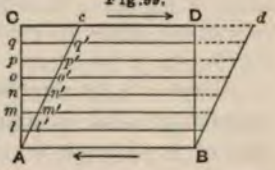
The force so applied to a fluid must not be one which is kept up equably over its whole surface—in other words, there must not be a "Hydrostatic Stress"—but it must act more effectively on one part of the mass of the fluid than it does on another.

A **rigid solid** is one which, when a stress is applied to it, experiences no deformation, no strain; and therefore in a perfectly rigid solid the fraction (strain/stress) = 0, and continues so for an indefinite period of time. This is an ideal; no substance is absolutely rigid; but we may form a sufficient idea of a rigid solid by considering an anvil on which a nail is placed: the anvil appears undeformed by the weight of the nail—it appears to be absolutely rigid. There is, then, no body absolutely solid, no substance absolutely rigid; the bodies which we call Rigid Solids are found to yield, and to yield continuously if we experiment upon them in sufficiently small masses (such as thin wires), and act upon them with sufficient forces for adequate periods of time. In practice a body is said to be Solid if its deformation remain practically or sensibly constant for a long time; it is said to be Rigid if the deformation associated with a given stress be exceedingly small, or, in other words, if the fraction (stress/strain), the *Coefficient of Rigidity,* be very large. A **soft solid** is one in which the coefficient of rigidity is very small—that is, it is a solid in which a small stress accompanies great deformation. A mass of jelly is very easily deformed; but if a moderate force, such as the weight of a caraway seed or currant, be applied to jelly even of very thin consistency, the deformation produced by it is approximately constant; the jelly does not flow over the small load, and is therefore a true solid, though it is soft.

It would seem at first sight rather an abuse of terms to declare that thin jelly is a solid, and that such substances as sealing-wax, pitch, and cobbler's wax are fluids; yet these latter are fluids because they flow, because they suffer continuous deformation under the action of a continuous force. A stick of sealing-wax supported at its ends yields continuously to its own weight; a mass of sealing-wax or pitch will flow down hill; a cake of cobbler's wax of a definite form will soon lose its sharpness of outline; a cake of this material placed in water, with bullets on it and corks under it, will be traversed by the bullets at the rate of about a quarter of an inch per month, and by the corks at a somewhat slower rate; the wax slowly flows round them as they sink or rise under the influence of their relative weights, just as water would much more rapidly do. These substances are, then, Fluids; but their flow is extremely slow, or, in other words, their **viscosity** or resistance to flow is extremely great. Treacle is another example of a viscous fluid; strong syrup, weak syrup, very weak syrup, cold water, alcohol, hot water, ether, are examples of liquids whose flow is successively more rapid, whose viscosity is less; and the last mentioned are said to be **mobile** liquids, though perfect mobility, the perfect absence of viscosity, is an ideal attribute not possessed by any actual fluid.

The Coefficient of Viscosity.—In Fig. 99 let ABCD represent a volume of fluid comprised between two planes passing through AB and CD, and let a shearing force represented by arrows act on the fluid for unit of time. The deformation, since the substance is fluid, goes on increasing; let A*cd*B represent a form assumed through the different layers (imagined separate as in Fig. 25) slipping over one another, the lowest, AB, being relatively at rest. If the velocity imparted to each stratum be proportional to its distance from AB, the ratios of their displacements ll', mm', nn', oo', to their respective distances, Am, An, etc., will be equal; *i.e.*, $\frac{ll'}{\mathrm{A}l}=\frac{mm'}{\mathrm{A}m}=\frac{nn'}{\mathrm{A}n}=\frac{oo'}{\mathrm{A}o}=\frac{\mathrm{C}c}{\mathrm{CA}}=\tan\phi$; or, in other words, this is a proper Shear.

Fig. 99.



This slipping of one imaginary stratum over another is retarded as by friction, for each stratum rubs against or is delayed by the one next to it; and thus the retardation acts as a force would do opposed to that which tends to produce movement—*i.e.*, to the shearing force—and it has to be measured over every unit of surface over which the retardation is effected. The shearing force itself is measured as a force exerted parallel to the planes passing through AB and CD (and therefore, in the general case, tangential to the surfaces of the different strata), and acting on each unit of area of these surfaces.

The ratio $\frac{\text{Shearing Force}}{\text{Shear per unit of time}} = \frac{\text{F}}{\tan\phi} = \mu$, the Coefficient of Viscosity. When $\phi = 0$, there is no deformation in unit of time, and the viscosity is infinite; when ϕ is very small, the deformation is very small in unit of time—that is, it is very slow, and the viscosity is very great. When the viscosity is very small, the deformation produced in unit of time is very great, or the obstruction offered to flow is very small.

If the distance AC $= 1$, and the displacement of CD in unit of time (*i.e.*, its velocity) be also $= 1$, then $\tan\phi = 1$, and the shearing force *is equal to* the coefficient of viscosity; whence the Coefficient of Viscosity may be measured like the coefficient of friction, in terms of a force—that is, by "the tangential force on the unit of area of either of two horizontal planes at the unit of distance apart, one of which is (relatively) fixed, while the other moves with the unit of velocity, the space between being filled with the viscous material" (Maxwell).

Relation between Rigidity and Viscosity.—The measure of Rigidity in the ideal perfect solid, the ratio of the deforming Force to the total Deformation produced by it, remains constant for any period of time during which the deforming force applied remains constant, and this ratio or fraction is independent of the unit of time. The measure of viscosity in a fluid, on the other hand, is the ratio of the deforming force to the deformation produced **in a unit of time.** Whether the unit of time chosen be great or small, the numerical value of the fraction $\frac{\text{Force applied} \times \text{Time } t}{\text{Shear produced in time } t}$ ought to be the same. Yet it is found that if an exceedingly small unit of time be chosen, the coefficient of viscosity may have some experimental value different from that found when the unit of time is larger. If, for instance, Canada balsam be stirred with a spatula, it will be found (Maxwell) to be doubly refracting. This shows that it is for the instant, and locally, in a condition like that of a stretched or a compressed solid, and that, though it is liquid if sufficient time (time of relaxation) be allowed it, yet under the action of impulsive forces its coefficient of viscosity is so high that it practically behaves after the fashion of a solid.

There is no substance perfectly solid, and hence when an apparently solid substance in a suitable form—for convenience, that of a wire—is exposed to a constant distorting force, though its form is found to be affected to a certain extent (say by lengthening or by twisting), and it appears soon to reach the limits of its distortion, which may then be measured, the apparent rigidity of the substance being thus ascertainable; yet in general it will be found that the prolonged application of a constant force induces a constant slowly-increasing additional distortion, the substance then acting as a fluid of exceedingly great viscosity. Further, we now know by the experience of manufacturing industry that lead and even iron, when exposed to sufficient pressure, can be made to flow slowly; and the operation of wire-drawing involves among the particles of the metal drawn, as they pass through the aperture in the plate, a relative movement which is similar to that of the particles of a flowing fluid.

When the viscosity of a fluid is infinite, there is no difference between that fluid and a rigid solid. It is supposed by some that the matter at the centre of the earth approximates to this condition. If the earth have a crust about 25 or 30 miles thick floating on a melted magma, at increasing depths the pressure (which near the centre must amount to about 45,000,000 lbs. per square inch) would compress the liquid so much that, though melted, it would have a viscosity so extreme that the mass would have the same relation to extraneous forces as a very rigid solid body would have. The earth would, in consequence of this, comport itself as if it had a solid nucleus floating in and merging by gradations of relative softness into a thin liquid layer on which the crust floats. (Osmond Fisher.)

Substances in the solid and liquid form are broadly distinguished from those in the **gaseous** condition by the two following characteristics. In the first place, the former have a free surface, while the latter cannot permanently retain a free bounding surface independent of the vessel containing them. Secondly, solids and liquids tend (apart from volatilisation) to assume a definite limited bulk and density; while gases always tend to assume an infinite volume and a correspondingly small density. If any definite quantity of a gas be confined within a limited space, it will always fill that space and press against the sides of the containing vessel; and it will subject that vessel to stress or pressure which the vessel must be strong enough to withstand. Thus a gas, with its **tendency to indefinite expansion,** has Elasticity of Volume: work has to be done in order to compress it, and when compressed it tends to restore the work done upon it. Under ordinary circumstances gases are prevented from expanding to an indefinite extent by the pressure of the mass of air which lies upon the earth's surface, which is drawn down towards and pressed upon that surface by the attraction of gravity, and which consequently compresses itself and all objects at the earth's surface with an "atmospheric pressure" of about 15 lbs. per square inch. Hence a closed flask containing air is subject to two equal and opposite pressures, whose resultant is nil; the air in the flask tends to burst it outwards; the air external to it tends to make it collapse: between the two pressures the flask has no need for strength. If, however, the air be in any way extracted from the flask, the external pressure will alone act, and the flask may collapse. In a steam boiler the internal pressure, the tendency of the steam to expand, is greater than the external pressure per square inch, and the boiler must be of sufficient strength to provide for the difference.

The pressure exerted by a confined gas is equal over the whole internal surface of the vessel containing it; and a pressure equal to the weight of a lbs. per sq. in. exerted on a piston forced into a cylinder of gas is communicated to every sq. in. of the inner surface of the cylinder, for the face of the in-moving piston is at any instant a part of the inner surface of the cavity containing the gas; and if any other part of the wall of the cavity—*e.g.*, the face of a second piston—be at the same time movable outwards, the excess of internal pressure will cause it to yield. If its area be the same as that of the in-moving piston, the out-moving piston would be acted on by an equal total pressure, and would move to exactly the same extent as the in-moving one.

There is an apparent paradox in the statement that a single-inch piston pressed in with a pressure equal to the weight of 60 lbs. per square inch will communicate a pressure of 60 lbs. to every square inch of a cylinder, however large; but there is no law of Constancy or Conservation of Force: there is one of Conservation of Energy, which in this instance is rigorously respected. The work that can be done by a second piston allowed to move outwards is (friction not being considered) equal to that done by the compressing piston, whatever be their respective areas: the range of movement and the area vary inversely: a smaller second piston may be pushed through a larger space than the first, a larger one through a smaller space.

In this way machinery may be driven at the distance of a mile: air is driven into a tube; at the other end of the tube there is a cylinder connected with the tube, and by the alternating admission of the pressure to one and the other face of a piston movable in this cylinder, the piston is caused to move, and the energy is thus applied at a distance.

If pressure p (*i.e.*, p dynes per sq. cm. of the bounding surface) will keep a certain quantity of gas within a space v, pressure $2p$

is found to be required to confine within an equal space twice as much gas: pressure xp will keep x times the quantity of gas within the same space. The densities in these examples are in the ratio $1 : 2 : x$, and the pressures are proportional to the densities (the same kind of gas being supposed to be used throughout) or to the quantities of gas forced into a given space.

The pressures being proportional to the densities acquired under their action,

$$p \propto \rho\,;\ \text{but } m = v\rho\,;$$
$$\therefore\ p \propto m/v, \text{ or } pv \propto m\,;$$

or for a given mass, $pv = c$; or $p \propto 1/v$, the usual form of Boyle's Law.

This statement may be otherwise expressed in the form of **Boyle's law,** that the pressure exercised by a given mass of gas varies inversely as the volume of the space within which it is confined; or that the space occupied by a given quantity of gas varies inversely as the pressure. Thus, if a quantity of gas occupying one cubic foot at a pressure of 15 pounds per square inch were compressed by a piston forced down with an additional pressure of 15 lbs. per square inch, the total pressure being doubled the volume would be halved; if, on the other hand, the pressure were diminished to half by the piston being pulled out with a force equivalent to 7½ lbs. per sq. in., the volume of the gas contained in the cylinder would be doubled. There are no perfect gases which absolutely obey this law at all temperatures and pressures; but a substance at a temperature and pressure far removed from those at which it will be condensed into a liquid approximates to this condition.

Now suppose that a quantity M of a liquid is placed in a vessel which it does not completely fill, every other substance, such as air, being removed from the vessel, which is then closed. The liquid does not fill the whole vessel; it has a free surface. Above this free surface there is a space, which becomes filled with part of the liquid substance, volatilised into a gaseous form. Let the quantity of liquid which has assumed the gaseous form be represented by y: the remainder, $M - y$, is still in the liquid form. The proportion volatilised (y/M of the whole) depends on the temperature as well as on the space which has to be filled. At another temperature some different proportion will be volatilised. When the liquid is heated this proportion rapidly increases. But as we have already seen, the pressure exerted by a confined gas on the vessel containing it depends directly on the amount of it. Hence in this case the pressure exerted rapidly rises as the temperature rises. Gas having this relation to the liquid form of the same substance, confined with it in a vessel otherwise empty, and in contact with it, is called the **vapour** of that substance; if it be compressed or cooled, it partly condenses into liquid. Even though not in contact with the liquid, if the gaseous form of a substance be compressed or cooled just so far that any further condensation or cooling will cause the deposition of some of it in the liquid form, it is said to be a vapour. In some cases a vapour condenses directly into a solid; *e.g.*, arsenious acid.

The term Vapour is often applied in a wider sense to the gaseous form of a liquid or solid substance—as, for instance, ether-vapour, chloroform-vapour, the vapour of arsenious acid; and then those vapours which are on the point of condensation are called **saturated vapours,** while those which can suffer a certain amount of compression or cooling without condensation are called **unsaturated vapours.** In this sense (with the exception, as yet, of pentafluoride of phosphorus) all gases are unsaturated vapours; for they can all be condensed by the simultaneous application of sufficient cold and sufficient pressure. Oxygen has been condensed; at a pressure of 300 atmospheres, and at the temperature of — 39° C. (Cailletet), there is no condensation, but when the gas is liberated it becomes foggy; according to Pictet it is liquefied at 320 atmospheres and — 140° C.; and then, upon allowing a jet of this liquid to escape into the air, the escaping jet of liquid oxygen becomes extremely cold and is partly solidified, while the remaining oxygen in the vessel becomes cloudy. Hydrogen treated according to Cailletet's method gives a mist. Pictet evolved it under a pressure of 650 atmospheres from a mixture of formiate and hydrate of potash; the hydrogen, on being chilled by a freezing mixture to — 140° C., and allowed to issue in a jet, appeared as a steel-blue stream of liquid, the light reflected from which, being partly polarised, revealed the presence of solid particles in the liquid, while the tube of exit became blocked with solid hydrogen. Ozone is with comparative readiness condensed by Cailletet's method to a bright-blue liquid. The term Vapour is used in still another sense—that is, a gas at such a temperature that by the application of pressure alone it can be condensed into a liquid. In this sense carbonic acid below 30°·92 C. is a vapour; above that temperature it is properly a gas, for no amount of pressure alone will liquefy it.

The Critical State.—When a liquid and its vapour are together in a tube otherwise unoccupied and are exposed to heat, there arrives a temperature at which the singular phenomenon of a blending of the liquid and gaseous (or vaporous) states is observed. If a capillary tube, for instance, be filled with liquid CO_2 and slightly heated, some of the carbonic anhydride will escape: the tube may be sealed up, and will then contain nothing but liquid CO_2 and the saturated vapour of CO_2. If they be heated to 30°·92 C., and if there be sufficient CO_2 present to produce a pressure above 73 atmospheres, the free surface of the liquid becomes blurred and merges into the superjacent gas: above this temperature the tube is full of what appears to be nothing but gas: if cooling be permitted there is a flickering seen in the tube, and the liquid and the gas again separate.

Some say that the liquid and the gas mutually dissolve each other; others (Ramsay) point out that the liquid CO_2 rapidly becomes lighter, while the confined vapour of CO_2 becomes denser, at higher temperatures, and that at the critical temperature and under sufficient pressure these two states meet and become undistinguishable.

If the CO_2 gas be exposed to a temperature above 30°·92 C. and be subjected to any pressure above 73 atmospheres, it will still be a gas: allow it to cool, the pressure being kept up, and it will be a liquid after it passes 30°·92 C.; and yet the transition is unobservable. If pressure and temperature be allowed to fall together, the flickering already mentioned is produced.

If the liquid originally fill the tube and then be heated to the critical temperature, the tube becomes filled with gas, but the precise mode of transition from the one state to the other cannot be observed. If the tube thus containing gaseous CO_2 at a high pressure be locally cooled, there is local condensation and flickering.

This temperature, 30°·92 C. for carbonic acid, is called the Critical Temperature. If the temperature of the gas be above 30°·92 C., no pressure can condense it into a liquid; if it be just below that point, a pressure of 73 to 75 atmospheres is competent to effect its liquefaction.

Water filling a sufficiently strong boiler might be exposed to a low red heat, 720°·6 C., and would then be transformed into a gas exercising such enormous pressures as to make any experiments upon it excessively difficult; yet it is known (see p. 358) to present this phenomenon at that temperature.

As for the gases, oxygen, hydrogen, and the like, whose condensation has only recently been effected, their critical temperature is very low in the thermometric scale (oxygen, −113° C., Wroblewski), and exceeding cold is a necessary condition of their condensation under pressure.

At the critical temperature, matter under sufficient pressure is in the Critical State; if heated a little more it is undoubtedly gaseous; if allowed to cool a little it is undoubtedly liquid, and is far less compressible; and if the pressure be kept up the transition is unrecognisable. By no optical test can the liquid just below the critical temperature and the gas just above that temperature be distinguished.

If a solid be dissolved in a liquid, and if the whole be heated under sufficient pressure to a temperature above the critical point, the liquid is now gas, and yet the solid remains dissolved in it (Hannay). Iodide of potassium, for instance, or chlorophyll, if dissolved in alcohol and treated in this way, will remain in solution in gaseous alcohol at 350° C.

There is thus under varying pressures perfect Continuity of the liquid and the gaseous state at all temperatures above the critical point; and this critical temperature is, in reference to each liquid, to be found by experiment.

When gaseous matter has been rarefied to a very great degree it assumes remarkable properties, of which the most striking is that such exceedingly rarefied gas or **ultragaseous** matter can be induced—as we shall see under Electricity, p. 603—to exercise pressure specially on localised areæ of the walls of the containing vessel, and by this concentrated pressure to produce mechanical and luminous effects characteristic of the so-called Radiant Matter.

The Ether.—We have already said that we can know matter only by those of its properties which we perceive by means of our senses. The existence of any form of matter is to us only an inference from the phenomena to which it gives rise; and if a large group of phenomena find their best or their only explanation in the assumed existence of a form of matter of an unfamiliar kind, the evidence for its existence is of exactly the same character as that on the ground of which we believe ourselves entitled to assert the existence of any kind or form of matter whatsoever. The phenomena of Light are best explained as those of undulations; but undulations—even in the most extensive use of the term, as signifying any periodic motion or condition whose periodicity obeys the laws of wave-motion—must be propagated through some medium. Heat while passing through space presents exactly the same undulatory character and requires a medium for its propagation. Electrical attraction and repulsion are explained in far the most satisfactory way by considering them as due to local stresses in such a medium. Current electricity seems due to a throb or series of throbs in such a medium when released from stress. Magnetic phenomena seem due to local whirlpools set up in such a medium. And the assumption of the existence of a single medium, with properties of great simplicity, will explain these varied phenomena and even co-ordinate them: thus the crest or the trough of a light-wave or a heat-wave is a point of maximum displacement or maximum tension—exactly the condition of the medium during the persistence of electric attraction or repulsion, that is, the Electrostatic condition; the middle point of the wave is changing its position or condition with rapidity—exactly the condition of the medium during the passage of a current, the Electrokinetic condition; thus light and radiant heat are explicable as electromagnetic disturbances of rapidly-alternating character; and this leads to the conclusion, sustained by experiment, that the velocity of light should be equal to the rate of propagation of an electric disturbance through a medium of this kind. We are led to infer, therefore, that there is such a medium, which we call the Luminiferous Ether, or simply the Ether; that it can convey energy; that it can present it at any instant partly in the form of kinetic, partly in that of potential energy; that it is therefore capable of displacement and of exercising pressure or tension; and that it must have rigidity and elasticity. Calculation leads us to infer that its density is 936/1000,000000,000000,000000 that of water (Clerk Maxwell), or equal to that of our atmosphere at a height of about 210 miles, a density vastly greater than that of the same atmosphere in the interstellar spaces; that its rigidity is about 1/1000,000000 that of steel—hence that it is easily displaceable by a moving mass; that it is not discontinuous or granular; and hence that as a whole it may be compared to an impalpable and all-pervading jelly, through which Light and Heat waves are constantly throbbing, which is constantly being set in local strains and released from them, and being whirled in local vortices, thus producing the various phenomena of Electricity and Magnetism; and through which the particles of ordinary matter move freely, encountering but little retardation if any, for its elasticity, as it closes up behind each moving particle, is approximately perfect.

Nothing of the nature of an air-pump can remove it from any given space; the most perfect **vacuum** conceivable must be defined as a plenum, a space fully occupied, but occupied by Ether alone.

Change of State.—Work must be done upon a solid in order to convert it into a liquid: energy must in some form be imparted to it. This form may be that of Heat, directly applied so as to fuse the solid. In such a case a definite amount of the energy imparted in the form of Heat apparently disappears (see Latent Heat, p. 335) for it does the mechanical work of liquefying the solid. If the liquid again assume the solid form, as in freezing, the process is reversed: the energy absorbed during liquefaction gradually reappears in the form of heat, which must be dissipated before the freezing can become complete.

If a solid body simply assume the liquid form without having external heat or other energy applied to it, the absorption of some of the heat of the body itself results in a cooling of the substance, as in the case of a freezing mixture, where certain chemical salts being dissolved in cold water, the resultant solution is found to be extremely cold.

Again, where two chemical elements combine, their combination is generally attended with heat, the elements losing their potential energy of separation. The supply of an equivalent amount of energy from without is necessary in order to reverse the process of combination—that is, to effect chemical separation or decomposition. When the processes of chemical combination and liquefaction go on together—the product of the combination of elements of which one or both are solid being itself liquid—the result may be that the cooling effect of the latter action exceeds the heating effect of the former; thus, in the union of carbon and sulphur to form carbon-disulphide, which is a liquid, the absorption of heat due to liquefaction is greater than the evolution of heat due to combination, and the action stops unless heat be supplied from without. On the other hand, in the combination of quicklime with water to form slaked lime, we find much heat evolved—partly due to the chemical combination, partly to the liquid water assuming a solid form.

The transformation of a solid into a gas, in a like manner, involves the expenditure of heat or some other form of energy in performing the mechanical work of volatilisation. Snow evaporates in a cold high wind; arsenic trioxide under ordinary atmospheric pressures is, without melting, volatilised by heat, while, if a sufficient pressure be applied, it melts before volatilising. Dr. Carnelley finds that in a similar way ice, if heated under an exceedingly small pressure, may be rendered very hot (180° C.), and will volatilise freely, yet without melting, unless the pressure be allowed to exceed a certain low maximum, which he calls the Critical Pressure, this being very low for water, very high for arsenic trioxide. A sheet of metal may be dissipated in vapour by an electric discharge, part of the energy of which becomes spent in producing this mechanical effect. Again, in chemical combination we often see the conversion of solids into gases. Carbon and oxygen combine to form carbon monoxide; of the heat which is evolved by the union of the elements, a large part is absorbed in rendering the solid carbon gaseous. If the CO produced be in its turn burned so as to form CO_2, none of the heat of combination of oxygen and carbon-monoxide is absorbed in doing mechanical work of this kind, and the amount of heat evolved in the second stage of the production of CO_2 is greater than that evolved in the first. Conversely, where two gases produce a solid, as chlorine and sulphuretted hydrogen do, the amount of heat liberated is determined not only by the amount of energy absorbed in decomposing H_2S, and by the amount liberated by the union of H_2 and Cl_2, but also by the fact that the sulphur is deposited in the solid form.

If a liquid were exposed to an indefinite and perfect vacuum, it would evaporate at once at any temperature above the absolute zero. If it be exposed to an imperfect vacuum, it will still evaporate readily, but not so readily as before, for its vapour must be able to force its way from the liquid and against the superincumbent pressure. If the pressure be great, the amount of heat which must be applied to the liquid in order to enable it to overcome this resistance and to enter into ebullition is also greater, and the Boiling-point of a liquid increases with the pressure.

Let a liquid be supposed heated in a vessel provided with a piston, by means of which pressure can be exercised on the contents of the vessel; the vessel being supposed of any sufficient length. The liquid is heated and converted into vapour; the vapour forces out the piston, and the external air pushes it in; the piston rests when the external and internal pressures are equal. If we press home the piston, the vapour is partly condensed: to retain it in the gaseous form we must simultaneously apply a stronger heat. This process may be supposed continued till at a certain high temperature (the "critical temperature") and great pressure we have the whole of the liquid evaporated, and its vapour compressed into the same space as the original liquid. If expansion be altogether prevented, this process is continuous, and the temperature at which water can be wholly converted into vapour under such circumstances is 720°·6 C.

Liquids are, as a rule, more bulky than the corresponding solids ; hence fusion, which involves expansion, obeys the same law as evaporation, which also involves expansion ; it is hindered by pressure, and the fusing point, like the boiling point, is raised by pressure. A few liquids—water, cast-iron —are denser than their solids. In such a case, an increase of pressure may be said to predispose the particles to set into the more compact and denser form, the liquid form, and fusion is facilitated by pressure. Thus the melting point of ice is lowered, that of most other solids raised, by pressure.

Change of state involves, then, either an absorption or a liberation of energy; and the amount of energy which must be supplied to a body in order to enable it to undergo a change of state depends on the pressure which tends to resist or to favour such a change, as well as on the intrinsic energy which it already possesses.

There is no known means of effecting any transformation of matter in any of its ordinary forms into the Ether, or *vice versâ*.

THE CONSTITUTION OF MATTER.

The question as to whether Matter is or is not infinitely divisible has been made the basis of much acute speculation; but it is only within this century that any serious proof has been adduced in favour of an Atomic Theory, or theory according to which matter is considered as made up of indivisible particles. According to this view, matter consists of particles or atoms, each of which it is impossible with our present appliances to divide, and the division of which, if it were possible, would probably result in the subversion of our ideas as to the apparently fundamental nature of some of the properties of matter.

Chemical Views.—The probability of this atomistic view was raised almost to the rank of certainty by the researches of successive chemical investigators. It was first found that as a rule every definite chemical substance in a state of purity had always the same constitution; that an analysis effected for one pure sample of, say, oxide of lead, was applicable to every other pure sample of the same substance. Hence the law of **Fixity of Proportions** in chemical compounds.

But it was remarked that the same substances often unite in different proportions to form compounds possessed of essentially different properties. Carbon and oxygen thus unite to form two well-known compounds, of which the percentage compositions are respectively :—

Carbon . . .	42·85	;	Carbon . . .	27·27.
Oxygen . . .	57·14		Oxygen . . .	72·72.

Analytical results tabulated in this way are not very instructive; but if the second example be multiplied by $\overline{42{\cdot}85/27{\cdot}27}$ we find the respective ratios to become

Carbon . . .	42·85	and	Carbon . . .	42·85.
Oxygen . . .	57·14		Oxygen . . .	114·28.

or in round numbers,

Carbon . . .	3	;	Carbon . . .	3.
Oxygen . . .	4		Oxygen . . .	8.

Here we find that the same quantity of carbon, united in the one compound (carbon-monoxide) with a certain quantity of oxygen is in the other (carbonic anhydride) united with twice the quantity of that element. From a large number of instances of this kind was evolved the **Law of Multiple Proportions**—that the same substances may form a series of different compounds by uniting in several fixed proportions which bear a whole-number ratio to each other. Nitrogen and oxygen thus form five compounds, in which the nitrogen and oxygen are present in the respective ratios of 14 : 8, 14 : 16, 14 : 24, 14 : 32, 14 : 40; and in this case the quantities of oxygen, united with a fixed quantity (14) of nitrogen, bear to one another the relative ratios of 1 : 2 : 3 : 4 : 5. Iron has two oxides, in which the iron and the oxygen bear to one another the respective ratios of 28 : 16 and 28 : 24; here the quantities of oxygen, united with the same quantity of iron, bear to one another the ratio 2 : 3.

Then, further, the law of **Chemical Equivalence** was formulated: chemical quantities, which are equivalent to the same thing as regards power of doing chemical work or forming chemical compounds, are equivalent to one another. One part by weight of hydrogen will combine with eight of oxygen (7·98165 ± ·00175); so will 108 parts of silver (107·896). 108 pts. of silver and 1 of hydrogen are mutually equivalent, for they can both do the same chemical work—they can enter into combination with 8 pts. of oxygen; and they are both equivalent to the 8 pts. of oxygen with which they can combine. If now it be found that 1 pt. by wt. of hydrogen can combine with 35·5 (35·478) pts. by wt. of chlorine, then the law asserts that the equivalent quantity, 108 pts., of silver should also, in its turn, be able to combine with an equal quantity, 35·5 pts., of chlorine. This law is a generalisation, based upon facts determined by the aid of the balance and independent of theory; and this law of equivalence, so based, though it be too sweeping a generalisation to be now accepted in its full sense, yet did useful service in its day in enabling tables of Equivalent Numbers or of Combining Proportions to be drawn up, and a system of Chemical Formulæ to be devised, based upon these equivalents. According to this system, the composition of water was symbolised as HO; this symbol might be read in words as—one equivalent of hydrogen and one of oxygen united to form one equivalent of water. The symbol of hydrogen peroxide was HO_2; one equivalent (1 pt. by wt.) of hydrogen combined with two equivalents ($2 \times 8 = 16$ pts. by wt.) of oxygen.

When such facts as these were known, a reasoned explanation of them was sought. None that offered was so plausible as Dalton's atomic theory, a revival of the old hypothesis of Leucippus, Democritus, and Lucretius, that matter consists of atoms, coupled with the proposition that the atoms of the different elements have different relative weights. According to this view the smallest mass of water must consist of an atom of hydrogen and another of oxygen, their relative atomic weights being 1 and 8; and these are connected as one might couple together a ball of wood and one of lead. More complex substances were produced by the union of a greater number of such atoms—as, for instance, HO_2, NO_5, (KO, HO), etc.; and the symbolic formulæ were then used to denote the relative number of such atoms entering into the formation of compound substances.

But it was found that the system of formulæ based upon the facts of equivalence did not work well when made to signify the relative numbers of atoms united to form a compound. The equivalent number for carbon was 6, because that quantity of carbon was equivalent (in carbonic oxide) to 8 of oxygen, which quantity was in its turn equivalent to the standard unity of hydrogen. In marsh gas 6 pts. by wt. of carbon are found to be combined with 2 of hydrogen—*i.e.*, with two equivalents; hence the formula, according to this system, must be CH_2. It is known, however, that one-fourth of the hydrogen can be replaced by half an equivalent ($17\frac{3}{4}$ pts. by wt.) of chlorine, forming $CH_{1\frac{1}{2}}Cl_{\frac{1}{2}}$: an expression intelligible though cumbrous when read in the language of equivalents, but absurd when read in terms of the atomic theory. This last formula had accordingly to be modified to C_2H_3Cl; and then the original marsh gas had to be supposed invariably to enter into reactions as $2CH_2$, or else its formula must be modified to C_2H_4. The latter is the more natural supposition. It was pointed out (Gerhardt) that throughout the whole of the chemistry of the carbon compounds similar reasoning shows that if the atomic weight of carbon be 6, the atoms always appear in reactions in even numbers; whence the inference is obvious that the at. wt. of carbon must be 12, and the proper formula of marsh gas is CH_4. In a similar way it was shown that the assumption that the atomic weight of oxygen, as well as its equivalent number, is 8, leads to the invariable appearance of O_2, or of an even number of oxygen atoms in every equation; whence the atomic weight of oxygen must be 16; and the Atomistic formula for water, as distinguished from the Equivalence-formula, must be H_2O. The atomistic formulæ now in use do not directly make use of the idea of equivalence: they denote the number of atoms of which the Molecule—a fruitful idea, due to Avvogadro—is made up. The symbol H_2O, for instance, signifies a molecule of water, made up of two atoms of hydrogen (at. wt. = 1) and one of oxygen (at. wt. = 16). When attention was first directed to this mode of representation, it was found to be entirely in accord with the half-forgotten researches of Gay Lussac on the relative volumes of gases which enter into combination. He had found that one volume (say 1 cubic cm.) of oxygen and two (2 cub. cm.) of hydrogen unite to form two volumes (2 cub. cm.) of water-vapour. The atomistic equation, on the other hand, is $O + 2H = H_2O$; that is, one *atom* of oxygen unites with two atoms of hydrogen to form a molecule of water. These two statements are closely parallel; and the molecule H_2O formed occupies in the state of water-vapour the same space as the original two atoms of hydrogen.

Similarly it had been found that the electric spark decomposed 2 c.c. NH_3 into 1 c.c. N and 3 c.c. H. The equation is

$$\begin{array}{ccccc} NH_3 & = & N & + & H_3 \\ \text{One molec.} & & \text{One atom.} & & \text{Three atoms.} \\ \text{2 c.c.} & & \text{1 c.c.} & & \text{3 c.c.} \end{array}$$

Here again the molecule of the compound, NH_3, occupies the same space as two atoms of hydrogen.

So forth; the general rule is that the molecule of any compound in the gaseous state occupies the same space as two atoms of free hydrogen.

Hence we may provisionally establish a general rule, subject to exceptions further to appear:—If in a chemical equation relating to gases we write "2 vols." under every complete molecule, and "1 vol." under every atom of any element entering into or resulting from the reaction in the free state, we learn the relative volumes of the gases concerned in the reaction. Thus, if alcohol-vapour be burned in oxygen,

Alcohol-vapour.		Oxygen.		Carbonic anhydride.		Water-vapour.
C_2H_6O	+	6O	=	$2CO_2$	+	$3 H_2O$.
One molecule.		Six atoms.		Two molecules.		Three molecules.
2 vols.		6 vols.		4 vols.		6 vols.

Thus a system of equations based on the atomic theory is found readily to give important information beyond what it was designed to give. This lends probability to the system.

The Molecule of a compound substance is the smallest mass that can exist in the free state. If we could break up a molecule we would sever it into its constituent atoms—as HCl into H and Cl—but we would destroy the substance on which we operated, as such. A molecule of hydrochloric acid contains 2, one of protoplasm from 700 to 1000 atoms.

What is the condition of elementary substances in the free state? Here such equations as the following come to our aid:—

$$CuH + HCl = CuCl + HH,$$
$$Ag_2O + H_2O_2 = Ag_2 + OO + H_2O;$$

and we learn that the molecule, even of an elementary substance, consists of two atoms, and we find by experiment that it occupies, like the compound molecules already discussed, the same space as two atoms—*i.e.*, one molecule—of hydrogen. All molecules, simple as well as compound, are thus seen each to occupy the same space; and conversely, the same space must be occupied by an equal number of molecules of whatever kind they be. This is the extremely important law known by the name of **Avvogadro.** All gases (at the same temperature and pressure) consist, within equal volumes, of equal numbers of molecules.

This is a general law, and its direct consequence is that the specific gravity of every gas, at a given temperature and pressure, as compared with that of hydrogen under the same conditions, is the relative weight of a molecule of the gas as compared with the molecular weight $(= 2)$ of hydrogen. Thus the molecular weight of alcohol, C_2H_6O, is $24 + 6 + 16 = 46$; that of hydrogen $= 2$; the single molecule of alcohol is twenty-three times as heavy as that of hydrogen, and accordingly the density of alcohol-vapour is twenty-three times that of hydrogen, other things—temperature and pressure—being equal.

There are some apparent exceptions. Mercury-vapour which, if two atoms formed its molecule, would have a molecular weight of 400 and a sp. gr. of 200, has a sp. gr. of only 100; hence its molec. wt. (twice its sp. gr.) is only 200, and its molecule contains only one atom. Cadmium and zinc have also monatomic molecules when in the state of vapour. Phosphorus and arsenic vapours have, on the other hand, an excessive sp. gr.; that of phosphorus is 62, and its molec. wt. must be 124, but its at. wt. is only 31; hence its molecule must contain four atoms. The molecule of arsenic is also tetratomic, while that of ozone is triatomic.

Hence, to provide for these exceptional instances, we must revise the rule provisionally laid down, and adjust it as follows: To find the relative volumes of gases entering or leaving a reaction, modify the equation, so that it represents no free gaseous atoms, but only complete gaseous molecules; then under every complete gaseous molecule write "1 vol." Thus—

$$\begin{array}{rcccc}
H_2 & + & O & = & H_2O \\
\text{becomes } 2H_2 & + & O_2 & = & 2H_2O. \\
\text{Two molecs.} & & \text{One molec.} & & \text{Two molecs.} \\
\text{2 vols.} & + & \text{1 vol.} & = & \text{2 vols.}
\end{array}$$

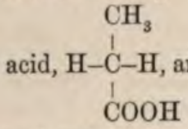
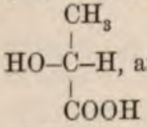
The equation

$$\begin{array}{rcccccc}
C_2H_6O & + & 6O & = & 2CO_2 & + & 3H_2O \\
\text{becomes } C_2H_6O & + & 3O_2 & = & 2CO_2 & + & 3H_2O. \\
\text{One molec.} & & \text{Three molecs.} & & \text{Two molecs.} & & \text{Three molecs.} \\
\text{1 vol.} & + & \text{3 vols.} & = & \text{2 vols.} & + & \text{3 vols.}
\end{array}$$

Again, the equation

$$\begin{array}{ccccccccc}
3CaP_2O_6 & + & 10C & = & Ca_3P_2O_8 & + & 10CO & + & 4P \\
\text{becomes} & & & & & & & & \\
3CaP_2O_6 & + & 10C & = & Ca_3P_2O_8 & + & 10CO & + & P_4. \\
 & & & & & & \text{Ten molecs.} & & \text{One molec.} \\
 & & & & & & \text{10 vols.} & & \text{1 vol.}
\end{array}$$

Another order of exceptions is presented in cases of Dissociation or Thermolysis. When NH_4Cl is volatilised, its vapour has half the sp. gr. indicated by the above theory; in other words, it occupies twice the theoretical volume. This is because a molecule of NH_4Cl is in reality split up into separate molecules of NH_3 and HCl (which may be partly separated by diffusion), each of which occupies the whole space that the original single molecule would have been able to fill if it had not been decomposed by the heat applied. Similarly, a molecule of calomel volatilised occupies twice its normal volume; for instead of a single molecule of Hg_2Cl_2 we have, as the result of dissociation, a molecule of $HgCl_2$, and another molecule, complete though monatomic, of mercury, each of these molecules independently taking up as much space as the original Hg_2Cl_2 would have occupied if it had not been decomposed. By these apparent exceptions Avvogadro's law is thus confirmed.

Upon this basis has been erected the modern science of Chemistry, one of the leading auxiliary ideas in which is that of the Atomicity of an atom—the number of atoms of hydrogen which an atom of any substance in question can combine with or replace. Whether the special manner of thought and expression of particular chemists be or be not adopted, the theoretical chemist can hardly express himself without making some use of the well-known Graphic Formulæ by means of which the relations of the atoms in a molecule may be indicated or suggested. Yet this mode of representation is sadly deficient, although exceedingly useful and suggestive. It gives a factitious representation in one plane of a statical condition of the molecule: it does not account for the energy possessed by a molecule in virtue of any one arrangement of its atoms, as compared with that possessed by a molecule of an isomeric compound in virtue of another disposition of atoms of the same kind and number; and, indeed, it scarcely touches as yet at any point the physical molecule or atom with which perfect knowledge would presumably show it to be identical. Still the attempt is being made to bridge over the gap—as, for example, by the researches of Le Bel and Van't Hoff, who trace out such relations as those between symmetry of the molecule, as in the case of propionic acid,  $H{-}\overset{\displaystyle CH_3}{\underset{\displaystyle COOH}{C}}{-}H$, and the absence of rotary power as affecting the plane of polarisation, on the one hand, and on the other between graphic asymmetry of the molecule, as in the case of lactic acid,  $HO{-}\overset{\displaystyle CH_3}{\underset{\displaystyle COOH}{C}}{-}H$, and the possession of this rotary power. But, on the whole, Chemistry and Physics, which should be parts of one dynamical science of matter and energy, are still separated by a wide gap, and one great stride which the Science of the future has to take is that of assimilating the theories of the physical and the chemical molecules, and thereby bridging over this gulf.

Physical Views.—Physicists have been obliged, independently of chemists, to develop mechanical theories of the molecule or the atom, as they have indifferently termed it. That such a thing does exist is manifest to them on several grounds. Not to speak of compressibility and porosity of matter as showing that it does not entirely fill space, we learn from Cauchy's investigations that if light be a wave-motion, there would be no dispersion, no prismatic colours of the spectrum, if the glass of the dispersing prism were continuous or were of a granular structure with indefinitely small grains. According to him, matter must be distinctly granular, whether it be discontinuous or not, and its granulations must not be greatly less in diameter than about $\frac{1}{10000}$ of the wave-length of the shortest wave of light—*i.e.*, about $\frac{1}{20000000}$ mm., or about $\frac{1}{500000000}$ inch. Sir William Thomson finds that there must be from 200 to 600 molecules in one wave-length. He also finds by his Electrometer that plates of copper and zinc exert a certain measurable attractive force upon one another. An indefinite number of plates would multiply this attraction to an indefinite amount; and if such plates were allowed to come together, the heat given out and representing their potential energy of separation would be indefinite, and they would combine after the manner of gunpowder. The energy observed to be given out in the form of heat during the formation of brass by the fusion together of copper and zinc is not indefinite: it corresponds to the mutual attraction of a number of plates not more numerous than 100,000000 to the millimetre. Hence copper and zinc could not be made into plates thinner than this, and plates of this tenuity would be only one molecule thick. A soap film cannot be stretched beyond a certain thickness without volatilising rather than yielding to farther extension at the same temperature; this limit appears to be reached when a thickness of 1/100,000000 mm. has been attained. Further, considerations derived from the kinetic theory of gases lead to the conclusion that a cubic cm. of solid or liquid contains a number of molecules which, though exceedingly large, is limited; and the distance between these is a quantity of the same order as those above mentioned. From these considerations Sir William Thomson concludes—Thomson and Tait, *Natural Philosophy*, vol. i. part 2, App. F, 1883, and *Nature*, July 1883 (which see specially)—that if a globe of water the size of a football (16 cm. diar.) were magnified to the size of the earth, the molecules or granules would each occupy spaces greater than those filled by small shot, less than those occupied by footballs.

But this tells us nothing about the nature of the atoms or molecules. It would at first sight be natural to conceive them as hard balls, but this would not explain their elasticity and mutual action: Faraday regarded them as "centres of force;" Macquorn Rankine as nuclei, each surrounded by an atmosphere in which there are whorls and currents of a complicated character.

The most interesting hypothesis is that of Sir William Thomson, who supposes each Atom of matter to be a Vortex-ring in the universal Ether. The ether itself we do not directly perceive; but this hypothesis would render our perception of matter a phenomenon of exactly the same order as that of light or radiant heat, viz., a perception of Matter as a Mode of Motion of the Ether.

If one look at a smoke-ring blown from a cannon, from a locomotive-engine chimney, from a tobacco-pipe, the lips of a smoker, or from an exploded bubble of phosphuretted hydrogen, it will be seen that the whole of the matter of the ring is in a state of rotation round an axis disposed in a circular form, and having no free ends. This is a Vortex-Ring; and such is that motion in the ether which is supposed to constitute a **vortex-atom.** A rotating ring of this kind in an imperfect fluid such as air must be the result of friction; but in a perfect fluid it could only originate by a special creation of some kind. Such a vortex-atom in a perfect fluid would have the following properties: it could move about in the fluid; its volume would be invariable; it would be indestructible; if struck by another it would be indivisible, but would present perfect elasticity, for though for the moment distorted, it would recoil and oscillate through its mean form: it would thus be capable of harmonic vibration, as the spectroscope shows the particles of matter to be; it would be capable of changes of form, becoming narrow and thick, or wide and thin; and it is practically the only form of motion in the ether which could remain in or near the same mean position, and at the same time be capable of being compounded with movements of translation. This kind of atomic structure would also account for what Tolver Preston calls the open structure of matter, which allows light, electric and magnetic stresses, and the action of gravity, to be transmitted through it. These properties of the vortex-ring explain well many of the observed properties of matter; but knowledge falls short, for we not only have the chemical atom and atomicity, but also physical mass and gravitation to explain before we can form any full theory of the inner structure of the Molecule.

The Kinetic Theory.—The next question is, Do these molecules remain at the same spot, rotating round it, or oscillating in its vicinity? or have they, in addition to whatever intrinsic motion they may be possessed of, a motion of Translation? The phenomena of Diffusion help us to arrive at a conclusion on this subject. If a solution of a coloured salt be placed in a vessel, and a layer of a colourless solution be laid upon the coloured stratum, the whole being left at rest for some weeks and protected from all disturbance, the plane of demarcation between the strata becomes blurred, the strata ultimately mix, and the solution becomes uniform.

This can only occur through a gradual travelling of the coloured solution into the colourless one, and *vice versâ*.

Again, if a jar of hydrogen and a jar of oxygen be brought into communication with one another, even though the former be uppermost, the gases will perfectly mix in a short time. This shows that the hydrogen rapidly travels into the oxygen, and *vice versâ*. The particles of matter, therefore, cannot be at rest, but must be in perpetual relative motion; and this is the Kinetic Theory of Matter.

Chemical analogy also illustrates this position. If steam be passed over red-hot iron filings, the iron takes the oxygen, and hydrogen passes off; if, on the other hand, hydrogen be passed over oxide of iron, it forms water-vapour, and reduced metallic iron is left behind. These actions, apparently so contradictory, are explained thus: There is a molecular agitation and a continued process of decomposition and recomposition of chemical molecules; the chemical atoms of iron, oxygen, and hydrogen are constantly changing their partners and forming new molecules; and in the first instance any molecules of hydrogen, in the second any molecules of steam, that happen to be formed are carried off in the current of gas which passes through the apparatus. The particles even of one and the same substance appear to be in this ceaselessly restless state of decomposition and recomposition: when the substance is heated, the molecules are easily broken up, but are not so easily formed again, whence we have the phenomena of Thermolysis or Dissociation; but even at ordinary temperatures the atoms associated within the molecules break asunder, and must but seldom happen to meet each other again. Agitation and break-up thus occurring within the molecules are incompatible with rest, and must necessarily be associated with violent translatory movements of the whole molecules.

In a gas, then, we must figure to ourselves a very large number of physical atoms, moving about with great velocity, striking one another and the sides of the containing vessel. Then the energy of any given quantity of gas, so far as that is due to movements of translation, will depend on the aggregate mass m and on the mean velocity $\overline{V}$; and it will be $\frac{1}{2}m\overline{V}^2$.

This mean velocity is the geometrical mean of all the individual velocities.

If we consider a cube of unit-volume, filled with any gas, and take any one internal face of it; that face, whose area must be unity, is struck by particles travelling with an average velocity $\bar{u}$ in a direction at right angles to that face, or having an average component of velocity $= \bar{u}$ in that direction, and having therefore a certain aggregate momentum. This momentum, with which the particles strike the wall during a unit of time, must be equal to the counter-pressure exerted by the wall of the vessel per unit of area;* the pressure p exerted on unit-area of the walls of the vessel by the gas is therefore equal to the momentum of the particles impinging on a unit-area of the wall in the course of a unit of time. But what is the amount of this momentum? It is the product of the number of particles which strike the unit-area wall in a unit of time into the average momentum of each.

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* This momentum would be lost to the gas within the cube were the particles conveying it not prevented by the counter-pressure of the wall from escaping: this loss ÷ time during which it would have been effected is the Rate of Change of Momentum (see p. 19) prevented during that time.

1. The number of the striking particles—
   If the gas contain N particles per unit of volume, and if these move towards the wall struck by them with an average rate $\bar{u}$ per second, the number of particles which must strike the unit-area wall in a unit of time is $N\bar{u}$.
2. The average momentum of each—
   The mass M of each particle is the same; the average velocity is $\bar{u}$; the average momentum of each particle is $M\bar{u}$.

The momentum with which the wall is struck is thus $N\bar{u}M\bar{u}$ per unit of area; and this $= p$, the pressure on the wall per unit of area. But the cube is one of unit-volume, its volume $v = 1$; the aggregate mass of the gas is $NM = v\rho = \rho$; whence $p = \rho\bar{u}^2$.

Next, what is the average velocity $\bar{u}$ in any one direction? The average velocity $\overline{V}$ (which does not depend on direction) is, if resolved into components $\bar{u}$, $\bar{v}$, $\bar{w}$, at right angles to one another,

$$\overline{V} = \sqrt{\bar{u}^2 + \bar{v}^2 + \bar{w}^2}.$$

But $\bar{u}$, $\bar{v}$, $\bar{w}$ are equal to one another, for there is no difference between one direction and another in respect of velocity: whence,

$$\overline{V} = \sqrt{3\bar{u}^2}, \qquad \text{and } \bar{u} = \sqrt{\overline{V}^2/3}.$$

Therefore the pressure per unit of area is $p = \rho\bar{u}^2 = \rho\overline{V}^2/3$; and consequently whatever be the volume v, the product $pv = \rho v\overline{V}^2/3 = m\overline{V}^2/3 = \frac{2}{3}\left(\frac{1}{2}m\overline{V}^2\right) = \frac{2}{3}$ the aggregate molecular-translational kinetic energy of the gas whose mass m is confined within volume v.

We know that at the same temperature the pressure is equal in all gases; in equal volumes of different gases the product pv must be the same; and hence the aggregate molecular kinetic energy of translation $(= \frac{3}{2}pv)$* must at the same temperature be equal in equal volumes of all gases.

If two gases have the same Temperature, the particles have the same mean molecular energy $(\frac{1}{2}M\overline{V}^2)$ of translation. This is a hypothesis; but if it were otherwise, two gases at the same temperature would change in temperature when mixed; for their average molecular energy would become equalised throughout.

If the aggregate kinetic energy of translation $(\frac{1}{2}m\overline{V}^2)$ be equal in equal volumes of two gases, and if at the same time the molecular energy of the molecules of each be equal (their temperatures being equal), it follows that the number of molecules must be equal in the equal volumes of the two gases, and hence Avvogadro's Law is true in the physical as well as in the chemical sense, being a direct deduction from the kinetic theory.

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* We here assume the absence of intermolecular forces. If there be such, independent of collisions, the molecular Kinetic Energy $= \frac{3}{2}pv + \frac{1}{2}\Sigma(Rr)$ (Clausius), where the last expression (the "Virial") is half the sum—a sum which for given values of p and v retains an appreciably constant value—of the products of the mutual distances r of every pair of particles into the corresponding mutual attractive force R.

If there be two gases whose respective densities at equal temperatures and pressures are ρ and $\rho_{,}$, Avvogadro's law shows that these unequal masses are divided among equal numbers of molecules: hence the mass of each single molecule must be proportioned to the density of its gas; for if M and $M_{,}$ be the respective masses of single molecules of the respective gases, $M = \rho v/N$ and $M_{,} = \rho_{,} v/N$; whence $M : M_{,} :: \rho : \rho_{,}$.

Thus the Molecular Kinetic Theory of Gases explains the **pressure** on the sides of the vessel containing a gas: it explains the tendency of gases to indefinite **expansion**: it explains **Heat** as the energy of molecular agitation; equality of **temperature** as equality of the mean energy of agitation in the several molecules. It also arrives at **Avvogadro's law**, and explains the numerical identity of ratio existing between the relative weights of the several kinds of molecules and the specific **densities** of the corresponding aggregate gases.

The equation $p = \rho \bar{V}^2/3$ given above yields $\bar{V} = \sqrt{3p/\rho}$, by means of which $\bar{V}$, the **mean velocity** of movement of the particles of any gas, may be found. Thus for hydrogen p, the pressure, is equal to the weight of say 76 cm. of mercury (density = 13·596), or to 1033·3 grms.-mass resting on every square cm. But the weight of 1033·3 grms. is mg; 1033·3 grms. × 981 cm. = 1013667·3 dynes. Again, ρ, the density of hydrogen, is ·0000895682 grammes per cubic cm. Hence $\sqrt{3p/\rho} = 184260$ cm. or 1842·6 metres per second, the average velocity of the particles of hydrogen.

Hence also the mean velocities of gases vary inversely as $\sqrt{\rho}$; or, which is an equivalent statement, the mean velocities of the particles of gases vary inversely as the square root of the molecular weight: whence oxygen-atoms have one-fourth the velocity of hydrogen-atoms because they are sixteen times as heavy. This is the law governing the relative speed with which the different components of a gaseous mixture will travel through a membrane.

The kinetic theory also informs us that when we double the number of molecules which move in a given space with a given mean velocity we double the number of molecules which strike the walls, and accordingly we double the pressure; or in other words, the pressure varies directly as the density of a given quantity of gas, this being another form of **Boyle's Law.**

It also tells us that if we mix a particles of one gas, b particles of another, c of a third, and so on, the average kinetic energy of all the particles being the same, or soon becoming equalised, the pressure produced by the a molecules of the first gas is proportional to their number, the pressure produced by the second gas is proportional to b, and so forth; or in other words, that in a mixture of gases the pressure produced by each component of the mixture is independent of the rest, and depends only on the amount of such component which is present in the mixture (**Dalton's Law**).

Again, when the temperature is increased, the energy of the particles is increased; each particle strikes both oftener and harder; the pressure experienced by the walls of the vessel therefore varies as the square of the velocity, and is proportional to the molecular energy of the particle—that is, to the absolute amount of heat-energy possessed by it. This if the volume be kept constant; but if the pressure be kept constant and the volume allowed to increase, then the volume varies as the "absolute temperature" (see p. 339). (**Charles's Law**, often attributed to Gay Lussac.)

The kinetic theory of gases also explains how it is that when a stream of gas passes through air, its progress is retarded by "**viscosity**;" rapidly-moving particles of the gas travel laterally into the air; slowly-moving particles of the air travel into the gas, and thus its progress is hampered. Similarly the viscosity of a gas will bring to rest a current set up within its own substance; but this viscosity is independent of the density in gases.

The theory also explains the **conduction of heat** in gases; rapidly-moving particles by collision part with some of their energy to others, which in their turn enter into collision with those beyond them: and we have already seen it explain the **diffusion** of gases.

The mutual impact of elastic solid particles would necessarily result in the ultimate transformation of the whole translational energy into energy of vibration; that of vortex-rings seems to imply no such result. The latter seems, therefore, the preferable form of the kinetic theory of matter, although it is as yet far from complete.

These molecules, thus travelling with such great velocities and entering into a practically infinite number of collisions with one another (in hydrogen 17750 millions per second), can never travel very far in an undisturbed path. At the ordinary temperature and pressure the *mean free path* of the molecules of hydrogen, which have the longest trajectory, seems to be about $\frac{1}{20000}$ mm., or a tenth part of the average length of a wave of light (Maxwell); $\frac{1}{10000}$ mm. (Crookes). The *diameter of molecules* is not the same in the case of all elements, but is on the average about $\frac{8}{10000000}$ mm. Thus the smallest visible organic particle ($\frac{1}{4000}$ mm. diar.) will contain about 30,000,000 atoms, which may be arranged in about 40,000 molecules. The *number of molecules* in a cubic cm. at the ordinary temperature and pressure is about 19,000000,000000,000000 (Maxwell), 1000,000000,000000,000000 (Crookes), not more than 6000,000000,000000,000000 (Sir W. Thomson).*

These numbers are arrived at by using a proposition formulated by Clausius, that

$$\frac{8 \times \text{free path of each molecule}}{\text{Diameter of each molecule}} = \frac{\text{Whole space filled by the molecules}}{\text{Their real aggregate volume}}.$$

From this the real aggregate volume, which does not differ very widely from that of the corresponding liquid, is found, and, if divided by the cubic space occupied by each molecule, gives the number of molecules.

When gas is rarefied the number of molecules in a given space is diminished. Let us suppose that the rarefaction is carried on so far that only one particle out of every original million is left in the space exhausted. The pressure is one-millionth of its original amount; but any molecule once in motion has one-millionth its former chance of encountering any other molecule, and consequently its average free-path is magnified a millionfold. The mean path would then be (Crookes) $\frac{1}{10000}$ mm. × 1,000,000 = 100 mm., or about 4 inches. By means of a good Sprengel pump exhaustion to the hundred-millionth of an atmosphere can be attained, and the mean free-path of the gas so rarefied would be about 33 feet. In our atmosphere at a height of 210 miles the single molecules are relatively so few (1000 to the cubic cm.), that each molecule might travel through a uniform atmosphere of that density for 60,000,000 miles without entering into collision; beyond a height of 300 miles the atmosphere is so rare (less than one molecule per cubic foot) that the particles might freely travel through such an atmosphere from one fixed star to another; while in the fields of space, at distances practically infinite from the earth or any other star, the number of cubic miles containing a single molecule would be represented by the figure 1 followed by 314 cyphers.

This opens up to us an extraordinary view of the nature of our atmosphere. We must,—though the process cannot be rapid, for each particle rising from the earth is retarded by gravity and falls back towards the earth,—

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* While there is no confusion as to the principles at issue, there is still a discussion in active progress as to the numerical details; and the reader who wishes to study the subject fully is recommended attentively to peruse all the articles by Prof. Clerk Maxwell (*Encyclop. Britann.*; *Nature*; *Philos. Mag.*; *Trans. Roy. Societies London and Edin.*, *Cambridge Phil. Soc.*), and by Dr. Crookes (*Transactions and Proceedings Roy. Soc. London*), to which he can get access, together with the literature to which he will there find reference.

constantly be losing particles of nitrogen and oxygen as we are dragged through space, and we may constantly be picking up new ones. If we entered regions of space in which there were no particles fit to make up our losses, it would be an interesting question how short a time would suffice altogether to deprive us of our atmosphere.

The region of space through which the earth is at present travelling contains much benzene vapour with ethyl-hydride and other alcohol-derivatives.

Thus our ideas on the subject of the constitution of matter have undergone a profound modification. Matter is discontinuous in the highest degree, for it consists of separate particles or molecules which are mutually non-interpenetrable; the special properties of the different states of matter depend on the number of molecules which are contained within a given space, as well as on the energy of movement which is possessed by each; and each particle is susceptible not only of translation as a whole, but also of vibration or rotation, and may besides be in a state of vortex-motion, upon the continuance of which its continued existence may depend.

MOLECULAR FORCES.

Hitherto we have conducted our reasoning on the implied assumption that the molecules had no mutual action, and we have arrived at results such as Boyle's law, Dalton's law, and others, which we have deduced from theory. Now we must confirm our theory by reference to facts, and we find this assumption overruled by such material discrepancies as the following. Boyle's law, though obeyed on the whole, is disobeyed by every gas when the pressure is so high or the temperature so low that condensation is not far off: this departure, though not extensive, is significant. All gases just about to become condensed are, except in the single case of hydrogen, more easily compressed than the law would indicate. Dalton's law is departed from by a mixture of gases condensible with difficulty: such a mixture is found to be even less condensible than the component gases, and the critical temperature is lowered. Charles's law is not obeyed throughout the whole range of experimental pressures and temperatures; at a high pressure any increment of heat produces a disproportionately large increment of pressure.

In fact, gases obey these laws only when their pressure is very feeble and their temperature at the same time high above the critical temperature—that is, when the molecules are comparatively far from one another. At ordinary temperatures and pressures the particles do affect one another not merely by mutual impact or mutual gravitation, but also by mutual actions or molecular forces effectively coming into play when the particles are at exceedingly small distances from one another. When the pressure is small, the free path is comparatively long, and the molecules are mutually removed from each other's influence: and the higher the rate at which the particles are moving, the less will be the proportionate effect of the molecular forces; or in other words, the higher the temperature the less appreciable will be the effect of intermolecular action.

When a gas is being compressed into a liquid we know that in the first place, in all gases except hydrogen, the particles are attracted slightly towards one another, and also that there is on the other hand a practical repulsion from one another caused by their energetic movement. We further find, however, that though the particles become approximated with relative ease while liquefaction is approaching, yet when the liquid state has been attained, and even before it has been attained, repulsion takes the place of attraction; the liquid when formed offers a relatively enormous resistance to compression. This is well seen in the case of carbonic anhydride merging insensibly from the gaseous into the liquid state; just before ceasing to be a gas it is very compressible, just after becoming a liquid it is relatively very slightly so.

Air obeys Boyle's law precisely, and the air-manometer is therefore correct at a pressure of 200 atmospheres; below that pressure the volume is in defect; above, it is in excess (Andrews).

In liquids the molecules are within the spheres of one another's action. This accounts for the viscosity of all, even of the most mobile liquids: the particles detain one another by their mutual attraction, and a flowing liquid is thus hindered in its flow by molecular friction. Molecular action also accounts for the fact that a stream of liquid has a certain tenacity and will not readily break: such is the condition of a stream of liquid in a siphon. Again, it explains why, under ordinary circumstances, the effects of molecular attraction are strikingly manifest in liquids only at the surface, and in the form of Surface-Tension. In the interior of a mass of fluid each particle is free to adjust its mean position under the influence of the surrounding molecules; the mean position which it assumes is that in which it is acted on equally on all sides, and there is then nothing to render the mutual attractions manifest. At the surface of a liquid mass, however, if it be a free surface, the particles can only be acted upon by others lying internal to them. The result is, as is shown in Fig. 100, a system of forces acting at right angles to the free surface of the fluid, and tending to reduce that free surface to the least possible area. We may, indeed, consider a drop of water as consisting of a quantity of water enclosed in a superficial skin of water which is under tension, and whose particles attract one another into the least possible superficial area; and since of all surfaces the sphere has the greatest content for the least area, the superficial film may be said to mould the drop to the spherical form, which in the case of falling raindrops is approximately perfect, as is shown by the rainbow. To these surface-tensions are also due the important phenomena of Capillarity.

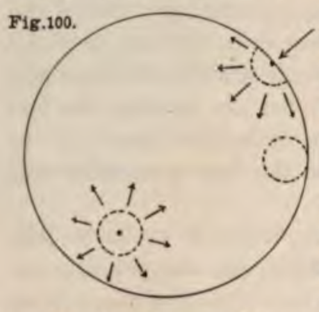
Fig.100.

Many of the properties of solids are also due to molecular forces. Such are toughness, hardness, and the like, which may be grouped under the generic name Strength of Materials; these depending probably, in part at least, on the proximity of the particles to one another. The molecular grouping of molecules is also very important, though very little can be said about it; but upon it depend not only the crystalline or amorphous condition of a substance and in part its strength, but also that stable or unstable equilibrium upon which the phenomena of elasticity or the properties of such things as Rupert's drops depend. These last consist of little masses of fused glass dropped into cold water; the outside is suddenly chilled and solidified while the interior is still in a state of fusion. The internal mass has to accommodate itself as it best can to the dimensions of the outer skin; it does so under tension, but the moment that the relations are disturbed by breaking off the narrow end, or even by the slightest scratch, the whole flies to powder: it is in a state of unstable equilibrium, and the slightest displacement precipitates a downfall of the whole arrangement. In the same way the slightest scratch in the interior of a large glass tube, especially if it have local variations of thickness,—even such a scratch as is produced by the fall of a crystal of quartz or a rub with the end of a iron wire,—will often shiver the tube; for which reason no rough treatment should be internally applied to such tubes with any metal harder than copper. This state of internal tension accounts for the danger in the use of cast-iron in structures.

Many solid masses have, however, their particles so arranged as to form conservative systems, which tend to restore any work done on them, and consequently are in stable molecular equilibrium ; the details of the molecular grouping are unknown, but in a perfectly elastic body, or practically in any solid body within its Limits of Elasticity, any displacement among the molecules produces a restitution-pressure equal and opposite to the distorting force or stress ; and it is observed that, as a general rule, the distortion is proportional to the distorting force (*Ut tensio sicut vis*, " Hooke's Law "), and hence the restitution-pressure is proportional to the distortion. This elasticity may in solids be observed more or less perfectly to obtain, whether the distortion be that of form or of volume ; while liquids have elasticity of volume alone, never of form.

To the same order of Molecular Forces must be attributed the effects of Cohesion between masses or particles of the same substance, and of Adhesion between those of different substances ; and also the phenomena of Chemical Affinity, the potential energy of chemical separation, and the liberation of energy attendant on chemical combination.

CHAPTER X.

SOLIDS.

THE special properties of solids are due to the relative contiguity of their molecules. Their definite free surface is due to the mutual attraction of their molecules, and is retained in virtue of the same forces which in the aggregate manifest themselves as causes of cohesion, tenacity, etc., and the result of which is that a solid can persist under the action of a stress not evenly applied—that is, of a stress which is not hydrostatic.

Cohesion is the mutual attraction of the particles of a solid for one another, and is measured by the amount of force which must be applied in order to overcome it. The term *cohesion* is generally applied to the mutual attraction of particles of the same substance, *adhesion* to that of different substances. When two pieces of white-hot iron or platinum are brought in contact they cohere by welding. When a piece of silver and a piece of platinum are brought in contact at 500° C. they adhere. If metals in the state of dust be mixed and exposed to a pressure of 7000 atmospheres they will form a firm metallic mass, and will even combine and form an alloy. Even sulphides and arsenides may thus be formed; for pressure promotes contact. Cohesion is manifested by two surfaces of glass, which, if ground exceedingly smooth and placed in contact, will cohere firmly; and the well-known Barton's cubes are little cubes of metal polished so smoothly that mere apposition causes them to cohere, the force of cohesion being so great that a string of a dozen may be supported in the air by this mutual attraction alone. Common graphite is ground to powder and purified by boiling with nitric acid and chlorate of potash: it is then washed and dried; the powder is placed in a mould and exposed to extreme pressure produced by a hydraulic press; after compression the black powder is found to have been converted into a solid mass of coherent pencil-graphite, which may be sawn into strips and used for pencils. If a leaden bullet be cut into two with a sharp and heavy knife, the two halves will cohere firmly if pressed together by their bright surfaces.

Hardness—Softness.—A body is said to be harder than another when it can be used to scratch the latter but cannot be scratched by it. In this sense the diamond is the hardest of all solids. The scratching body must not have too sharp a point, for this would prove a pin to be harder than glass, which is not the case. Hardness is a property that cannot be measured. All that we can do is to make up a list of substances in their relative order of hardness, and to express the hardness of any particular substance by stating its place in that series. The standard series, due to Mohl, is the following:—

1. Green laminated Talc. 2. Crystallised Gypsum. 3. Transparent Calcspar. 4. Crystalline Fluorspar. 5. Transparent Apatite. 6. Pearly cleavable Felspar (Adularia.) 7. Transparent Quartz. 8. Transparent Topaz. 9. Cleavable Sapphire. 10. Diamond. Flint scratches quartz with difficulty, but is easily scratched by topaz: hence its hardness is set down as 7·25 on this arbitrary scale. The rapidity of movement of the attacking substance is a matter of practical importance: thus the sand-blast (a stream of sand rapidly blown from a tube) is capable of cutting through rocks and even through steel with relatively great rapidity; and the same result is seen in the mechanical operation of filing.

Mr. Edison finds that platinum wire may be rendered as hard as steel pianoforte wire by heating *in vacuo*, keeping up the vacuum, and gradually increasing the temperature. The particles of platinum have all air removed from their interstices, and they cohere very firmly by welding.

Hardness—Fragility.—This is a distinct use of the word Hardness. In this sense the diamond possesses little hardness, for if struck a blow with a hammer it flies to pieces.

Malleability, the property of yielding to the hammer without breaking at the edges.—Gold can be hammered out into leaves extremely thin. A half square-inch of gold of the thickness of letter paper is hammered out to 81 square inches; each square inch of this thin sheet is again hammered out into 81 square inches, of which each one is in its turn again hammered out to 81 inches. Antimony, on the other hand, flies to powder at the first blow of the hammer.

Plasticity.—Some solids can be moulded, as lead in a bullet-mould.

Ductility.—Some metals can be drawn through fine apertures in a draw-plate, and wires can thus be formed: other metals are incapable of this, for they snap. The order of ductility is—Gold, Silver, Platinum, Iron, Copper, Palladium, Aluminium, Zinc, Tin, Lead. Platinum wires of exceeding tenuity, such as are adapted to the eye-pieces of microscopes for micrometric work, are made by constructing a thick silver bar with a thin platinum core, drawing this out to an extreme fineness, and dissolving off the silver by steeping the drawn wire in nitric acid.

Extensibility—Inextensibility.—Some substances can, like indiarubber, be extended greatly by the application of a stretching force: others, like baked clay, very little. The ratio of the extension produced to the extending force is the "extensibility;" the former being measured by the ratio of the increase in length to the original length.

The general rule is that a wire made of any substance, the area of whose transverse section is s square cm., and whose length is l cm., suffers, when stretched by a force of F dynes, an absolute elongation e, which is (1) proportional to the length of the wire, (2) proportional to the stretching force applied, (3) inversely proportional to the transverse section s, and (4) proportional to a special coefficient E, the Coefficient of Extensibility, a small fraction, which must be determined experimentally for each substance. Thus $e = \mathrm{E}.\mathrm{F}l/s$.

The following coefficients of extensibility (reduced to C.G. measures) are given partly on the authority of Unwin, partly on that of Wertheim. A rod of each substance whose transverse section is 1 square cm. is stretched by a force of one dyne.

The proportionate extensions (measuring the coefficients of extensibility) are, in the respective cases of

Cast Steel tempered,	one	(2520,000000 × 981)th	of the whole length.
Wrought Iron . .	,,	(2000,000000 × 981)th	,,
Copper	,,	(1050,000000 × 981)th	,,
Wood	,,	(10,000000 × 981)th	,,
Leather	,,	(175,000 × 981)th	,,
Fresh bone	,,	(230,466000 × 981)th	,,
Tendon	,,	(16,341,000 × 981)th	,,
Nerves	,,	(1,890,000 × 981)th	,,
Living muscle at rest	,,	(95,000 × 981)th	,,
Arteries	,,	(5,200 × 981)th	,,

What weight would be necessary in order to double the length of a piece of steel bolt 1 sq. cm. in sectional area, if the strain could be effected without rupture? The elongation e would be equal to the original length, $e = l$, $l = \mathrm{EF}l$; $\therefore$ $\mathrm{F} = 1/\mathrm{E} = (2520{,}000{,}000 \times 981)$ dynes. If the sectional area were 1 sq. mm., $\mathrm{F} = s/\mathrm{E} = (\frac{1}{100}\text{ sq. cm.}) \div (\frac{1}{2520{,}000000 \times 981}) = (25{,}200{,}000 \times 981)$ dynes = the weight of 25,200000 grammes.

French engineers are in the habit of reducing these inconveniently large physical constants by expressing extensibility in terms of the number of kilos. weight which would be required to double the length of a bar whose sectional area is one square millimetre: the resultant numbers are $\frac{1}{100000}$ of those obtained when the extensibility is measured in terms of the number of grammes' weight which would be required to double the length of a bar whose sectional area is one square centimetre.

Muscles are more extensible when they are in a state of contraction than when they are at rest; and if a muscle when loaded by a certain weight be stimulated to contraction, the mere effort to contract may so diminish the resistance to extension or increase the extensibility that the contracting effort may be more than counterbalanced by the mechanical stretching of the muscle produced by the weight hanging upon it, and the overloaded muscle may actually stretch when stimulated to contract. Muscles also become a little less resistant or more extensible under the action of a given load shortly after death.

There is no substance of which wires or rods could be loaded with indefinite weights, or even with such weights as would double the length: there is for each substance a special limit of tenacity or cohesion, when extension can go no further, and the rod is ruptured. This **breaking weight** measures the cohesion, and the following table represents, according to Wertheim, the number of grammes which must be hung on a rod of 1 sq. cm. section in order to break it—

Bone	800,000
Tendon	625,000
Nerve	135,100
Veins	18,500
Arteries	13,700
Muscle	4,500

Thus a nerve whose section is ¼ sq. cm. could bear a stretching force equal to the weight of 33·7 kilogrammes or over 5 stone; but the danger of stretching an artery or a vein by mistake is obvious. There is a great difference in the breaking weight of the same tissue in persons of different age and habit. Wertheim found that the fibula of a young man of thirty had a breaking weight of 1,503,000 grammes per centimetre, while that of the same bone in an old man of seventy-four was reduced to 432,500.

Compressibility follows the same laws as extensibility. Within narrow limits the coefficients of compressibility and of extensibility have the same value. Excessive compression leads to crushing, and each substance has its own Crushing Weight, found by experiment on masses of determinate size.

Rigidity—Flexibility.—In every rod undergoing flexion, if this be due to the weight of a mass suspended from a free end, there must be a certain extension of the upper aspect of the rod, a compression of the lower, and a Neutral Line between, which retains its original length. If the flexure be due to weight pressing down the middle of the rod which is supported at its extremities, the extension is in the lower aspect of the rod, the compression in the upper. In the former case a cut in the upper aspect would weaken the rod; in the latter the same effect would only be produced by a cut on the lower aspect. Flexion may bring about compression and extension beyond the range of the breaking or crushing strengths, and the body may thus be broken. If this occur before there has been any perceptible flexion, the body is said to be **brittle**: if it allow a considerable range of flexion it is said to be **tough**—it bends much before breaking. The crystalline or granular or fibrous structure of a substance has much to do with its brittleness or toughness. For example, tin, which is very crystalline, is very brittle; wrought-iron axles, which are at first fibrous and very tough, are subject to a molecular rearrangement facilitated by vibration, and become crystalline and brittle.

Elasticity.—"Elasticity is the property in virtue of which a body requires force to change its bulk or shape, and requires a continued application of the force to maintain the change, and springs back when the force is removed; and if left at rest without the force, does not remain at rest except in its previous bulk and shape" (Sir William Thomson).

There are two properties, **Resistance** and **Restitution,** which must concur in any given body before it can be said to be elastic.

The **coefficient of resistance to extension,** K, is the reciprocal of the coefficient of extensibility; *i.e.*, $K = 1/E$; and it is the fraction $\frac{\text{Force acting}}{\text{Proportionate Elongation produced}}$.

From the equation $e = E.lF/s$, substituting $1/K$ for E, and supposing s and e/l to be both unity, we get $K = F$. Whence K is the number of units of force which would have to be applied to a rod (of the substance whose coefficient of resistance is K) whose sectional area = 1 sq. cm. in order to double its length, if such doubling were possible. A steel rod of 1 sq. cm. section is lengthened $\frac{1}{2520000000}$ by a weight of 1 gramme—*i.e.*, by a force of 981 dynes. A force of one dyne would lengthen it by $1/\overline{981 \times 2,520,000,000}$. The resistance of steel is accordingly $\{1 \text{ Dyne} \div 1/981 \times 2,520,000,000\} =$ 2,472120,000000 dynes in C.G.S. measures. This is the value of K for steel, and it measures the amount of force which would be necessary to produce an elongation equal to the original length of a rod of that metal, whose transverse-sectional area is 1 sq. cm. To produce any less proportionate elongation e/l, the force necessary, F, is $K.e/l$, if the rod have a cross-section of 1 sq. cm.; $K.se/l$ if the cross-section be s sq. cm.

The Work done in producing extension is the product of the average resistance overcome into the space through which it is overcome.

If K be the coefficient of resistance, and if a rod whose cross-section is s sq. cm. be exposed to a tensile force F, stretching will go on until the ultimate resistance arrived at is in equilibrium with the stretching force F. When this is the case, $\mathrm{K}.se/l=\mathrm{F}$. The *average* resistance encountered by the tensile force is $\frac{1}{2}\mathrm{K}.se/l=\frac{1}{2}\mathrm{F}$. The space through which the resistance is overcome is e. The work done is average resistance × space $=\frac{1}{2}\mathrm{F}e$.

The coefficient of resistance to **compression** is, within small limits, equal to that of resistance to extension, and obeys the same laws.

Torsibility of a body is measured in the simplest case—that of a rod or wire—in terms of the *angle* through which a unit of force, applied at the distance of 1 cm. from the axis of the rod or wire, can twist it. The **resistance to torsion**, T, is the reciprocal of this angle.

The Work done in producing torsion is the average resistance (or half the "twisting moment") multiplied by the angle of twist.

If the pressure p be applied at the end of a lever r, the twisting moment is pr: if the angle through which the wire is twisted be ω, the work done is $\frac{1}{2}pr\omega$.

The torsion-restitution-pressure $(=p)$ is proportional (1) to ω the angle of twist, directly; (2) to r, the distance of the point at which it acts, inversely; (3) to T, the Resistance to torsion as above defined. It is therefore equal to $\mathrm{T}\omega/r$, and the twisting moment pr is equal to $\mathrm{T}\omega$, whatever be the distance of the point of application of the twisting force p.

A bar suspended by its midpoint on a wire capable of twist, and acted upon by a twisting moment pr, will rotate and cause the lower end of the wire to rotate with it; if, however, the upper end of the wire be at the same time twisted in an opposite sense, to so great an extent that the reverse twisting moment due to the torsion ω of the wire itself becomes equal to pr, there is then no change in the position of the suspended bar at the lower end. The restitution-pressure at the point of application of the pressure p to the bar is $\mathrm{T}\omega/r$; this is equal to p, which it holds in check. Any other pressure $p_{,}$, similarly applied would be equal to $\mathrm{T}\omega_{,}/r$; whence $p:p_{,}::\omega:\omega_{,}$; forces may be compared by observing the ratio between the angles of opposite twist which must be given to the one end of a wire or fibre in order to prevent those forces, similarly applied, from causing twist at the other end.

The Coefficient of Resistance to **Shear** is the same as the coefficient of rigidity, which is a constant independent of the compressibility. The work done in producing a shear may be specified in two ways—as the product of the average resistance (*i.e.*, of half the force applied) into the amount of the shear, or as the force applied into the amount of displacement of the centre of figure of the mass sheared. Either of these pairs of terms gives the same product, $\frac{1}{2}F \, . \, PC \, . \tan PCP'$ (see Fig. 25).

Power of Restitution.—These resistances are not the only criterion of Elasticity. A body may resist extension, compression, torsion, shear, and yet not be elastic. In order that it may be perfectly elastic, it must have all the following properties:—

(1.) It must offer a definite resistance to distortion.

(2.) The distortion is not permanent, and if the deforming pressure be removed, the distorted body springs back to its original form or bulk.

(3.) The distorting pressure must be continuously maintained in order to keep up the distortion.

(4.) As long as a distorting pressure is kept up, there is a counter-pressure or **restitution-pressure** (P) developed and sustained in the elastic substance. As this holds the deforming pressure (F) in check, and is in equilibrium with it, thus setting up a condition of stress in the substance, it must be numerically equal to it; $P = F$.

(5.) The restitution-pressure does not become diminished by lapse of time.

The restitution-pressure P is ultimately equal to the deforming force F: but $F = K.se/l$, whence $P = K.se/l$, and the Restitution-Pressure is proportionate to the Displacement. If the distortion $se/l = 1$, $P = K$, whence $P = 1/E$, and the restitution-pressure or restitution-force then exercised is represented by a number, the Coefficient of Restitution (or "coefficient of elasticity"), which is equal to $\frac{1}{\text{Extensibility}}$, or to $\frac{1}{\text{Compressibility}}$. If the distortion se/l have any other value than unity, the restitution-pressure is $K.se/l$.

Under any given distortion within the limits of restitutive power, the restitution-pressure is equal to the product of the Coefficient of Restitution into the distortion; the coefficient of restitution being numerically identical with the reciprocal of the compressibility or of the extensibility. It is usual to profess to measure the elasticity of a solid by a "*coefficient of elasticity*," which is stated to be equal to the resistance to distortion. There is an equality, a numerical identity, between the Resistance to distortion and the Coefficient of Restitution (upon which the amount of restitution-pressure depends), provided that any one system of units is strictly adhered to, that the body is perfectly elastic, and that the distortion is unity. It seems, however, strange to set up a method of measuring elasticity based on a tacit fundamental assumption that the bodies dealt with are perfectly elastic.

If there be two bodies, of which one has a low, the other a high coefficient of restitution, and if the same displacement be effected in both, the restitution-pressures in the two substances differ in the same ratios as their respective coefficients: and in these two bodies the relative amounts of work stored up in the form of tensional or potential energy also differ in the same ratio. Elasticity thus presents two aspects, the Statical and the Dynamical. On liberation of a strained body, the whole of the energy stored up in it may be restored in the kinetic form.

This restitution may be due to the solid body being a conservative system of particles, a small displacement amongst which acts as a disturbance of masses in stable equilibrium: by such a displacement an aggregate force is called into action which tends to produce restoration to the original form or bulk. In an elastic body the greater the displacement or distortion the greater the restitution-pressure, and that in direct proportion (**Hooke's Law**).

Perfect and Imperfect Elasticity.—A body is perfectly elastic when any given stress produces no permanent set or deformation, restitution being always complete. It is imperfectly elastic when it does permanently retain such a set. It is said to be strained beyond the **Limits of Elasticity** when it is so far strained that it retains such a set: it is said not to be strained beyond the limits of elasticity when it is not deformed so far that it cannot exactly return to its original form or bulk. When the limits of Elasticity are narrow, as in the case of lead (which, though exceedingly easily bent so as to take a permanent set, can yet be induced to enter into vibration, and must therefore be elastic within narrow limits), the body is said to be "imperfectly elastic," or to possess little **elastic Toughness**. When it can suffer distortions within wide limits without taking up a permanent set, it is said to have great elastic toughness; and a body which has infinitely wide limits of elasticity is said to be perfectly elastic. There is no body perfectly elastic, but any body may within the limits of its elasticity be considered as a perfectly elastic body.

In popular language a body is said, like indiarubber, to be very elastic when it has great elastic toughness—*i.e.*, when it can be distorted through wide ranges without taking up a permanent set; but this use of the word should be discouraged in favour of that use in which it is made to signify conjoined powers of resistance to deformation, and of restitution of shape, of bulk, and of work done upon the elastic object.

The elastic toughness exemplified in a Toledo sword-blade must be distinguished from the ordinary ultimate toughness or breaking toughness; the former may be much less than the latter.

Residual Restitution with **Deferred Restitution-pressure.**—When a body which has been distorted is left to itself without vibration, it may, when it has come to rest, be fixed between supports; it then exerts no elastic pressure; but in the course of a little time it will be found to be exerting an elastic pressure which has been in the meantime slowly developed, and which tends to restore the body more nearly to its normal condition. Mechanical disturbances—such as vibration, shaking, jarring, etc.—which allow the molecules to glide past one another, facilitate the development of this deferred restitution. Boltzmann found that successive torsions, differing in amount and in sense, caused the subsequent successive emergence of deferred restitution-pressures whose order of succession was the inverse of that of the torsions which had given rise to them.

Vibrations due to Elasticity.—When a body is distorted, not beyond the limits of elasticity, and liberated, the work done upon it is restored. The body exactly regains its original form or bulk, but at the moment of complete restitution the energy possessed by the body (if perfectly elastic) has wholly assumed the kinetic form, and the body passes rapidly, if it be free to do so, through its mean form or bulk, and suffers an equal distortion or alteration of volume in the opposite sense. Again a restitution-pressure is developed, and the body swings back through its mean position. This is repeated, and thus we have vibrations produced as the result of elasticity. The force bringing back every particle towards the mean position is proportional to the distance from that mean position, and this is the criterion of harmonic motions. Hence in a solid body, which is a system of particles, any displacement sets up an intramolecular restitution-pressure, which results in harmonic motion (Fourier-motion) of the separate particles, and the particles of a disturbed tuning-fork or stretched string may execute harmonic vibrations, particles equidistant from one another generally assuming equal differences of phase in their respective S.H.M.'s. The whole body executes, like a pendulum, isochronous vibrations, as, for example, the vibrating mainspring of a watch.

Viscosity of Elastic Solids.—When an elastic body has entered into vibration it appears more or less rapidly to lose its energy; its vibrations wane away. This waning away is due to the "*viscosity*" of the solid: the energy of vibration becomes converted into heat. The amplitude of each successive oscillation bears to that of the one immediately preceding a constant ratio. If the ratio between the first and second oscillations be $1 : v$, the third will be v^2, the nth will be v^{n-1}. On account of this viscosity a tuning-fork cannot be made of lead or zinc, the vibrations of which too rapidly die away. This "Viscosity" is what is frequently understood by the term **imperfect elasticity**: the restitution of form or bulk may be perfect, but that of energy is not, for some of it is dissipated in the form of Heat.

Fatigue of Elasticity.—When a tuning-fork is kept (as by an electromagnetic arrangement, p. 648) continuously vibrating for a long time, it stops almost instantaneously when the exciting cause is removed. The steel requires periods of rest: if it be kept continuously vibrating it has a tendency to become viscous, and to return comparatively slowly to its mean form after each displacement.

Velocity of propagation of a wave-motion $= \sqrt{K/\rho}$.—The restitution-pressure developed in consequence of a displacement varies as K, the coefficient of restitution; the acceleration produced by the restitution-pressure varies as the restitution-pressure; the velocity in the circle of reference (in S.H.M.) varies as the *square root* of the acceleration; the velocity of propagation varies as the velocity in the circle of reference: therefore the velocity of propagation varies as the square root of K.

Given the same elastic pressures and the same work done upon two bodies whose respective densities are ρ and $\rho_{,}$, the energy being equal, the respective velocities produced must vary inversely as the square roots of ρ and $\rho_{,}$. Hence v varies as $\sqrt{K/\rho}$; and it can be shown that no multiplier intervenes, and that v is equal to $\sqrt{K/\rho}$.

This is the case of a sound-wave travelling through a solid substance. A sound-wave is propagated through steel of sp. density $= 7{\cdot}85$, with velocity $v = \sqrt{K/\rho} = \sqrt{981 \times 2520{,}000000 \div 7{\cdot}85} = \sqrt{314{,}919{,}745{,}223} = 561177$ cm. per second $= 5611{\cdot}77$ metres per second.

The property of Elasticity is not inconsistent with brittleness: glass has very narrow limits of pliability, and is accordingly brittle, but within these limits it is eminently elastic.

Physiological Examples of Elasticity.—The whole ligamentous system affords examples, and many of the bones also possess this property. The ligaments of the elastic arch of the foot, the vertebral ligaments, and the intervertebral discs acting against the down-dragging weight of the viscera; those ligaments which by their very molecular constitution (however this may be accounted for) are always on the stretch, such as the elastic ligament of the eye, the filled arteries, the ligaments of the *symphysis pubis*; the combined flexion and twist of the ribs in inspiration and their elastic restitution in expiration; the ligaments of the lamellibranch shell, the tracheæ of insects,—all furnish examples of Elasticity.

The Mechanical Advantages of Elasticity.—These can be studied in a well-hung vehicle with light springs. Any sudden jolt or jar is not communicated to the body of the vehicle with its original abruptness, but gives rise to a wave-motion which lifts the body of the carriage and allows it to oscillate till it returns to relative rest. If a person jump, landing on his feet, the shock is partly broken by the elastic arches of the feet; but further, before it reaches the brain it has to pass through a succession of elastic discs, the ultimate effect of whose intervention is a gradual and not an abrupt arrest of the downward movement of the head. Were it not for this the brain would be ruptured by exceedingly small leaps.

Work is directed by elastic intermediaries so that it can become useful and not harmful. Jolts and jars—which, as we have seen under Momentum, involve the disappearance of Energy in doing harmful mechanical work—are converted into smooth wave-motions. Thus energy is saved, as well as mischief prevented. If a person run over a gravelly road with a heavy vehicle attached to his person by a non-elastic cord, he will feel greatly bruised. If he interpose a steel spring or a thick piece of indiarubber between himself and the vehicle, the pain is infinitely lessened, and the actual energy expended is about 25 per cent less (Prof. Marey and M. Hirn); work has not been spent in jolting and jarring himself and the vehicle.

The use of elastic intermediaries suggests itself in all cases where jolts of any kind would be injurious. Compare the effects of an involuntary movement made by a patient whose limbs are under extension by a weight and non-elastic cord with the effect of the same movement when a light spring intervenes between the limb affected and the extending weight. The spring persists and keeps up the tension, but it yields to the momentary twitch; the weight rises a little and sinks back to its former position.

If a piece of thin cord, tied round a somewhat heavy mass of iron, be pulled with a jerk, it may snap without lifting the heavy mass; whereas, if an indiarubber band be interposed somewhere between the hand and the iron, the same jerk may first extend the indiarubber band which, in its turn, may then lift the heavy mass.

Strength of structures as depending on their form.—This is the special study of the Engineer. Here we can only state a few principles.

Galileo found that a given weight of material if disposed in solid bars presents less resistance to crushing or bending than the same material arranged in the form of tubes, provided that the walls of these tubes be not excessively thin. The following table is from Weisbach's *Engineering Mechanics*:—

	Resistance to Breaking.	Resistance to Crushing.
Massive cylinder, rad. $= 4$ Mass $= \pi r^2 l\rho = 16\pi l\rho$	1000	1000
Hollow ; radii 5 and 3 Mass $= 25\pi l\rho - 9\pi\lambda\rho = 16\pi l\rho$	1700	2125
Massive cylinder, rad. $= 5$	1000	1000
Hollow, radii 5 and 4	870·40	870·40
5 and 3	590·40	590·40

Hence, mass for mass, the hollow tube is stronger: diameter for diameter, the solid is the stronger. The strongest tube for all purposes has the relative radii 11 and 5.

Examples of this kind of structure we find in the hollow stems of plants, in the feathers of birds, in the long bones of the human body.

Economy of material may be carried still farther by the adoption of the lamellar or trabeculated structure. We see in lattice-girders how the judicious arrangement of struts which support each other makes a structure strong enough for all practical purposes, though very light; often much stronger than if it were burdened with the excessive weight of its own substance which, if it were solid, it would have to support.

In the spongy structure of bones we find a similar arrangement. In the upper end of the femur we find a disposition of horizontal, vertical, and oblique trabeculæ, which together form a rigid triangular framework supporting the weight of the body. In the astragalus we have a comparatively light and porous structure, but the trabeculæ are so arranged as to resist and to distribute the downward pressure of the body and the compressing pressure exerted by those bones, the os calcis and the scaphoid, which abut against it in the arch of the foot.

CHAPTER XI.

OF LIQUIDS.

THIS chapter may be divided into three parts treating of (1) Molecular Actions, (2) the Statics of Liquid Masses, (3) the Kinetics of Liquid Masses.

1. MOLECULAR ACTIONS.

Cohesion.—If a ring of iron wire be dipped into a solution of soap, it will be seen on taking it out that the cohesion of the liquid causes a film to be formed, which remains stretched across the ring. The force of cohesion is also manifested by a drop of water hanging from a glass rod. Such a drop may be gradually increased in size till at a certain maximum its weight overcomes its cohesion, the water breaks asunder, and the drop falls. A thin rod of glass or wire of metal may similarly be fused at the end, and the fused drop may be increased in size by continued fusion until the molecular forces can no longer counteract its weight. A little water on the end of a thick glass rod will retain a piece of paper placed in contact with it, even though some grains' weight be suspended from the paper.

Surface-Tension.—It has already been remarked that the molecular forces are most strikingly manifest at the surface of a liquid, and that every liquid may be regarded as bounded by a superficial skin or film, which behaves like a stretched membrane, and which in time reduces the contained fluid to that form which gives to the greatest cubical content the least superficial area. The tension of this superficial film is the Surface-Tension of the fluid. A raindrop, a shot falling from a shot-tower, assume the globular form because the sphere is the simplest geometrical form which fulfils these conditions. A bubble of air in water assumes a spherical form—not perfectly so on account of the resistance to its ascent. The most convenient way of studying the various forms assumed by masses of fluid under the influence of surface-tension is to relieve them of the effect of gravity by floating them in liquids of their own specific density, with which they will not readily mix.

A mixture of alcohol and water is made of the same specific density as olive oil. Masses of olive oil placed in this fluid will neither rise nor sink, but will assume the globular form. If they be not free to assume the globular form, but have limiting conditions imposed upon them,* they may assume geometrical forms of great interest, all having the smallest superficial area possible under the given conditions. If, for example, into such a globular mass of oil, an oiled circular disc of iron be suspended, having a diameter greater than that of the mass, the mass of oil will spread over each face of the disc, and will form on each side of it a segment of a larger sphere. If such a disc be brought up to the globular mass by one face only, the oil will not pass round the edge of the disc. The curvature of the segments of spheres produced may be modified by adding or removing oil by means of a pipette. If a short oiled cylinder with open ends be put into the dilute alcohol, and a mass of oil inserted by means of a pipette, the oil will fill the cylinder and form a kind of biconvex lens of oil; by means of a pipette oil may be taken from the mass till it becomes a biconcave lenticular mass ; or if the operation be stopped at the right instant, a plane-ended cylindrical mass of oil will be obtained. If a circular ring be immersed in a large mass of oil, and some of the oil be then removed, the mass will assume a lenticular form. If a little iron framework be constructed, representing in outline the sides (one inch) of a cube, and hung into a mass of oil which is then gradually removed, the mass of oil will have part of its periphery determined by the iron framework, and will assume the appearance successively of a cube with convex sides, of a cube with plane sides, of a cube with concave sides.

But we can study the effect of surface-tension to even greater advantage when we diminish the mass of a fluid without decreasing the area of the superficial film. This we can do by using soap films or collodion films.

Soap Films.—Plateau's method. 1 part fresh moist Marseilles soap (much better, pure oleate of soda) cut into very small pieces ; dissolve with moderate heat in 40 parts by weight of distilled water. Filter through moderately-thin filter paper. Mix thoroughly 15 volumes of this solution with 11 volumes of Price's glycerine ; let the mixture stand for seven days. On the eighth day cool down to 3° C. for six hours ; a considerable deposit is formed. Filter through porous paper, but take the precaution of placing in each filter along with the solution a little closed stoppered bottle full of ice. This will prevent the re-solution of the precipitate formed by cold. The first filtrate is turbid, but this must be refiltered till it is limpid. Films and bubbles made with this solution last for eighteen hours if kept under a glass shade in very slightly moist air.

Collodion Films.—(*Gernez.*) Ether 89 parts by weight, absolute alcohol 5½, photographic gun-cotton 5½; dissolve. Decant. To 100 parts by volume of the clear solution add 70 to 100 parts of pure castor oil. This mixture is tenacious enough to permit the use of frameworks 8 cm. in diameter.

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* Refer to an exceedingly charming work by M. Plateau, *Statique des Liquides soumis aux seules Forces moléculaires*, a treasure-house of experiments devised by a *savant* afflicted with total blindness.

If a roughened ring be dipped into either of these mixtures, and taken out, a plane film will be found stretched across it. A pipette (whose point has been dipped into the same mixture) may be employed to blow bubbles and place them on this film, and then to enlarge or diminish these bubbles. Such films and bubbles stretch themselves into the most singularly beautiful forms when iron frameworks forming the complete angular periphery of cubes, pyramids, cylinders, and so forth, are substituted for the roughened ring above described; and these forms may be infinitely varied by modifying the size of the bubbles placed on the films, or by breaking with a hot needle the connection of the film with one or more of the peripherical bounding lines; in the latter case the most beautiful skew-surfaces are formed, all presenting the least area possible under the limiting conditions.

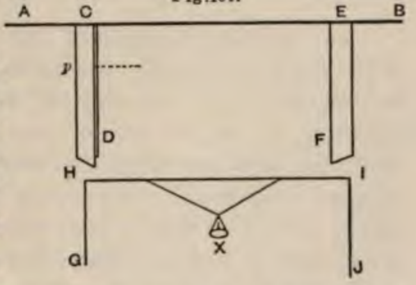
If on a simple film stretched over a ring, a piece of silk thread (moistened beforehand with the same solution) be laid in such a way that the thread crosses itself at some one point on the film, and if the film be pierced inside the loop of thread, the loop will fly open and form a perfect circle: for the rest of the film tends to occupy as small an area as possible. If a drop of alcohol be laid within the loop, the loop flies open in the same way; although the film is not broken, yet its surface-tension is diminished.

If a shallow dish containing mercury be tilted over, so that the mercury is on the point of pouring out; if then some of the mercury be drawn over so as to start a flow, the stream will drag the mercury out of the dish.

If a piece of camphor be placed on clean water it partly dissolves in the water. A strong solution of camphor has less superficial tension than a weak solution, which in its turn has less tension than pure water. At any part of the surface where the solution happens to be more dilute, there the weaker solution, having the greater tension, pulls the camphor towards itself. The camphor accordingly flies about the surface of the water. This is generally prevented by allowing the finger to touch the water, unless the finger be beforehand specially purified; so slight a trace of fatty matter communicated to the water prejudices its surface-tension to so great an extent. If a drop of ether be suspended by a glass rod close to a thin layer of water, the rest of the water is observed to flee from the spot; the surface-tension of the rest of the water is unchanged, but just under the drop of ether the tension is diminished by absorption of the ether-vapour. A thin layer of water into the centre of which a drop of alcohol is thrown leaves the alcohol for a similar reason. If a glass of strongly alcoholic wine be tilted so as to moisten the side of the glass, the film of wine left will gradually lose some alcohol, and becoming more aqueous it will have a greater superficial tension than the wine; it will pull itself up the sides of the glass and gather into drops. A thin layer of water on a metallic plate, the midpoint of which is heated, withdraws to the edges.

Measurement of Surface-Tension.—A soap or collodion film has two surfaces, and if the film be not too thin, these are independent of one another. Consequently the tension of a film is double that of a single surface of the same fluid and of the same area. The tension of a film can be measured directly. A little framework, consisting of a transverse bar AB, and two grooved slips CD and EF, allows the piece of wire GHIJ to slip freely up and down the grooves. The wire is pushed home, and a quantity of the liquid is applied between HI and CE. The little pan X may be loaded with sand until the wire HI is pulled out to a certain distance Cp from AB. When it is in that position, the film has an area CE . Cp. The weight of the total mass suspended on the film and the tension over the area CE . Cp are equal to one another. If the total weight applied be increased, the area assumed by the film is increased in direct proportion. The total weight F is distributed over the breadth CE; whence, if T represent the superficial tension across *unit of length* of CE, then F = T . CE. The energy of the film is the work done upon it; force F has pulled the film through a space Cp: the energy is F × Cp = T . CE . Cp. The energy may also be represented as the product of S (the energy per unit of area) × CE . Cp (the area) = S . CE . Cp. Hence

Fig. 101.



$$T \,.\, CE \,.\, Cp = S \,.\, CE \,.\, Cp$$
$$T = S.$$

The energy per unit of area (which in the case of a soap film is 54·936 ergs per sq. cm.) is numerically equal to the surface-tension across each unit of length (which in the same case is 54·936 dynes per cm.). These are independent of the form of the film, and depend only on its actual area, not on its curvature. For each single surface T and S, as found by experiment on films, must be halved.

This Modulus of Superficial Tension, T, is, in all the instances which we have considered, that of a surface between the liquid and the surrounding air. In the case of pure and perfectly clean water and air, this modulus is equal for each surface to 81·96173 dynes per sq. cm., very nearly three times the superficial tension of a single surface of soap solution in contact with air. The tension of the bounding surface separating olive oil and air is 36·8856 dynes; that of the surface between olive oil and water is 20·56176 dynes, per cm. At the meeting-place of oil, water, and air, these three surfaces meet; the tension of the water-air surface decidedly preponderates, and the edge of an oil-drop floating on water is drawn out indefinitely. If a drop of water be placed on chloroform—the respective tensions being water-air 81·96173, chloroform-air 30·6072, and chloroform-water 29·5281—its surface-tension (water-air) at first preponderates and pulls it into a drop. When water, air, and clean glass are placed in contact there is again a triplet of tensions, the resultant of which pulls the water over the glass, which is thus wetted by the water. The water tends to stand so that its surface makes a certain angle with the glass; this is the **angle of capillarity**, 180° between water and glass, 45°·30 between mercury and glass.

In the case of water this angle is such that the upper surface of water in contact with glass is concave; in the case of mercury the upper surface is rendered convex.

On the curvature of the upper surface, thus determined, depends the direction in which the contractile tension of the superficial film acts. The concave surface of water tends to contract and become flat, and it does so in proportion to the curvature imposed on it.

The narrower a capillary tube is, the greater is the curvature of the surface of any liquid standing in it, and therefore the greater is the contractile tendency of that surface. The effect of this tendency is, in the case of water, to exert an upward pull, to neutralise to some extent the downward effect of gravity, and to support a column of water in the tube. Hence water stands at a higher level in a narrow tube whose lower open end is dipped in water than it does in a wider one; and the height of the column supported is inversely proportional to the radius of the tube at the spot where the curved surface of the liquid is situated. Mercury under similar circumstances stands at a lower level.

This tendency of a curved surface to exert traction or pressure on a fluid may be seen in a conical capillary tube; if a drop of water be introduced, the smaller concave surface will pull the drop towards the apex; if a drop of mercury, the smaller convex surface will push the mercury from the apex.

Capillary phenomena are thus phenomena of surface-tension; and it is futile to explain the rise of sap in plants or the passage of fluids through minute vessels by "capillary attraction" when there is no free surface. An experiment which may, on the other hand, illustrate these movements, consists in oiling the interior of an open capillary tube, filling it with water and dipping the end of the tube in oil; the attraction of the sides of the tube for oil will cause the oil to run along the tube and to drive out the water; this, however, is not an exclusively capillary phenomenon.

If two plates of clean glass be set to stand vertically and parallel to one another in a shallow dish of water, the water will rise up the sides of each to a height *half* that which it would attain in a tube whose diameter is equal to the distance between the plates. If the two plates have two vertical edges in contact, the liquid will rise indefinitely where they are in contact; at other parts it rises to a height inversely proportional to the local distance between the plates, and it thus presents the outline of an equilateral hyperbola.

Just as the surface of a liquid is raised against a fixed plane of clean glass, so a floating vessel of clean glass may by surface-tension be pulled down so as to lie more deeply in the liquid than its specific gravity would lead us to expect. A floating hydrometer always gives an abnormally low reading on this account; it is pulled into the liquid, so that the liquid appears to be lighter than it really is. If a little vapour of ether be poured on the surface of the liquid so as to diminish the surface-tension, the hydrometer rises. If the water have any grease on its surface, the same effect follows. If the hydrometer be greasy, it is repelled and stands abnormally high in the liquid. Hence great confusion and inaccuracy may result from films of grease on the glass or on the fingers of the manipulator.

Objects which are wetted by the liquid in which they float are thus apparently attracted by it; those which are not so are apparently repelled. Two wetted objects floating on water seem to attract one another; two objects floating on a liquid which does not wet them both seem also to attract one another. This may be seen by throwing upon the surface of water a number of wooden balls, of which some are smoked with lampblack, while others are purified first with soap and water, then with pure water; the smoked balls approach each other, the clean ones approach each other, but the clean balls appear to avoid the smoked ones.

We may mention another consequence of surface-tension. A jet of water issuing from a rectangular orifice is most acted upon by surface-tension at its narrow edges. These are pressed together; they meet, and when they do so, spread out laterally; the same action is repeated, and the whole jet is a succession of flat segments at right angles to one another. At first sight such a jet seems to have a screw form.

The distances at which molecular forces act are not immeasurably small. Quincke found that while water stands against glass at one angle, against silver at another angle of capillarity, yet against glass coated with silver it stands at such an angle as to show that the influence of the glass is felt through the silver when the layer of silver is less than ·000,005 cm. thick; this thickness being one-tenth of the average length of a wave of light, and being further (Meyer) very much the same thing as the mean free path.

Superficial Viscosity.—This is a property of the superficial film of liquids, which is independent of the surface-tension. If a magnetic needle be so adjusted as to have its lower surface in contact with the surface of a solution of saponine, it will remain in any position, in defiance of the directive force of the earth's magnetism. On the surface of most other fluids it will move into the magnetic meridian, but the whole superficial film of the liquid will move with it, as may be shown by strewing lycopodium over the surface. The superficial film is, as a rule, exceedingly viscous as compared with the interior mass; it is consequently hard to move or to break. If a liquid have great superficial viscosity and small surface-tension (as in the case of soap and water), a bubble rising through the liquid may raise the surface film, which the tension is not able to break: the bubble may therefore persist. If a wire ring, bearing a soap film, be swept rapidly through the air, the air may fill and stretch the film, and separate part of it in the form of a complete bubble. A bubble rising with very great rapidity through a fluid may tear off some of the viscous superficial film and form a complete bubble: this is seen when a mixture of olive oil and strong sulphuric acid is vigorously stirred.

Pure water has great surface-tension, which is able to overcome the superficial viscosity. Thus pure water will not froth. Some liquids, such as a solution of gum arabic or of albumen, will froth when shaken, but their superficial viscosity is not sufficiently great to enable bubbles to be blown with them. Alcohol, sulphuric ether, sulphide of carbon, and some other liquids, have a superficial viscosity less than their internal viscosity, and consequently, when alcohol is mixed with a superficially viscous liquid, it neutralises its relative superficial viscosity, and frothing is rendered impossible. Hence the practice of adding a few drops of spirit in order to check frothing in pharmaceutical operations.

To this toughness of the superficial film, the floating of an oiled needle or the walking of an insect on water must be in part ascribed. The depth of the dimple produced by the needle is not sufficient to account, by displacement, for the support afforded to so heavy a body: the superficial tension is diminished by the oil: the tenacity of the surface film plays its part in supporting the needle. To the same cause we must attribute the smoothing of the surface of a rough sea when oil is poured upon it: the new surface has great superficial tenacity and small superficial tension, and is not readily broken up into surf. The new surface of the sea is relatively rigid; waves press against it from beneath, but their energy is spent in producing eddies below.

The superficial film of a liquid is thus seen to be a seat of energy and to be physically different from the interior.

A bubble in bursting does so with an audible sound: it scatters particles of its substance and of the contained gas to a height of three or four feet; this happens during the effervescence of sewage which is undergoing fermentation.

Cohesion-Figures.—If the surface of a tumblerful of salt water ($\frac{1}{2}$ teaspoonful to the tumbler) be touched with a pen not too full of ink, the ink will, in falling through the liquid, assume very remarkable vortical movements. A shower of rain falling on a troubled sea produces similar vortex-rings, which are carried down into regions of comparative stillness, and moderate the turbulence of the water by equalising its distribution of momentum. The forms assumed by drops of water or of mercury falling on a flat surface, at the instant when they spread out and break, are very remarkable, and may be seen when the spreading drops are momentarily illuminated by the electric spark. The edge of the spreading drop breaks up into thinner and thicker nodes and loops which vibrate: very roughly the result may be seen in a cooled splash of candle-wax.

Solubility of Solids in Liquids.—When a solid is dissolved in a liquid, work is done in overcoming its cohesion. This cohesion is overcome by the adhesion between the solid and the liquid. Ice put into sulphuric acid has its superficial particles successively torn off, and a mass of dilute sulphuric acid (which on account of liquefaction assumes a low temperature unless heat be supplied) is produced. Such union may or may not be associated with a play of chemical affinities; in the case of ice and sulphuric acid there is a tendency to the production of definite hydrates of sulphuric acid, the formation of which is accompanied by the evolution of a certain amount of heat. If sulphate of magnesia be placed in water it will be dissolved to a certain limited extent; if the salt be added in excess above this limit it will not be dissolved; when this limit has been reached the solution is a **saturated solution.** This limit is expressed by the **coefficient of solubility,** a number indicating the quantity of solid which can be dissolved and remain in solution in unit-mass of the liquid at the particular temperature for which the coefficient is or ought to be specified. A saturated solution can dissolve no further quantity of the same salt, for the adhesion of such a solution to the salt is no longer greater than the cohesion of the salt itself. If the cohesion of the salt be lessened by heat, more may be dissolved; and as a general rule, with but few exceptions—hydrate of lime, sulphate of soda, phosphate of magnesia—salts are more soluble in hot than in cold water. The adhesion of a liquid to the solid which it holds in solution may be relatively great or feeble; and its relative amount may be indicated, though not measured quantitatively—(1) by a high or low coefficient of solubility; (2) by the amount of energy which must be imparted to the molecules in order, by boiling, to tear the water away from the salt, or, in other words, by the high or low boiling-point of a saline solution; (3) by the relative effect of charcoal filters in retaining the salts of a saline solution while allowing the water to pass, a property made use of in the analysis of poisons; and sometimes (4) by the detachment of the liquid from the solid by a stronger molecular attraction, as in the case of iodide of starch, a solution of which is precipitated by acetate of potash, the water leaving the iodide of starch and adhering to the salt.

When a saturated solution is cooled, the adhesion between the liquid and the solid diminishes, the coefficient of solubility diminishes, and the solid segregates in a separate form: thus hot saturated solutions may be set aside to cool, and on cooling they crystallise, the materials dividing into crystals of the salt and an ordinary cold saturated solution of the same. Sometimes, as in the case of sulphate of soda, such a solution (though cooled down to a temperature at which it cannot permanently retain all the salt which it holds in solution) does not crystallise, but forms a **supersaturated solution.** Such a solution is in a state of unstable molecular equilibrium, and the instant it is touched with a crystal of the same salt or of an isomorphous substance, or by the dust of the air containing the same substance, or by an oil (especially if somewhat oxidised), or by a bubble of gas soluble in the liquid, or when it is exposed to the least vibration, the whole molecular arrangement topples over, and the excess of salt assumes the solid form with great evolution of heat.

A similar delay in solidification occurs in the case of melted phosphorus, which can be kept fluid at 10° C. (its solidification point being 44°·2 C.) for weeks, especially if the water lying above it contain a trace of potash hydrate or of nitric acid. The slightest shake or contact with a piece of phosphorus determines solidification.

Miscibility of Liquids.—If a bottle be filled with oil and water, and shaken, the layers separate as soon as the disturbance ceases. Alcohol and water treated in the same way mutually dissolve each other, and mix perfectly in any proportions. Ether and water will each take up a certain proportion of the other, and will separate into two layers, the one a saturated aqueous solution of ether, the other a saturated ethereal solution of water; these two fluids are miscible only in certain proportions. Very often, as in the case of alcohol and water, there is a contraction of volume and evolution of heat, there being some potential energy of separation somehow liberated by the approximation of mutually attracting molecules of the different substances.

Imbibition.—Porous objects, such as a lump of sugar, blotting paper, a heap of sand, a sponge, a lamp-wick, absorb liquids with a rapidity which depends on the nature of the porous substance itself and on that of the liquid absorbed, and which is greater if the materials be warm. This takes place by reason of an attraction between the solid and the fluid (which Chevreul called *affinité capillaire*), and heat is evolved when this attraction is satisfied, as in the case of a wetted rope, which rises in temperature from 2° to 10° C. (part of this effect being due to the concurrent shrinkage of the rope). When a porous body which has thus taken up a quantity of liquid is subjected to pressure, the whole of the liquid can by no means be squeezed out; some water still remains, which can be evaporated away. Imbibition will fill the pores of a solid with a fluid, but will not set up a permanent current in those pores unless, as in the case of a lamp-wick, there be constant removal of the fluid at one extremity of the porous object while imbibition goes on at the other.

Diffusion—Jar-diffusion.*—If a phial, filled to within say half-an-inch of the top with a saline solution, be placed in a jar; if water be poured into the jar so as to surround the phial, and if more water be cautiously added until the phial is covered with a layer of water of about half-an-inch in depth, the whole being set aside in a quiet place, the solution in the phial will diffuse into the surrounding water. The quantity of substance diffused into the water in a given time depends (1) on the length of that time; (2) on the quantity of substance in the phial solution, being (within narrow limits) proportional to its strength; (3) on the temperature, being greater at a high temperature; (4) on a Coefficient of Diffusibility special to each substance. Other things being equal, urea and salt diffuse twice as fast as sugar, sugar twice as fast as gum arabic, gum arabic more than four times as fast as egg-albumen.

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* See Graham's *Chemical and Physical Researches.*

Sugar travels as far in a column of water in two days as albumen in fourteen. The following numbers indicate the relative times necessary for the process of diffusion to convey in water through equal distances equal amounts of the several substances :—Hydrochloric acid, 1; chloride of sodium, 2·33; sugar, 7; sulphate of magnesia, 7; albumen, 49; caramel, 98 (Graham).

The rate of diffusion of all substances is increased by moderate heat, but in those substances whose coefficient of diffusibility is small, it is more increased by heat than it is in those substances which are already very diffusible. Hence the greatest proportionate differences in diffusion-rates are found in the coldest solutions.

Some liquids, such as water and sulphuric acid, ether and chloroform, mercury and molten lead, diffuse into one another with considerable rapidity.

If a mixture be placed in the diffusion-phial, the approximate rule is that each component of the mixture is diffused out at its own rate, and independently of the others. There is, however, a departure from strict adherence to this rule, in the sense that the ordinary differences of diffusibility are exaggerated in such a mixture. If the phial contain a double salt, such as alum, diffusion may effect chemical decomposition: sulphate of potash and sulphate of alumina are separated, the former being diffused more rapidly.

A high boiling-point of any solution (which indicates adhesion of water to the salt dissolved) is associated with rapid diffusibility of the same salt; but there is no close relation between the rapidity of diffusion of a salt and its solubility.

Colloids and Crystalloids.—On surveying a number of objects which have a wide range of relative diffusibilities, we see that at one end of the scale we have such things as urea and chloride of sodium, and at the other such things as starch, gum, gelatine, albumen. The former are bodies of rapid diffusibility, have generally a certain chemical stability and a crystalline form, and are called **Crystalloids.** The latter are bodies of slow diffusibility, have very probably large molecules compounded of groups of their simplest molecules, have in general (with rare exceptions, such as the blood-crystals) the non-crystalline amorphous glue-like character which gives them the name of **Colloids,** and are for the most part in a state of unstable equilibrium when in the moist condition. Colloids have great cohesion, and adhere firmly to other colloids: thus isinglass heated with acetic acid forms a cement which adheres firmly to glass; and when they dry they tend to contract firmly, so that a strong solution of gum arabic, drying in a test tube, will sometimes break the tube. They often, when drying up, extrude their contained water, and form clots, on the surface of which the water presents itself in drops. Colloids also in many instances possess the power of taking up alcohol or olein in the room of their water of constitution. This property is possessed even by such a substance as colloid silicic acid.

An animal tissue, which is in great part composed of colloids, may have its water expelled and replaced by alcohol by dint of repeated washing in that liquid.

Colloids being very slightly diffusible are tasteless; they do not reach the nerve-ends. For the same reason they are very indigestible—*e.g.*, gelatine—unless peptonised; peptones being, by exception, diffusible though otherwise colloidal.

If a layer of pure jelly be laid on a layer of jelly charged with soluble salts and also with caramel, the salts will diffuse into the upper layer of solid jelly nearly as fast as if it were pure water; the caramel will not travel at all. If a film of colloid matter (starched paper) be placed between a saline solution containing colloid matters and a mass of pure water, the colloid septum will offer little obstruction to the passage of the salts into the water, but will prevent the colloid matter from passing through. Colloid matter is thus impervious to the diffusion of other colloids, but does not hinder the diffusion of crystalloids.

Diffusion through Membranes—Osmosis.—If three layers of liquid, chloroform, water, ether, be placed in a closed bottle and set aside, it will be found that in course of time the ether travels into the chloroform, but that the water does not to any appreciable extent allow the chloroform to pass into the ether. The ether dissolves to some extent in the water and diffuses through it: it is removed from the water by the chloroform: step by step the water passes the whole of the ether through its own substance. A thin caoutchouc membrane lying between alcohol and water allows the alcohol to pass through it into the water; but the reverse passage of water into the alcohol is barred. If an organic septum be used it is wetted, and the water passes into the alcohol. If hydrochloric acid and water be separated by an animal membrane, the hydrochloric acid passes through in greater quantity: both fluids wet the membrane, the hydrochloric acid is most attracted. Hence molecules may travel through a septum devoid of perceptible pores as well as through one in which pores exist.

If the membrane employed be porous, we have the process of Osmosis. The membrane becomes penetrated by that one of the two liquids ("liquid A") for which the walls of its pores have the greater attraction or affinity. When each small capillary column of the liquid A comes at the farther surface of the membrane into contact with the liquid B, the molecules of liquid B diffuse into it. Thus the end of the column of liquid A comes more to resemble that liquid B which is less attracted by the walls of the pore, and it is extruded from the pore and pushed into liquid B. This process is continuous, and a stream of liquid A is drawn through each pore of the membrane into liquid B. Down the centre of the stream there is, however, a backward diffusion-current of molecules passing from the liquid B. This happens if the pores be wide enough to allow the centre of the stream to be comparatively out of reach of the immediate influence of the walls of the channels, an influence which we have seen to extend to a distance of $\frac{1}{20000}$ mm. or $\frac{1}{250000}$ inch. If the liquid stream be not too rapid, these molecules will make their way against it into liquid A. If the channels be very narrow the liquid stream is relatively accelerated; thus the ratio between the amount of water that passes through a porous membrane into a saline solution and the amount of salt that passes in the opposite direction is increased by diminution of the pores. This ratio is called the **Endosmotic Equivalent.** It is not a constant, but depends on the nature of the membrane, and even with the same membrane it differs according to its thickness or state of freshness, and may be increased by tanning with tannin or chromic acid, which diminish the size of the pores.

Thus for a membrane on one side of which is dry common salt, on the other side water, if the membrane be a piece of cow's pericardium, for every grain of salt which passes into the water, 4 grains of water pass into the salt; with a piece of cow's bladder, the endosmotic equivalent is 6. If on one side of an animal membrane there be placed a strong solution of sulphate of magnesia and on the other a quantity of blood serum, the fluid of the blood serum will pass into the saline solution, taking some albumen with it, and some sulphate of magnesia will pass into the blood serum.—(Milne-Edwards.)

The mechanical structure of the membrane has a marked influence on the process; thus water will pass more readily inwards through frogskin, more readily outwards through eelskin.

The matters already moistening the membrane also affect the rate of transmission; thus albumen more readily passes through a membrane previously moistened with alkalies. If between alcohol and water there be arranged a membrane previously soaked in oil, the membrane cannot be wetted, and the alcohol now passes into the water.

If the saline solution be in a state of movement relative to the membrane, the particles are drawn away from the membrane, and the diffusion-stream is hindered; if the water into which the salts are passing be constantly renewed, the molecular diffusion is accelerated.

Heat increases the rapidity of Osmosis. An electric current (the "electrodes" being on opposite sides of the membrane) has the singular effect of, as it were, pushing the liquid bodily through the membrane towards the negative electrode. Even gelatine and the fatty matters of milk can be thus driven through a membrane.

If a mixture of different substances be exposed to osmosis through a porous membrane, the colloids will remain or will pass through in very small quantities, the crystalloids pass through freely. This is the basis of the process of **Dialysis.** Various mechanical arrangements for carrying out dialysis suggest themselves: a phial with the bottom cut off or a wide glass tube, over the lower end of which a piece of membrane is stretched; the material to be dialysed being placed in this, and the whole suspended in water. The most convenient arrangement in many respects is a piece of parchment paper (the leaks in which are stopped with albumen coagulated by heat) laid upon a wooden ring, into which a smaller ring is thrust so as to form a dish with a membranous bottom; this is floated on a mass of water, and the substance to be dialysed placed in a thin layer on the dish. The crystalloids (strychnine, etc.) pass into the water, the colloids (mucus, etc.) remain in the dish. This method is peculiarly applicable to the separation of poisons from animal matters.

If the mixed solution exert pressure upon the membrane, colloids as well as crystalloids may be found to pass in considerable quantities through that membrane along with the fluid forced through by the pressure.

If peroxide of iron be dissolved in a solution of perchloride of iron, and the whole be then dialysed, the chloride of iron will pass through the membrane, leaving the colloid oxide of iron behind in solution. Neutral Prussian blue (as used in microscopical work) is also a colloid, and may be purified in the same way: so is sucrate of copper, a soluble compound of copper oxide with sugar, which is reduced on heating. Albumen may also be obtained in a relatively pure form by separating it by dialysis from the greater part of the salts that it may contain.

If the membrane used be the gastric or intestinal membrane, taken after death, it is found that curare or snake poison will not pass through it, while they are absorbed readily by the dermis or by serous membranes. They seem not to wet the former; hence the selective absorption of poisons has a certain physical basis.

Absorption by the dermis is seen to be a physical process; the walls of the vessels, both lymphatic and venous, are known to be physically permeable to osmose, and the salt being placed on a vascular region is quickly absorbed, the osmose being accelerated by the flow of liquid in the vessels. Substances brought in contact with the pulmonary epithelium are also very rapidly absorbed. Lymph acts towards blood as water does towards a saline solution, and the tendency of osmotic action is to carry the fluids of the body into the blood-stream. Repletion of the vessels checks this tendency. Adhesion between water and oil is greatly increased if a little alkali be dissolved in the water. When the mucous membrane is covered with bile it has much more affinity for oil globules, which are, besides, each endowed by emulsionising with an aqueous or soapy covering, which makes them act like minute masses of water, and enables them not to experience any relative repulsion when carried with the rest of the aqueous stream.

Osmose is thus related to capillary affinity and to diffusion, but it bears no exact numerical relation to either of these, for it depends on the relation between the pores and the solid parts of the membrane, upon the nature of the material (colloidal or otherwise) of the membrane, upon the width of the pores, upon the temperature and electrical condition, upon the mutual action of the fluids, and in physiological cases (Milne-Edwards, *Physiologie*, tome V) it seems to depend on the influence of the nervous system.

2. THE STATICS OF LIQUID MASSES.

Liquids are incapable of resisting a change of shape when acted on by force which is not equally applied over the whole surface, and they **flow** when thus acted on, unless supported on all sides.

All soft masses which cannot in the aggregate permanently resist a change of shape are practically liquids, and are subject to hydrostatical laws.

It is often convenient in discussing the equilibrium of liquids to imagine little elements of the liquid, floating in and forming part of the liquid, to become solidified, or otherwise to become separately recognisable while not altering their other relations to the surrounding mass. Then, if the liquid as a whole is at rest, each of these little elements of mass must also be at rest.

This being so, the forces acting on each little element of mass must be in equilibrium, and their resultant must be nil. This can only occur (since each fluid element is subject to pressure on all sides, as may be understood by considering the rush of fluid from all sides that would occur if the little element of mass were suddenly annihilated) if the pressure on all sides be equal; and since the element may be reduced to a material point, the proposition follows that at any point in a liquid the pressure in all directions is equal.

The pressure at any point of the surface of a liquid at rest must be at right angles to the surface. If it were not so, it must be oblique; being oblique, it would be resoluble into a component at right angles and one parallel to the surface. The latter could not fail to act, the surface being that of a liquid; hence the liquid would not be at rest; whence there is no such component, and the pressure is at right angles to the surface. Conversely, when a liquid is at rest, the pressure which it exercises on the vessel containing it is at right angles to the walls of the vessel, for the walls of the vessel coincide in aspect with the surface of the liquid.

If in a liquid at rest, expressly supposed to be not under the influence of gravity, two elements were imagined to be in contact, and yet to be subject to different pressures, there would at the point or surface of contact be a relative difference of pressures which would necessarily cause movement of the liquid; but the liquid is supposed to be at rest; hence there can be no difference between the pressures of any two contiguous elements, and the pressure throughout a weightless liquid at rest is everywhere the same **(Pascal's Principle).**

If pressure be applied from without to some of the particles of a liquid, and if that liquid be free to change its shape, it will do so; if it be not free to flow, the particles pressed on will press against contiguous particles, and these against their neighbours; thus the pressure becomes equalised throughout the whole of the liquid. This is the principle of the so-called Transmissibility of Fluid Pressures. The pressure applied to any area of the surface of a liquid not free to flow becomes equally felt over every equal area of the surface.

If a wide cylinder with a piston whose area is a square inches tity of fluid that would lie vertically above the base if the column of fluid were perfectly cylindrical and of the height b, but which, owing to the form of the vessel, does not so lie.

The total pressure on the base of a vessel containing liquid depends on the height (b) of the liquid and the area (a) of the base, the density ρ of the liquid, and g the local acceleration of gravity, but does not depend on the actual weight of the liquid employed.

The total pressure $P = ab\rho g$.

If a flask filled with water be fitted with a cork in which a long narrow tube is fixed upright, a very small quantity of water poured into the tube will be competent to burst the flask.

This proposition—that the same amount of water may produce widely-differing amounts of pressure on the vessel in which it is contained, these amounts depending on the form of that vessel—is said to be a Hydrostatic Paradox; the only paradoxical element about it is, however, its discrepancy with a certain uninformed intuitional belief in the Conservation of Force.

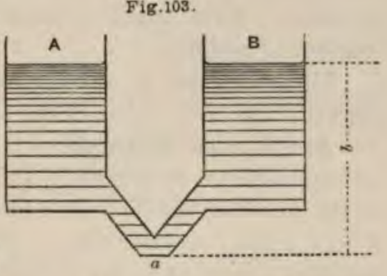
A slack bag containing liquid, and set to rest upon a plane surface, exerts a pressure upon that surface which is equal to the product of the area of contact × the height of the centre of gravity of the liquid. So for semi-fluid masses.

When the surface of a liquid is exposed to the atmospheric pressure of 760 mm. or 76 cm., it bears on each sq. cm. of surface the weight of 76 cub. cm. of mercury, or 1033·3 grammes; this is equal to $(1033{\cdot}3 \times 981)$ dynes: or if the barometer stand at x cm., the pressure on each sq. cm. of surface is $(13{\cdot}596x \times 981)$ dynes. This number of units of force per sq. cm. may be expressed by the symbol Π. Then the total atmospheric pressure on area a sq. cm. is $a\Pi$. The liquid pressure on the area a at the mean depth b cm. would have been $ab\rho g$ if there had been no pressure at the surface. When the atmospheric pressure acts at the surface of a liquid, the total pressure on any plane, whose area is a and whose mean depth below the surface is b, amounts to $(a\Pi + ab\rho g)$.

When the human body (as in ordinary circumstances) has the head in the highest position, the blood in the head is exposed to the ordinary atmospheric pressure. If the head be downwards, the pressure on the blood vessels of the head is increased by the weight of the column of blood in the inverted body, and hence there is congestion. If the body float submerged in a liquid of its own sp. density, head up, the pressure on the blood vessels of the head is the ordinary atmospheric pressure increased by the weight of the column of liquid immediately overlying the head; but if the head be suspended, though the increased depth cause a correspondingly-increased external pressure on the head, yet the equally-increased internal pressure of blood balances this effect, and there is no congestion. This may be illustrated by a loop of thin indiarubber-tubing filled with water: suspended in air, the depending part is distended: suspended in water, it is relieved from distension.

Communicating Vases.—"Water seeks its own level." If there be two communicating vessels containing the same liquid, the lowest part of the communicating channel may be considered as a common base: its area is a. Regarding it as the base of vessel A (Fig. 103), we see that the pressure on it must be $ab\rho g$, and the height of the liquid in A is b; regarding it as the base of vessel B, the pressure (which must be the same, for the liquid is at rest) is equal to $ab_1\rho g$: whence $b = b_1$, the height of the liquid in the two vases must be the same, and the level must be the same in two communicating vases, whatever be the shape of the communication, so long as the communication-pipe is continuously filled with liquid. This implies that sufficient time for assuming equilibrium is allowed.

Fig. 103.



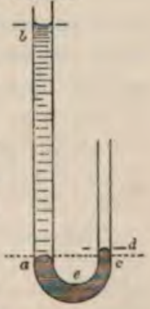
If the fluids in the two communicating columns be not of the same sp. density, the effect is an inequality in the heights of the columns, which vary inversely as the sp. density.

The two pressures are $ab\rho g$ and $ab_1\rho_1 g$; these are equal;

$$\therefore b\rho = b_1\rho_1, \quad \text{or} \quad b : b_1 :: \rho_1 : \rho.$$

In Fig. 104 the column of water ab and that of mercury cd balance one another because they produce an equal pressure on the base e. If a U-tube contain water, of which that in one limb is heated while that in the other remains cool, the liquid in the hotter limb will stand at a higher level than that in the cooler. The relative specific densities of fluids may be estimated by methods based on this principle.

Fig. 104.



The accuracy of the "water-level" may be interfered with by capillarity. If both limbs of a U-tube be narrow, but unequally so, the liquid will stand at a greater height in the narrower limb.

If a U-tube be taken of which the narrower limb is the shorter, the quantity of water placed in the tube may be regulated so as to afford the following three conditions:—(1) The shorter limb filled with water, the upper surface of which is concave, and the water standing at a lower level in the wider tube; (2) The shorter limb completely filled with water the upper surface of which is plane, and the concave surface of the water in the wider tube at nearly the same level, but a little higher; (3) The shorter limb completely filled with water the upper surface of which is convex, and the water standing at a higher level in the wider tube, its surface being concave.

Every liquid tends to set the whole of its surface at right angles to the force of gravity.

When a cylindrical vessel containing a liquid is rotated round its longitudinal axis, the surface of the liquid assumes a parabolic form which is maintained constant as long as the rotation is uniform.

Thus the form of the free surface of liquids is affected by gravity, by molecular forces, and by rotation.

Archimedes' Principle.—If an element of mass of a liquid be supposed to be solidified, this will not affect its equilibrium in the midst of the fluid of which it had previously formed part; it will neither rise nor sink. Even though its nature be altered, provided that it does not become either lighter or heavier, it will neither sink nor rise: it has apparently lost its weight. If it become heavier than the liquid it will sink; if it become lighter it will rise. Gravity has no effect in making a body rise or sink in a liquid except in so far as there is a difference between the density of the liquid and that of the body suspended in it. This leads to Archimedes' principle:—"A body suspended in a fluid apparently loses as much weight as is equal to the weight of the mass of fluid which it displaces." The application of this principle to the study of sp. density we have already seen.

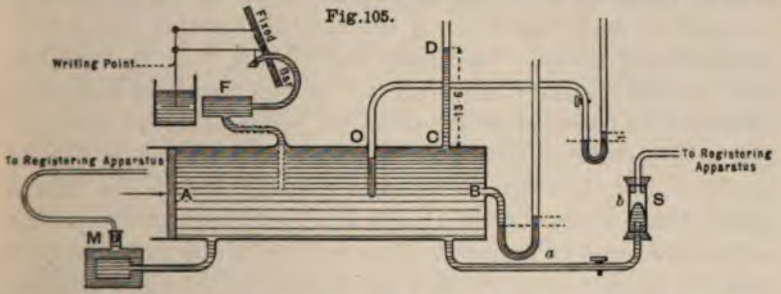
A body lighter than water may be loaded with just so much mass as will sink the light body without that additional mass itself entering the liquid; the whole will then float, the lighter body displacing a bulk of water equal to its own bulk; the weight opposing the buoyancy of the water being the weight of the body *plus* that of the load placed on it; and the ratio

$$\frac{\text{weight of body}}{\text{weight of body} + \text{load}} = \text{sp. density of the floating body.}$$

That a body, even though sufficiently light to float, tends to sink in water until the weight of the water displaced becomes equal to the weight of the whole body, may be shown by a very simple experiment. Take two similar phials, two small elastic bands, and four nails which must not be too heavy. With these may be constructed a couple of rough models representing a person with his arms kept down by his side, and a person whose arms are elevated above his head. On putting these models into water the difference of floating capacity will be very obvious.

Measurement of Pressure.—The pressure to which the surface of a liquid is exposed can always be measured by the height of the liquid column which that pressure can support. If in Fig. 105 the water contained in the cylinder AB be exposed to a certain pressure communicated by a piston at A, and if a side tube (a piezometer tube) placed at C be in communication with the liquid, water will rise in the tube until there is equilibrium. This equilibrium is between the Pressure of the fluid in AB (together with the atmospheric pressure acting through A), tending to push upwards the column of water CD, and the Weight of that column, which (together with the atmospheric pressure acting on D) tends to make it sink back into the cylinder. The whole outward pressure P exerted by the liquid on the orifice C must be equal to the weight of the column CD.

Fig.105.



The latter is (if the area of the orifice at C be a, and H the height of the column) equal to $g \times$ the mass of the column $= a\,H.\rho.g$. As this is distributed over the area a, on every unit of area of the surface of the fluid its amount must be $H.\rho g$. Whence the outward pressure at C is, per unit of area, equal to $H\rho g$.

Let us suppose that the column CD is one of water, 13·596 cm. high; the pressure per unit of surface is $H\rho g = (13{\cdot}596 \times 1 \times 981)$ dynes per sq. cm.

If at B a U-tube (a manometer tube) be fixed, containing in its bend a quantity of mercury, the mercury will stand at the same level in both branches as long as the internal pressure and the external are equal; but if the internal pressure be increased, the mercury will be depressed in the branch nearer the cylinder, and will rise in the other.

In the case supposed it would (setting aside the difference of pressure due to difference of level between C and B) sink through $\frac{1}{2}$ cm. in the nearer and rise through $\frac{1}{2}$ cm. in the farther limb: a difference of 1 cm. of mercury being thus established. This column of mercury is that whose weight balances the internal pressure: its weight is (1 cub. cm. × 13·596 × 981) dynes, acting upon every square centimetre. Hence—

The pressure on the surface of the liquid in the cylinder, AB of Fig. 105, may be equally well represented in brief phraseology as a pressure of 13·596 cm. of water, or one of 1 cm. of mercury.

Exploration of the pressure in the interior of a stationary liquid mass.—In Fig. 105 let there be an aperture in the walls of the cylinder at O; through this aperture pass a tube which exactly fits it. The inner end of this tube is furnished with a flexible and elastic cap. The outer end is connected directly or by means of indiarubber—or, better, of leaden tubing—first with a stopcock (the bore of which is the same as that of the tube), and then with a manometer tube. Before the tube is passed through the orifice O the level of the mercury in the manometer must be adjusted. This is done while the stopcock is open, by pouring mercury into the manometer tube and bringing it to an exact level by the addition or subtraction of mercury in the outer limb; the stopcock is then closed, and the tube adjusted with its elastic closed end in the body of the cylinder. The stopcock is then opened; the pressure of the fluid in the cylinder on the indiarubber cap (if it differ from the atmospheric pressure) alters the shape of the cap, and the mercury in the manometer assumes a difference of level which indicates the pressure in the interior of the cylinder. If the cap be so small that it is collapsed by a given pressure *p*, it cannot be used to record pressures of greater amount than *p*. This defect can be remedied either by using a larger cap or else by using capillary manometers of uniform bore, in which the displacement of a very small quantity of mercury (and therefore a small compression of the indiarubber cap) will serve to indicate high differences of pressure. If the cap be at all inflated before it is inserted within the cylinder, the elastic recoil of the cap adds an unknown quantity to the internal fluid pressure, and the readings of the instrument are untrustworthy, unless special contrivances are made use of for ascertaining the exact effect due to this cause.

Figure 105 F shows the essential parts of another instrument by which the pressure in the cylinder AB may be measured; it is substantially identical with Bourdon's Steam Gauge. A hollow tube of elastic metal having an elliptical cross-section, bent into the shape of a Ω, and filled with liquid (alcohol,

glycerine, water, or oil), suffers changes of shape under the influence of changes of pressure in the contained fluid. When the internal pressure increases, the Ω straightens out; when it decreases it becomes more curved.

The pressure increasing, the cross-section tends to become more circular (the circle being a figure of greatest area for least circumference): the surface and the mean curvature are constant; the curvature across the tube increasing, that along the tube diminishes, and the tube straightens out.

Such a tube is continuous with a box or cavity containing liquid, which may in its turn be continuous with the liquid of the cylinder when the surface-pressure has to be found, or may be connected merely with an indiarubber cap like that inserted as an explorateur in orifice O of the same figure.

For physiological work this principle is applied in Fick's Federmanometer, in which the Ω-tube is filled with alcohol, and the tubes which intervene between it and those bloodvessels in which the blood-pressure has to be determined are filled with a solution of bicarbonate of soda of a sp. gr. of 1·083; this being (Cyon) the strength of solution which most markedly checks any tendency to coagulation.

A given amount of bend of the Ω-tube may be interpreted as signifying exposure to a certain amount of pressure, if the instrument be previously graduated by finding the relation between certain known pressures and the distortions produced by them.

The instrument S in Fig. 105 is the sphygmoscope of Marey. The tube *a* is closed by an elastic cap which projects into the lumen of the wider tube *b*; *a* and its cap are filled with liquid, which is continuous with that of the cylinder; the pressure within the cylinder forces the fluid into the cap until the elasticity of the cap and the pressure of the liquid are in equilibrium: the air in the tube *b* is compressed, and the pressure is communicated to a manometric capsule or other registering apparatus, the displacement of the lever of which may be made by preliminary graduation to indicate, in terms of mercury-column, the value of the pressure to be measured.

The instrument M is the manomètre métallique inscripteur of Marey. An elastic metallic capsule filled with liquid, which is continuous with that of the cylinder AB, plays in this instrument a part which, in principle, is exactly the same as that of the elastic cap in the sphygmoscope S.

Measurement of Variable Pressure.—If the pressure in the cylinder AB of Fig. 105 be variable—as, for example, if the piston A oscillate—the various manometers represented in the figure will give oscillating readings. The manometers at B or O and the piezometer at C are subject in action to the defect that, when a single momentary increase of pressure produces a rise of the liquid or of the mercury in the column, the column does not return promptly to its mean position when the additional pressure is taken off, but oscillates like a pendulum for a period of time more or less protracted, until at length friction and viscosity bring it to rest. If the piston A oscillate, its movements are not faithfully reproduced by the oscillations of the mercury manometer, for the latter depend on (1) the weight of the column of liquid lifted at each displacement from the mean position; (2) the variations of internal pressure tending to make the column assume new mean positions; and (3) on friction; and they are the result of the composition of two sets of oscillations, the one due to the variations of pressure in AB (and agreeing with these variations in period, but not in form or amplitude or phase), while the other set, the pendulum-oscillations of the manometer-column (which may even overpower the former if the mass of mercury be great or if the tubes be wide and offer little resistance), are due to the inertia of the mercury, but vanish if the frictional resistance be very great. The oscillations of the mercury may be checked by making one part of the manometer-tube capillary (Marey's manomètre compensateur), or by interposing a stopcock (Setschenow) the orifice of which can be narrowed till all oscillations are cut off, the instrument then recording merely the slow variations of mean pressure.

Fick's instrument F is damped (prevented from oscillating in virtue of its own elasticity) by connecting with the writing levers a disc immersed in glycerine, as shown in the figure: the viscosity of the glycerine causes all secondary oscillations rapidly to die away. The result is that the Federmanometer is very trustworthy as a recorder of the general form of the variations of pressure in AB. The sphygmoscope and the metallic inscriptor, not having much inertia to combat, render accurately the general form of the variations of pressure, especially if in the liquid surrounding the elastic capsules in the latter instrument there be lightly packed a number of bits of sponge to check elastic vibrations of the capsules; but all the different forms of pressure-indicators, with the exception of those shown at O, C, and B, require preliminary graduation before their indications can be held to denote the absolute value of the pressures; and further, this preliminary graduation must be frequently repeated.

3. THE KINETICS OF LIQUID MASSES.

Streams.—When a liquid flows in a stream, its particles do not become separated from one another to any perceptible extent, and the liquid usually preserves its mean density. The liquid moves as a whole and has inertia, as may be seen in a rapid and full stream leaping over a chink into which a slow or meagre stream would be pulled by gravity.

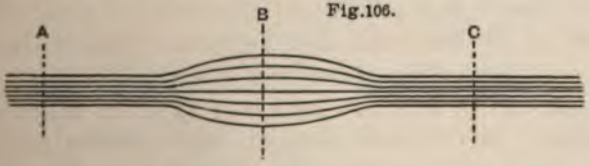
This principle is sometimes made use of in order to prevent an excess of rain-water entering drain-pipes; a sloping gutter has chinks in it, opening into the drainage system: when the gutters become flooded, the water rushes over these chinks, and the comparatively pure water is directed elsewhere than into the sewage, the excessive dilution of which may be considered as a commercial evil.

When once a steady stream-flow has been set up, it can in general be maintained by the maintenance of the supply of liquid and of the propelling force.

Steadiness of flow is favoured in actual cases by viscosity, by a free bounding surface, by converging solid boundaries, by a stream passing round a curve with its greatest velocity externally.—(Osborne Reynolds.)

In any steady stream there may be drawn a series of imaginary lines, which represent the direction of movement of the elements of liquid through which they pass. These lines are called Stream-lines, or Lines of Flow. As long as a stream retains the same breadth and form, these lines may be considered parallel to one another; if the stream widen out they diverge; if it contract they converge. Such lines are shown in Fig. 106, which represents a steady stream of frictionless liquid flowing either from A towards C, or from C towards A.

Fig.106.



Law of Continuity.—If we consider successive sections, equal or unequal, taken across a liquid stream, it is plain that the amount of liquid which crosses each section, during any given interval of time, is equal in each case: otherwise there would be congestion at some part of the stream. In Fig. 106 the amount of liquid which crosses A or C in a second must be equal to that which crosses B: in other words, the Amount of Flow across all sections of a liquid stream is the same. This may be otherwise expressed by saying that at any part of a stream the velocity varies inversely as the area of section at that part: if the stream be broadened out so as to have a tenfold cross-section, its velocity is decreased to one-tenth. The statement of this law is due to Lionardo da Vinci.

Forces producing flow.—If a perfect liquid, exercising no intramolecular friction and no friction on the walls of the canal or tube conveying it, were once set in motion (say in a closed circuit or circular tube), it would go on moving without the continued application of force. The energy actually expended on the liquid in producing its movement would remain in the fluid-mass, the velocity of which would consequently remain unaltered. Such a liquid might be exposed to a severe hydrostatic stress—as, for example, if such a perfect closed stream were contained in a continuous flexible tube exposed to the weight of a mass of liquid in which it was deeply immersed at the one level—and yet the flow would not be affected. A hanging loop of tubing containing a circulating liquid, of which the lower part is exposed to a greater pressure than the upper, will present a turgidity of the lower part of the loop if the tubing be distensible, while if it be rigid there will be no expansion of the stream; in the latter case the flow will not be affected; in the former the expansion of the stream affects the local velocities, and therefore the distribution of the energy of the system, but the mean velocity may remain constant. In the case of a suspended loop of distensible tubing the indirect effect of gravity is thus to diminish the velocity of the lower part and to increase that of the upper part both of the descending and of the ascending parts of the stream; but on the amount of flow it may produce no effect.

Flow, on the one hand, and hydrostatic pressure uniformly applied, on the other hand, are thus seen to be perfectly distinct conceptions, and in a perfect fluid they might be independent of one another; but in every physical fluid viscosity and friction come into play, and flow can only be kept up by maintaining a difference of pressure within the fluid, considered as a whole from end to end. As the flow is kept up, so is it started: a column of liquid in equilibrium may be made to flow by locally increasing the pressure or by locally diminishing it.

If the pressure at a point A be p_A, and that at a point B be p_B, the difference of pressures between these two points is $p_A - p_B$. The difference of pressure per unit of distance is $(p_A - p_B)/\mathrm{AB}$. The force producing the flow depends on this ratio; and the greater this ratio, the greater (but not in direct proportion, see p. 287) is the velocity produced.

Small velocities are associated with small differences of pressure; or, in other words, with relatively great distances between points whose difference of pressure is equal to any predetermined quantity, say a unit of force. When the velocity is great, the points between which the difference of pressure is unity are relatively near to one another. The theory of Flow from this point of view resembles that of Potential: surfaces of equal pressure correspond to equipotential surfaces; Lines of Flow or Stream-Lines correspond to Lines of Force.

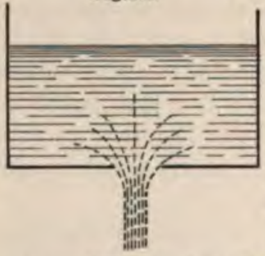
When the liquid is driven through a long uniform tube there is, at the orifice of inflow, a certain initial pressure; at the other, the orifice of outflow, there is no pressure at all. If the liquid be driven by an equal force through a shorter tube, the pressure vanishes in the same way, but does so more rapidly, and—since a greater difference of pressure per unit of length is associated with greater velocity—the velocity is greater than in the longer tube. The shorter the tube the greater the velocity, other things being equal. The shortest tube possible would be a plain aperture in the side of the vessel from which the liquid issues. In this case the liquid at once assumes the greatest velocity which it can acquire under the action of a given pressure.

"Head of Water."—In the case of a vessel containing water which passes out through an aperture, the pressure driving the particles through the orifice is the hydrostatic pressure on that orifice; it is therefore equal (if the area of the orifice be a and the height of the surface of the liquid above the centre of figure of the orifice be H) to $a \,.\, \mathrm{H} \,.\, \rho g$. The height H is known as the Head of the liquid producing the pressure, and the "Head of Water" is a term familiar to hydraulic engineers.

Torricelli's Law.—If v be the constant velocity of outflow of a stream passing out of a vessel under the pressure of a constant head H, $v = \sqrt{2g\mathrm{H}}$. If the aperture be in the sides of the vessel, the liquid issues with velocity v at right angles to the walls of the vessel; its horizontal velocity becomes combined with a new downward fall due to gravity, and the liquid travels in a parabolic path, forming a continuous parabolic Jet. The form of the parabola indicates the proportion between v and g; and thus v is found to differ very little (one per cent) from Torricelli's value, $v = \sqrt{2g\mathrm{H}}$. It is somewhat greater the more convex the wall of the vessel. The amount of outflow per unit of time is not, however, the product of the area of the aperture into the velocity; it is only about $\frac{62}{100}$ of that amount.

The **Vena Contracta.**—The issuing jet may be observed (especially when it is directed upwards) not to be perfectly cylindrical, but to diminish in diameter from the aperture to a spot called the vena contracta, whose position is sometimes somewhat difficult to define. This conical form is due to the fact that the onward flow of liquid is not confined to that part of the fluid which exactly faces the aperture, because the lateral parts of the liquid converge on the orifice; thus the most external stream-lines of the jet, which are at first tangential to the wall of the vessel, assume a direction at right angles to this wall, changing their direction gradually, and therefore presenting a curved form as shown in Fig. 107. From the vena contracta onwards the jet is approximately cylindrical, and presently breaks up into drops, which (especially if any vibration affect the vessel from which the jet issues) are found to be oscillating in form, each becoming alternately a prolate and an oblate spheroid. The unaided eye cannot perceive these separate drops, but recognises the vein as continuous though troubled. When, however, the jet is instantaneously illuminated by the electric spark, and its momentary shadow upon a screen observed, the existence not only of these separate drops, but also of others of a smaller size occupying intermediate positions, may be demonstrated with ease; for the instantaneous impression on the retina persists for the sixth part of a second, and the shadow of the jet appears stationary on the screen. The jet may also be looked at through a Stroboscopic Disc, a rotating disc provided with equidistant narrow apertures. Through each aperture a glimpse is caught of the jet in a certain position. If the rate of rotation of the disc be properly adjusted, each successive glimpse is caught just when each falling drop has had its place taken by its successor; and thus, on the whole, under such a succession of glimpses the jet appears to be stationary.

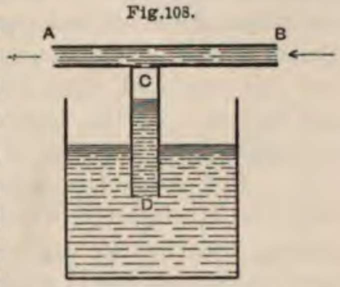
Fig.107.



This phenomenon is one of free fall in the air, for the break-up into drops depends greatly on surface-tension ; a liquid cylinder of excessive length and with a free surface first assumes an undulating contour, and then breaks up into separate vibrating drops, as Plateau has shown. The vibrations of liquids in tubes are therefore not to be explained as phenomena of this kind.

Ajutages.—The amount of outflow from an aperture in the wall of a vessel is greatly influenced by the form of the ajutage or mouthpiece through which the liquid passes. This may be made so as to present the same form as the jet itself, and if it be prolonged just as far as the vena contracta, the amount of outflow becomes equal to the product, area × v × time; not because the outflow is itself altered, but because the area of the orifice of outflow is reduced so as to become equal to (amount of outflow/vt), the terms of this ratio being unaltered. If the ajutage project inwards, the outflow and the velocity are materially diminished. If the ajutage project outwards, being cylindrical, the cylinder, if its walls be wetted by the liquid, is completely filled by it, the jet is cylindrical, and the outflow is greater than when there is no such ajutage. The liquid is drawn towards the sides of the cylinder, and conversely, the sides of the cylinder are drawn towards the liquid. Hence there is no pressure exerted on the walls of the tubular ajutage; on the contrary, there is suction, and if any part of the walls of the tube be mobile, it will be drawn into the stream.

Fig.108.



Lateral diminution of pressure.—If through a tube of the form shown in Fig. 108 there pass a current of liquid in the branch AB under a pressure which is barely sufficient to keep up a stream filling the tube, the mutual attraction of the walls of AB and the liquid will put the liquid in AB in a state of tension and diminish the pressure in AB. In the side tube CD a certain column of liquid can be supported in consequence of the diminished pressure in AB. If this rise to the point C, the upper layers of the column DC will be constantly carried off by the stream BA, and thus a stream is set up in the direction DC. If the pressure in the main pipe AB be too great, liquid will be driven down CD.

The former action is by some considered as explaining the flow of lymph up the thoracic duct.

The same kind of suction-effect may be perceived in the older forms of washhand-basins connected with a house drain-pipe by a simple bent tube or trap ; a downrush of liquid along the main pipe produces a deficiency of pressure, which allows the atmospheric pressure communicated through the basin to drive the liquid which seals the trap into the drain pipe, and thus to leave a channel patent to the entry of sewer gas.

The peculiarly beautiful forms presented by jets under various circumstances are described and figured by Savart in the *Annales de Chimie et de Physique*, vols. 54 and 55.

If two vessels having an aperture in each of the same size, shape, and at the same level, be so arranged that these apertures are exactly opposite one another and close together: if liquid be poured into the one vessel, it will run into the other. The vessels may then be removed to a certain distance from one another, and the liquid will continue to pass from the one vessel into the other, through a tube formed of its own superficial film, until the same level is nearly attained; then the liquid begins to flow out of both vessels, and the two jets, meeting, spread out into a sheet which is driven back and fore between the two orifices as the liquid in the one or the other vessel stands for the moment at the higher level.

Energy of Jet.—If Torricelli's law held perfectly good, that $v = \sqrt{2g\text{H}}$, the velocity would be the same as if every particle had fallen from the surface of the liquid to the orifice, and had passed out of the orifice with a velocity due to its fall through the height H.

The outflowing jet would thus convey with it kinetic energy equal to $v^2/2$, or to $g\text{H}$, per unit of mass.

This would be absolutely the case were it not for friction and viscosity. If the level of liquid be maintained constant by a continued supply, the velocity is constant. At the instant when the whole of the original liquid has passed out through the orifice, the experiment may be stopped. The liquid which has passed out has conveyed with it energy $= \frac{1}{2}mv^2 = mg\text{H}$ if Torricelli's law be true. At the commencement of the experiment it had potential energy (mass m at an average height of $\frac{1}{2}\text{H}$) of $\frac{1}{2}mg\text{H}$ only. It has therefore gained energy $= \frac{1}{2}mg\text{H}$. This energy has been lost by the water which has replaced it, and sunk from the surface to an average depth of $\frac{1}{2}\text{H}$ below the surface, thus losing potential energy $\frac{1}{2}mg\text{H}$ without any compensating gain of energy in any other form.

The same principle is illustrated in the following experiment. One cork of a Woulff's bottle completely filled with water is fitted with a piece of glass tube drawn out so as to form a jet; the other cork admits a tube leading from a vessel containing mercury; the mercury is caused to fall into the bottle. Some of the water which already fills the bottle is driven out with great velocity in a thin stream. The mercury sinking through the water loses energy proportional to its density ($mgH = v\rho gH$); the water forced out acquires this energy, and hence has a great velocity imparted to it.

Velocity of Outflow.—Torricelli's law shows that the velocity with which a liquid issues through an aperture varies as the square root of the head of water or the pressure; or that the head of water necessary to produce a certain velocity in a free stream of liquid, subject to no resistances, is proportional to the square of the velocity.

Any pressure exerted on a liquid may be measured as Head of the same liquid or of water.

Example.—The pressure at the bottom of a column of water 1033·3 cm. deep is equal, per sq. cm., to the weight of 1033·3 grammes. The atmospheric pressure on the surface of a liquid is equal to the same. Hence the atmospheric pressure is equal to that of a head of water of 1033·3 cm. Water at a depth of 1033·3 cm. under a water-surface exposed to the atmosphere is exposed to as much pressure as if it lay at a depth of 2066·6 cm. of water under a free surface not exposed to the atmosphere. In a vessel filled with water, 1033·3 cm. deep, provided with an aperture in its lower surface, this aperture communicating with a vacuum, and the upper surface of the liquid communicating with the atmosphere, the velocity of outflow will, according to Torricelli's law, be $\sqrt{2 \times 981 \times 2066{\cdot}6} = 2013{\cdot}6$ cm. per sec., while, if the lower aperture were also in communication with the atmosphere, the effective head of water would be 1033·3 cm. and $v = \sqrt{2 \times 981 \times 1033{\cdot}3} = 1423{\cdot}8$ cm. per sec.

The pressure produced by compression, as in pressing home a syringe, the negative pressure produced by rarefaction, as in pulling up the handle of a syringe, may all be measured in the same way. In general, when a fluid is acted upon by a number of pressures corresponding to the respective heads of the same fluid H, $H_{,}$, $H_{,,}$, etc., the velocity of outflow of the fluid through an orifice is $v = \sqrt{2g\,(H + H_{,} + H_{,,} + \text{etc.})}$

The speed of outflow, $\sqrt{2gH}$, does not depend upon ρ, the density; all liquids—ether and mercury—issue with equal velocities under the action of equal heads of their own substance.

Recoil.—The law of action and reaction perfectly applies to liquid jets and to the vessels from which they issue. The Hydraulic Tourniquet is an example: a cistern containing water and capable of rotating on an axis: pipes ending obliquely issue from its sides: water runs out of these pipes: and by reaction they are driven backwards. Since they are not fitted to an immovable cistern, but to one free to rotate, the whole rotates, and thus the contrivance may be used to convey water-power, the water constantly running into the rotating cistern, and running out of the obliquely-set exit pipes.

Resistances.—When a fluid stream passes through a tube or a channel it experiences different retarding resistances, which convert energy of motion into heat, and of which the following are the chief:—Surface Adhesion, Surface Friction, Inequalities of the Surface of the bounding solid, Eddies, and Fluid Viscosity.

Surface Adhesion.—If a liquid wet the walls of the tube or channel through which it passes, the layer of liquid which is in contact with the walls does not change except by molecular diffusion and exchange. It remains *in situ* while the liquid flows past; in other words, there is infinite friction between this layer and the walls wetted by it. This gives great scope to the viscosity of the fluid, though it itself, when once the flow has been set up, does not directly cause any waste of energy. While the walls are being wetted there is a slight liberation of heat, due to the satisfaction of the mutual molecular attractions between the liquid and the walls.

Surface Friction.—If the liquid do not wet the tube through which it passes, the surface of the moving liquid and the walls of the vessel rub against one another, and energy is lost in overcoming this friction. Loss of kinetic energy is also caused by roughnesses on the walls of the tubes or channels, which give rise to little eddies or whirlpools.

Eddies are produced when a moving fluid is subjected to unsymmetrical retardations. The cases in which eddies, whirlpools, vortex-rings, rolling and tumbling water, and the like, are produced are extremely numerous. Water flowing in a tube which suddenly widens or suddenly narrows generally presents such eddies at the point of sudden enlargement or contraction.

The production of eddies is favoured by mobility of the liquid, by variations of velocity at different parts of the cross-section of the stream, by rigid bounding walls, by diverging boundaries, by curvature with the greatest velocity internally.—(Osborne Reynolds.)

Viscosity.—When a disc or cylinder suspended in a fluid is caused, by twisting the supporting wire or wires, to enter into oscillations, it is found that the oscillations soon die away; though they continue isochronous, their amplitude diminishes; and the amplitudes of any two oscillations stand to one another in a constant proportion. If the disc or cylinder be wetted by the fluid, the layer immediately in contact with the solid remains in contact with it; this film, moving with the solid, sets in motion the film next in contact with it, and that in its turn sets the next in motion. Each film goes through a displacement somewhat less extensive and more retarded than the one gone through by the film which sets it in motion. Continuous rotation of the disc or cylinder would in time cause the whole fluid to rotate; but the influence of an oscillating disc travels a very short distance, for half an inch away from the disc the fluid remains undisturbed. Within this small distance the liquid performs oscillations which in period resemble those of the oscillating disc, but which in amplitude are less, and in phase more retarded, the greater the distance from the disc. This lagging behind on the part of the liquid has the effect of dragging on the disc and of gradually bringing it to rest.

If the disc be wetted the retardation is independent of the nature of the material of the disc, for there is no velocity lost by friction between the solid and the liquid. If the disc be not wetted, there is distinct friction (external friction) in addition to the viscosity (internal friction).

The Coefficient of Viscosity serves as the means of measuring the viscosity of a substance. We have already seen that it is equal numerically to the force which is necessary to maintain a flow of one layer of one unit-area past another of the same area with a relative velocity of one unit, the distance between the layers being unity, and the space between them continuously filled with the viscous substance.

If F be the force required to keep up the flow of two layers past each other, their area being each a, their respective distances from a plane of reference being $r_{,}$ and $r_{,,}$ and their distance from each other therefore $r_{,} - r_{,,}$; if their respective velocities be $v_{,}$ and $v_{,,}$, and their relative velocity $v_{,} - v_{,,}$; and if the coefficient of viscosity be η, $F = \eta . a\,(v_{,} - v_{,,}) \,/\overline{r_{,} - r_{,,}}$.

In the case of water at 0°·6 C. this coefficient is 0·0173, at 45° C., 0·05833, at 90° C., 0·00339 (Meyer), while that of air,* which obeys the same laws, is ·00017 $(1 + 0{\cdot}00733\,\theta)$, where θ is the C. temperature; all expressed in C.G.S. units.

Though the density of air is $\frac{1}{770}$th that of water, its viscosity is as much as $\frac{1}{100}$th that of water. For brass, η is about 300,000,000,000. Moist air is more viscous than dry air: hot air is more viscous than cold air.

Hot water is less viscous than cold. Most saline solutions are more viscous than water, saltpetre solution being an exception. Most saline solutions are more viscous the more concentrated they are, saltpetre solution being again an exception—(Meyer).

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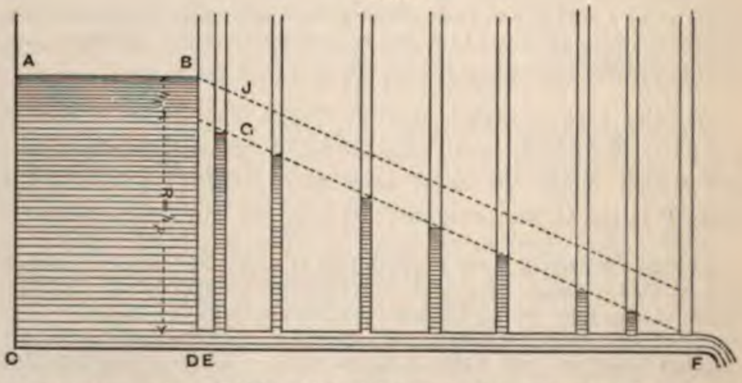
* O. E. Meyer, *Pogg. Ann.*, vol. cxlviii., 1873.

The experimental determination of the coefficient η by means of observations made with the aid of an oscillating disc involves much mathematical computation, and it is often quite sufficient to record the so-called **Logarithmic Decrement** or *log. dec.* special to each liquid. Let us suppose that the oscillating disc or cylinder first turns through the angle ω; that at the next oscillation its deviation from its mean position is $\frac{99}{100}\,\omega$; that at the third it is $\frac{99}{100} \times \frac{99}{100} \times \omega$; and so forth. Then each successive angle is equal to the one immediately preceding multiplied by $\frac{99}{100}$; its log. is equal to the log. of the preceding angle of oscillation *plus* that of $\frac{99}{100}$, or *minus* the log. of $\frac{100}{99}$; that is, *minus* ·0043648. Such a constant difference in the logarithms of the successive angles of oscillation is the *log. dec.* for the particular substance whose viscosity it measures. Under Poiseuille's law (p. 292) we shall find a simple method of measuring the value of η.

Effect of Viscosity on a stream of liquid.—The external layer is at rest. The axial parts of the stream are less influenced by viscosity. The velocity of the axial part of the stream is greater than that of the peripheral; the fall of pressure is therefore greatest in the centre of the current. The fall of pressure being greatest in the centre, the external parts of the stream tend to move into the centre, and to have their velocity accelerated. In capillary tubes the axial stream travels with a greater speed than the average as determined by Poiseuille's Law, to be presently stated.

Constant flow through uniform rigid pipes.—The pressure which is necessary to keep up a continuous flow of water

Fig.109.



in a uniform pipe, EF in Fig. 109, may be produced by a total head H of water in a vessel (a pressure-vessel, ABCD in the figure), this height H being maintained constant. The water is observed to issue from F, with a constant velocity v; this velocity would (if there had been no resistances) have corresponded to a head $h_v = v^2/2g$; this may be considered (so far as the velocity and the kinetic energy of the outflowing stream at F are concerned) to be the effective head of water at F, the orifice of exit: it may be called the **"velocity-head."** This velocity-head, GJ, is equal in all parts of the tube.

The hydrostatical pressure in the immediate neighbourhood of F is necessarily null; that at E, just within the pipe EF, is less than the pressure ($= a\mathrm{H}\rho g$ per unit of area) corresponding to H, the original head of water; it corresponds to a head h_p (the **pressure-head**), which differs from H in the first place in consequence of a certain slight waste of head caused by the formation of eddies between D and E, and in the second place differs from H by the amount of the velocity-head itself. If we neglect the effect of these eddies we may say that the velocity-head and the pressure-head are together equal to the total head: $h_v + h_p = \mathrm{H}$.

The hydrostatic pressure in the tube (if the tube be uniform) dies away uniformly, as is shown by the level assumed by the water in the successive piezometer-tubes of Fig. 109.

If the tube were lengthened there would be a similar—but necessarily a slower—dying away of the pressure; the velocity would be less throughout the tube; the velocity-head being less, the pressure-head would be greater: there would therefore be a greater pressure at E.

The hydrostatic pressure at any part of a stream measures the resistance which has yet to be overcome. If there were no resistance (as in the imaginary case of a perfect liquid) there would be no lateral pressure, no pressure-head; and the whole of the original total head H would be taken up in producing a velocity $v = \sqrt{2g\mathrm{H}}$.

The greater the velocity of a stream, the greater the resistance encountered by it within a tube of given dimensions. The resistance at any point thus depends not only on the dimensions of the tube between that point and the orifice of outflow but also on the velocity of the stream.

The relation is $\mathrm{R} = l\,(a\,.\,v^2/d + b\,.\,v/d^2)$ (Haagen); R being the measure of the resistance, l and d the length and diameter of the tube yet to be traversed by the stream, a and b constants to be found by experiment.

R is not a number of units of force, but it is the height (in cm.) of a lateral column of water whose weight can be supported by the stream-resistance.

Its weight is Rg dynes, and the local resistance at any point of the stream is therefore Rg dynes of force per sq. cm. of transverse section of the uniform stream passing that point.

Given that the tube has a certain length l, and diameter d, and a certain constant driving head of water H, the velocity v must so adjust itself that the three equations

$$\mathrm{H} = h_p + h_v \qquad (1.)$$

$$h_v = \frac{v^2}{2g} \qquad (2.)$$

$$\mathrm{R} = h_p = l\left(a\frac{v^2}{d} + b\frac{v}{d^2}\right) \qquad (3.)$$

shall all hold good. If the tube be exceedingly long, the resistance becomes proportionally very great and the velocity very small: yet in a tube of any assignable length there would be a constant velocity, and the pressure would uniformly (though slowly) diminish from one end of the tube to the other. The **pressure-line,** GF of Fig. 109, would in such a case—a case of low velocity—have a gentle slope. When the tube is very short, the resistance is initially small and rapidly falls; thus great velocity is associated with steep slope of the pressure-line.

If the driving pressure increase or diminish (the dimensions of the tube remaining unchanged), the velocity produced by it and the resistance brought into play both increase or both diminish. If the dimensions of the tube be altered while the driving pressure remains unchanged, the resistance and the velocity will vary in contrary senses: increased resistance, diminished velocity; diminished resistance, increased velocity. If the resistance be increased by increasing the length or lessening the diameter of the tube, the velocity and the amount of flow cannot remain constant unless the driving pressure be also increased. (Hypertrophy of the heart when the placental is added to the ordinary circulation.)

If the hydrostatic pressure be found to have increased (higher columns being supported in the piezometers), the plain inference is either that the driving pressure has been increased, or else that the peripheral resistance has been increased by narrowing or lengthening or perhaps by roughening the tube. If the pressure be found to have been diminished, either the driving power or the resistance, or both these, must have been also diminished.

If more than one of these elements vary, the result may be either accumulation or compensation of effects. Higher head or narrowed tubes both increase the pressure; with lowered driving pressure on the one hand and narrowed or lengthened tubes on the other, the pressure may remain the same, though, in this case, the velocity is diminished. Hence it is necessary to observe both the pressure and the velocity in order to investigate the local condition of any stream.

Flow due to variable pressure in uniform rigid tubes.—If the driving pressure be reduced, the pressure-head becomes a greater, and the velocity-head a less, fraction of the reduced total head; the velocity-head is thus lessened not only in proportion to the diminution of driving pressure, but in a still greater ratio. Conversely, if the driving pressure be increased, the velocity-head is increased in a greater ratio. Since the velocity is proportional to the square root of the velocity-head, it is not the velocity but the square of the velocity which is a little more than doubled by doubling the driving force. Hence a curve indicating the variations of velocity agrees in general form, but not in its amplitudes, with a curve indicating the variations of driving pressure.

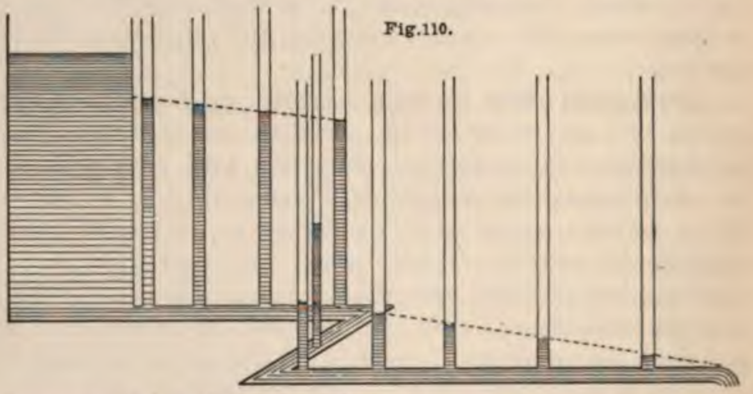
Interrupted flow through uniform rigid pipes.—*a.* The driving force may be applied intermittently, and may cease during the intervals. A perfect incompressible fluid, treated in this way, would move like a solid rod struck endwise by a hammer: all its particles would move simultaneously, and liquid would pass through the orifice of exit without any interval of time. A physical liquid is hurled upon itself, compresses itself, and resiles. Thus the suddenness of outflow at the orifice of exit is somewhat modified; but even with physical liquids the more rigid the tube the more abrupt is the onflow. (Atheromatous arteries.)

b. The pressure being continuous, the flow may be suddenly stopped by an obstruction, say by a stopcock suddenly closed. Beyond the stopcock the liquid runs on somewhat and rarefies itself, or even produces a vacuum near the stopcock; it returns and oscillates until it comes to rest. Between the driving pressure and the stopcock there is a sudden increase of pressure. If a house water-tap be suddenly turned off when water is running from it, a jar or jolt may be given to the water in the pipes which may be audibly perceived throughout a large building. This jolt is due to the sudden stoppage of the water, which has already acquired momentum. The water compresses itself, rebounds and oscillates, producing waves of condensation and rarefaction which travel back into the mains. The pressure in the pipes is greatly increased by this mode of treatment; an original pressure of 30 lbs. per sq. in. may be raised to one of 120 or 130 lbs.

This principle is economised in the Hydraulic Ram. A stream of water is alternately cut off and allowed to flow: every cut-off enables the stream, whose pressure is thereby greatly increased, to force a valve which it could not otherwise force, and water is thus driven into a small chamber containing a limited volume of air. This air is compressed, and its elasticity enables it to force the water out through a narrow jet, at a pressure nearly equal to the greatest pressure experienced by the liquid during the cut-off.

Flow through bent tubes.—Bends increase the resistance and diminish the proportionate velocity. The forward momentum of the liquid is destroyed, the reaction of the walls is called into play, and by the elasticity of the liquid and of the walls a new path is given to the liquid. Energy is consumed in this process, particularly in producing eddies in the stream, and the piezometer-tubes show that the pressure in the water, which is about to meet the obstacle, is much greater than in that which has just left it (Fig. 110). If the driving pressure be applied intermittently, the liquid between the driving apparatus and the rigid bend may be sharply compressed before it can pass round the bend; it is driven against the bend like a solid, and if the bend be at all extensible it is driven forward. (Locomotive pulse.)

Fig.110.



Flow in tubes not of uniform diameter.—We are apt to think that when a fluid passes from a wide into a narrow tube the pressure is increased; and conversely, when a fluid runs out of a narrow into a wide tube, that it is relieved of pressure. The reverse is the case. To understand this we must consider the flow as already set up and constant. The law of continuity shows that when a rapid stream passes into a wide channel, it travels more slowly. The velocity-head suffers a diminution, and the pressure-head increases: the kinetic energy possessed by the rapidly entering narrow stream is partly spent in dashing that stream against the comparatively stationary layers in the wider channel, and is thus partly converted into potential energy. A certain degree of compression is thus produced, and a corresponding pressure, which is additional to the hydrostatic pressure already existing. Conversely, when a stream narrows it runs more rapidly: its kinetic energy becomes greater (mass for mass), and there is a tendency to stretch or rarefy the narrowed and accelerated stream. This tendency to rarefaction, or even to tearing asunder the stream, corresponds to a defect of pressure in the narrower tube.

In the case of a liquid passing from a narrower channel into a wider, we have a flow from a place of lower pressure into one of higher. This apparently anomalous flow is explained by the fact that the pressure, even in the wider channel, can never (on account of the gradual disappearance of pressure-head in the production of heat) exceed, but must always be less than, that due to the original pressure-head, h_p.

Flow in branched rigid tubes.—If the total cross-sectional area of the branches do not exceed that of the main tube from which they spring, the parietal surface-area of the stream is increased; this increases the resistances, and the velocity falls. If the total cross-sectional area do exceed that of the main tube, the channel is widened and the resistances are relatively diminished: they may even be diminished by this cause more than they are increased by the increase of the total surface. The resistances are, on the whole, absolutely diminished in this case, and the velocity of the whole system may be absolutely greater than that in an unbranched tube of corresponding length.

If we compare two branched systems: the one large, containing many branches, each of which would, if the stream were driven through it alone, offer much resistance, but all together affording the stream a wide bed for its flow; the other system small, containing few branches, of which each is capable of offering only a small resistance, but which, by their small number, cause the stream to flow in a narrow bed; it is possible that the driving pressure necessary to produce a given velocity may, in these two cases, be the same. The advantages of the aggregate wide channel in the first system are neutralised by the great resistances; the advantages of the small resistances in the second system are counteracted by the narrowness of the channel.

Thus both small and large animals have approximately the same blood pressure in the aorta.

Where branches are given off, the pressure either increases or begins to fall off less rapidly, because the velocity diminishes; where the branches reunite, the pressure rapidly falls off. If the whole system be quite symmetrical, the pressure in the middle of the system is greater than half the initial pressure.

The pressure in the capillaries is more than half the initial pressure in the aorta, though their joint sectional area is very great and their resistance accordingly very small; and the pressure very rapidly falls as the veins unite. At the same time the velocity increases as the sectional area diminishes, and the amount of flow into the auricles is equal to that from the ventricles.

When, in a system of branched tubes, some of the branches are relatively shorter or wider, the amount of flow through these is to some extent relatively more rapid.

When in such a system the flow is once fairly established, if the branches as a whole become narrowed, the resistance is increased and the velocity falls. If some only be narrowed, while the driving pressure remains the same, the velocity in the remaining branches may be increased, for the channel is narrowed, being wholly or partly restricted to the latter. The pressure in those which are narrowed is increased; but the pressure in the unnarrowed branches may also be increased, for the peripheral resistance of the system as a whole is rendered greater, and the velocity-head is diminished. The effect on the unnarrowed vessels may thus be the same as if the driving power had been increased.

Flow through capillary tubes.—Poiseuille found that the law regulating the velocity of the flow of fluids through tubes is materially altered when the diameter is very small. Through capillary tubes he found that the volume of fluid flowing in unit of time is $V = k \,.\, r^4 H/l$, where V is that volume, r and l the radius and length of the tube, H the head of fluid, and k a constant to be determined by experiment. This determination is effected by actually observing the number of seconds taken to drive a given volume of liquid through a capillary tube of given length and diameter. The constant k does not depend on the nature of the walls of the tube, if the walls be wetted by the liquid; it depends only on the nature of the liquid and on the temperature. Water near the boiling point passes five times as rapidly through capillary tubes as water near its freezing point. We can do no more here than assert with a reference * that theory indicates that (on the assumption that the layer of liquid in contact with the solid wall is at rest, an assumption verified by the fact that it is immaterial what is the nature of the substance of the tube, provided that it is wetted by the liquid) the coefficient $k = \pi\rho g/8\eta$: whence it is easy to find the value of η, the coefficient of viscosity, for any liquid at any given temperature.

Since $k = \pi\rho g/8\eta$, the volume of liquid passing through a capillary tube in time t is $\pi\rho g . r^4 \mathrm{H} . t/8\eta l$ or $\pi r^4 . t . \mathrm{P}/8\eta l$, where P is the pressure, in dynes per sq. cm., upon the surface of the liquid as it is delivered into the capillary tube; the pressure at the other end of the tube being *nil.*

The Velocity v = Volume of fluid flowing across any section in unit of time ÷ Area of that section. Hence, in a capillary tube, $v = k . r^4 . \mathrm{H}/l \div \pi r^2 = k . r^2 . \mathrm{H}/\pi l = \mathrm{P}r^2/8\eta l$.

The flow in capillary tubes is proportional not to the square, but to the fourth power of the radius; the velocity is proportional not to the square root of the pressure, but to the pressure itself.

The resistance in capillary tubes varies directly as the velocity; in wide tubes approximately as the square of the velocity. This seems discrepant; it is due to the formation of eddies in the wider tubes; in a capillary tube the flow is steady.

But what is a "capillary" tube? For water it is a tube under 1/50-inch in diameter. Would it be the same for treacle? No; a long inch-wide tube behaves with treacle as a 1/50-inch tube does with water; the flow of treacle through it obeys Poiseuille's Law. Professor Osborne Reynolds has made the very important discovery that steadiness of flow and obedience to Poiseuille's law cease only when the expression (Velocity of stream × Width of stream ÷ Viscosity of fluid) has reached a certain critical value. Too great a velocity, too wide a stream—in either case the stream breaks up into eddies and the movement is like that of water in a wide tube; but even in a wide tube—not too wide—the effect of great viscosity may keep the above expression down below its critical value and the flow may be steady like that of water in a capillary tube. If the stream be wide enough the above expression will be above its critical value; a Mississippi of treacle would flow turbulently round any obstruction;

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* Helmholtz and Piotrowski, *Sitzungsberichte d. Wien. Acad. Math. naturw. Cl.* XL. 1860; and O. E. Meyer, *Wiedemann's Ann. d. Physik u. d. Chemie*, 1877, vol. ii., and literature there cited.

lava-streams flow like water. One and the same tube may be made, by increasing or diminishing the velocity of flow through it, to play the part of a wide tube with eddies or of a narrow tube with steady flow.

Measurement of the pressure at any point of a stream.—This cannot be effected by cutting the tube and fixing a manometer in it. The result in that case would be a determination of the original driving pressure or head of liquid, for the condition becomes statical and the liquid seeks its level. The manometer must be fixed laterally and at right angles, and the flow must be allowed to proceed without any hindrance. This being seen to, any one of the forms of manometer already described may be used.

Measurement of the velocity of a stream.—It is needless to point out that the velocity cannot be calculated from a single observation of the pressure at any point.

The velocity may be observed by direct or by indirect methods with more or less accuracy.

A. Direct Methods.—*a. Optical.*—The velocity of bodies floating in the stream may be measured by observing the distance traversed by one of them in a given time.

Objection to this method.—The velocity of floating bodies does not necessarily represent the velocity of the stream.*

A body of the same sp. density as the liquid moves in the axial stream: the larger it is, the more it is delayed by the peripheral layers and the more slowly it moves. It moves without rolling, unless it gets into the peripherical layers and is twisted out of them into the centre of the stream.

A body heavier or lighter than the liquid is pressed against the upper or lower wall. It rolls in the peripherical layers, for it is urged forward by unsymmetrical forces; and it is retarded. Within certain limits, the larger it is the less it is retarded; but it always travels more slowly than a body of the same density as the liquid.

Of two heavy bodies the heavier moves more slowly; of two light bodies the lighter moves more slowly; an effect due to rolling friction.

A disc which rolls travels at the same rate as a sphere of the same density and radius; if it travel in the axial stream the velocity is the same as that of a sphere or cylinder of the same radius. If a heavy or light disc lie flat against the wall, the friction is increased and the speed diminished.

Bodies nearly filling a tube approximate more nearly in speed than when they are small in relation to the tube; if heavy they tend to check the stream and to permit the pressure to accumulate behind them: if light they tend to roll rapidly, and thus to diminish the pressure behind them.

If the density of the liquid be diminished, the bodies (*e.g.* red corpuscles), which had been just light enough to float in the unaltered liquid, sink and roll in the lower part of the stream. If it be increased, all float. In either case, or if the sp. gr. of the floating bodies be altered, these bodies block the bends of the tube and check or slow the stream (albuminuria, cholera, pernicious anæmia, fatty embolism, injection of milk into the veins). When the stream is slowed, if the particles be viscid they adhere to the sides of the tube and the stream flows past them: after a while they may be torn off and proceed.

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* For the facts mentioned in these paragraphs I am indebted to the kindness of Prof. Hamilton of Aberdeen, who rejects Schklarewsky's assertion that the same granular substance may float in the axial or in the peripheral stream, according to the *nature* of the other granules with which it is associated in the stream.

b. Chemical.—This is a method principally employed in physiological work and suggested by Hering. A soluble chemical substance easily recognised is introduced into the stream: at a certain distance the stream is tapped, and samples of the liquid are there collected at regular intervals of time. The interval of time which elapses before the chemical substance can be detected in the liquid at a given distance affords a datum from which the mean velocity can be roughly calculated.

c. Volumetric.—1. The vessel may be cut and the amount of outflow measured. This is objectionable (1) because the resistance is diminished and the velocity increased, and (2) if the stream be a closed circuit, opening into it causes loss of liquid and a modification of the driving pressure.

2. A tube filled with liquid may be placed in the course of the stream. The liquid in the tube is driven into the stream, liquid from the stream taking its place. The time taken to empty the tube is observed. The objections are (1) resistance interposed, and (2) difficulty of recognising the exact instant at which the liquid is wholly replaced.

3. In Ludwig and Dogiel's Stromuhr, used by physiologists, there are two large cavities; the one nearer the heart is filled with oil; the cavity nearer the periphery is filled with defibrinated blood (the introduction of which into the animal's circulation does little harm). When the stream flows, the oil passes from the proximal chamber into the peripheral one: the defibrinated blood of the peripheral chamber passes into the animal: the proximal cavity of the stromuhr becomes filled with the fresh blood of the animal. Then by a play of stopcocks (effected just when the oil is on the point of escaping into the vessels of the animal) the stream in the instrument is reversed, and the animal's blood flows into the chamber now occupied by the oil, the oil passing back into the chamber which it had originally occupied, and the blood which had freshly entered that chamber being returned into the circulation. The oil-chamber is always functionally in the rear. This may be repeated several times, and thus the amount of time taken to pass a certain large volume of blood through the instrument may be ascertained. The animal suffers on the whole no loss of blood, and there is no material increase (2 to 5 mm.) of resistance in the circuit of fluid; while if the stream be periodically reversed with attention and promptitude, relatively great accuracy is attainable in the determination of the mean velocity.

B. Indirect Methods.—The mechanical effects of a stream of liquid are derived from its forward momentum.

a. "Hydrostatic Pendulum."—If some object be attached by its upper end to the walls of the tube and swing freely in the stream, the quicker the flow the more will the lower free end be displaced. A box containing such a contrivance (the so-called hydrostatic pendulum of engineers) may be inserted (as in Vierordt's Haemotachymètre) in the course of a stream; or the pendulum itself may consist simply of a needle thrust through the elastic walls of the tube or (as in Chauveau's Haemodromomètre) through elastic parts of the wall of the tube. In the latter case, as the immersed end of the needle is driven by the stream in one direction, the external end moves in the opposite direction, and the elastic walls of the tube exercise a constant pressure upon it, tending to adjust it in its normal position at right angles to the wall of the axis, the whole arrangement being very sensitive to variations of velocity.

Chauveau's instrument has been so modified that at one and the same part of the circulation the pressure may be found by a sphygmoscope, and the velocity ascertained by a hæmodromometer which, being coupled with self-registering apparatus, has acquired the name of hæmodromograph. The condition of a stream cannot be thoroughly investigated unless the pressure and the velocity are both ascertained.

b. Pitot's Tubes.—If a piezometer-tube be prolonged into the axis of a stream, and if it be bent at the submerged end so that its lower orifice faces the stream directly, the liquid will be forced up in it to a certain height greater than that which corresponds to the lateral pressure at the same part of the tube, and varying with the velocity. If its lower orifice be turned away from the stream, the column of liquid is lower than it would have been in a plain piezometer. If two such lateral tubes be fixed near to one another in the walls of a main tube, the mouth of one facing, that of the other turned away from the stream, there will be set up a difference in the heights of the columns in these tubes; this difference depends entirely on the velocity and varies with it. This principle has been applied by Marey (*Trav. du Lab.*, 1875, p. 347) in the formation of a registering instrument of great excellence, but in physiological work unfortunately not suitable, because coagulation is promoted by the projection of the lateral tubes into the lumen of the main tubes, the vessels through which the blood passes.

All instruments in which indirect methods are applied must be graduated by exposing them to the action of streams of various known velocities, and marking the corresponding positions at which the index stands.

Work done in keeping up a stream.—The initial head H would (if no energy were wasted in overcoming resistances, etc.) produce a velocity $v = \sqrt{2g\mathrm{H}}$, and the kinetic energy imparted to a mass m of fluid would be $\frac{1}{2}mv^2 = mg\mathrm{H}$. This is the whole energy lost by the water falling out of the cistern, and this is independent of the amount of the resistances. The work done in keeping up a stream is therefore independent of the length of the pipes; the pressure at any point is not so. If we know m and H it is easy to calculate the total work done by the driving apparatus: if we do not know H, but do know h_p (the height of the maximum piezometer column), the equation

$$\text{Work done } (= mg\mathrm{H}) = mgh_p + \tfrac{1}{2}mv^2$$

enables us to find it when we know the velocity of outflow.

This equation is arrived at by combining the two equations, $\mathrm{H} = h_p + h$ and $h_v = v^2/2g$.

If the left ventricle of the human heart propel at each systole 180 grammes of blood at a mean pressure equal to that of 12·8 cm. of mercury (that is, the sp. gr. of blood being 1·055, at a head H of 164 cm. of the same liquid, blood), during each systole the left ventricle does work equal to $mgH = 180 \times 981 \times 164 = 28{,}959{,}120$ Ergs = weight of 29,520 grammes raised 1 cm. or ·2952 kilos lifted through 1 metre.*

If the liquid leave the tubes with actual velocity V, its kinetic energy is then only $\frac{1}{2}mV^2$. The difference $(mgH - \frac{1}{2}mV^2)$, having been spent in overcoming resistances, has been transformed into heat.

Streams in elastic tubes.—If the inflow be continuous the internal pressure expands the tube, and continues to do so until the tube exerts a contractile restitution-pressure equal to the expansile pressure of the liquid. Then the stream flows on as in a rigid tube.

If the inflow be intermittent the case is different. We may first suppose the liquid to be injected instantaneously, and the tube to expand as a whole. In such a case, a sudden inflow creates a pressure which expands the walls of the tube, in addition to forcing onwards a certain quantity of the liquid. Work is thus done upon the walls of the vessel. These, being elastic, tend to restore the work done upon them. When the pressure due to the inflow is relieved, the primitive form of the tube is gradually resumed: the potential energy of the stretched walls is transferred to the liquid in the form of kinetic energy. The stream is thus kept up until the original form of the tube is restored.

If there be a quick succession of influxes, each successive inflowing quantity may enter the elastic tube before its predecessor has left it. If the frequency of inflow be sufficiently great, the outflow may be uninterrupted though variable in velocity and amount. The rate of succession necessary for continuous outflow depends on the width of the tube—being greater as this is greater—and also, in the reverse sense, on the extensibility of the walls of the tube and on the mechanical resistance offered to the outflow. The greater the resistances, or the greater the extensibility of the tube, the greater will be the proportionate size of the dilatation or pouch of the elastic tube, and the more continuous will be the outflow: the more deliberate, therefore, may be the succession of influxes necessary to keep up a continuous outflow.

Primary waves in elastic tubes.—The tube does not in fact dilate and contract as a whole, nor does the liquid at each inflow enter instantaneously. The pouching is local, and the inflow more or less gradual. The more gradual the inflow, the less the width and the greater the length of the pouch produced: the more abrupt the inflow, the wider and shorter will the pouch be. The pouch contracts and drives the liquid onwards: this action dilates the walls of the tube beyond the pouch: the dilatation travels onwards, and liquid travels with it. The contraction of a pouch can never produce another quite equal to itself in width: and so, as the dilatation travels along, it becomes narrower and longer. In this case the direction of movement of the liquid as a whole and that of the dilatation are the same.

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* Nearly 150 foot-pounds per minute, or $\frac{1}{220}$ horse-power on the average; about $\frac{1}{80}$ h.-p. during contraction: its own weight raised about 22,000 feet per hour.

If instead of a sudden inflow of liquid due to pressure there were a sudden outflow due to suction, there would be a local collapse of the walls of the tube. The walls, returning to their original form, will cause a stream to be set up, which travels towards the orifice of suction. The contracted *form* of the tube will travel in a direction opposed to that of this stream.

The dilatation or the contraction of the tube, as it travels, forms a wave, the so-called Pulse-Wave—positive, and travelling in the same direction as the liquid, in the case of an inflow and dilatation; negative, and travelling in the opposite direction, in the case of a suction of the liquid and a contraction of the elastic tube.

The farther down the tube the later the arrival of this pulse-wave. The velocity of propagation of a wave of this kind depends on K, the elasticity-coefficient, and on a, the thickness of the wall: the greater these are the greater is that velocity. It also depends on d, the diameter of the tube, and on ρ, the density of the liquid: the greater these are the less is v, the velocity of propagation, the distance traversed by the wave in one second.

The law is (Moens: *Die Pulscurve*, Leiden, 1878) $v = 0{\cdot}9\sqrt{g\mathrm{K}a/\rho d}$.

The elasticity varies in the case of arteries; the more expanded an artery is the more elastic it is, and therefore a full pulse travels more rapidly than one of small expansion. The length of such a wave = time of inflow × rate of propagation. In the case of the pulse the former is $\frac{1}{3}$ sec., the latter is from 10 to 18 metres per sec. The length of the dilatation in the arteries would be 3·33 to 6 metres if the arterial system were long enough to contain a whole wave. The arterial system is never during life wholly relieved from pressure, and is in a state of permanent though variable distension. The elasticity and self-contraction ("arterial tension") of the arteries are opposed to the expansile internal blood-pressure, and at each instant these are either equal to it or are in the act of coming into equilibrium with it.

The form of a simple pulse-wave.—The more abrupt the disturbance, the steeper will be the front of the resulting pulse-wave, and the more abrupt will be the expansion of any part of the tube at which the pulse-wave arrives. The greater the elasticity, the less will be the height of the wave; the greater its length, the gentler will be the pulse-rise. The greater the resistance, the more abrupt will be the dilatation and the more slowly will it disappear.

Secondary waves in elastic tubes.—When by a sudden access and sudden removal of pressure a primary wave is produced in an elastic tube, the distal end of which offers no resistance to outflow, the liquid is not found on removal of the pressure to stop just when it has regained its position of equilibrium. In virtue of its inertia, it overshoots that mark and passes beyond that position, leaving the tube somewhat collapsed behind it. The tube being elastic regains its form, and thereby exercises suction: a back-rush occurs which in its turn, and for the same reason, is again excessive; and a system of secondary waves is thus set up, of which usually only the first is very important.

The form of the physiological pulse-wave.—The pulse-wave presents first a sudden rise, a steep-fronted primary wave, due to a rapid contraction of the ventricle; then a series of equidistant secondary waves, of which there are commonly two, seldom three, sometimes only one, and sometimes that one (Moens) so strongly marked as to resemble the primary wave in height ("dicrotic pulse"), a result specially observed when the tension of the vessels is small and their "coefficient of elasticity" great. Between the primary wave and the first secondary wave there is a sudden sinking and recovery of pressure which give the appearance of an interpolated undulation. This is (Moens) due to the cessation of the ventricular pressure and to the folding back of the valves under the influence of the pressure in the elastic blood-vessel (aorta or pulmonary artery).

The distance between the primary and the secondary waves increases as they travel. In arteries, the higher a wave the faster it travels: the primary wave travels faster than the secondary. In caoutchouc tubes, the coefficient of elasticity does not vary with the distension as it does in arteries, and there is no such relative retardation observed.

Reflected waves in elastic tubes.—If the tube be wide down to the orifice of outflow, which is itself narrow, pouching takes place at the distal end of the tube. There will be a greater or less recoil, which is the greater the greater the elasticity of the tube; and this produces reflected waves in the stream; but the outflow through the narrow distal orifice may under such circumstances be singularly uniform.

The reflected wave returning may meet the hinder part of the same oncoming wave, and may complicate its appearance with secondary undulations (see Marey, *La Circulation du Sang*, 1881).

This does not wholly explain secondary oscillations of the pulse, because the capillary system (which presents a wide channel for the onflowing stream) does not present this kind of resistance.

Amount of outflow from distensible elastic tubes.—Marey showed that when an intermittent inflow was distributed by a λ-tube and divided between a rigid and a distensible-elastic tube (the proximal ends of which were provided with valves to check regurgitation, and the distal orifices of which were narrowed to increase the resistance), the flow from the distensible tube was (if the intermittent inflow were sufficiently frequent) not only continuous, but also absolutely greater in amount than that from the rigid tube. This has been explained as depending on the diminished mean resistance offered by the widened tube; consequently a given initial total-head, H, may have a larger proportion of its own amount devoted (as velocity-head) to imparting movement to the liquid, though, as a result of the widened area, the actual mean rate of flow across each unit of sectional area may be less than that in the rigid tube.

If the distensible elastic vessel became rigid it would be necessary, in order to keep up the same onflow in it, to increase the driving power. [Hypertrophy of left ventricle in atheroma.]

CHAPTER XII.

OF GASES.

Density.—The standard of density of gases is, for chemical purposes, Hydrogen $=1$: sometimes air is taken as the standard, in which case the density of hydrogen is $\frac{8837}{127590}$. It is for most physical purposes better to adhere to the C.G.S. system, according to which air has a density $\rho = {\cdot}0012932$, and hydrogen a density $= {\cdot}0000895682$.

As a rule the density of a gas is determined by first weighing a vessel—a glass or copper vessel or a collapsed indiarubber bag—devoid of contents, and by again weighing it when it contains a known volume of the gas in question.

Elasticity.—Gases, as we have seen, have elasticity of volume alone; and in this they are perfect. In gases the Pressure outwards (acting across each unit of area) is equal to the Resistance to a compressing force acting inwards (across the same area); the Pressure is at any given temperature numerically equal to the Elasticity-coefficient; $p = K$.

Compressibility.—The extent to which a gas can be compressed is indefinite, provided that its temperature is above Andrews's critical point (p. 217); if the temperature be below this point, compression may liquefy the gas.

Boyle's Law, already stated, is that the volume of a gas and the pressure acting on it vary inversely.

Float on water a little glass bulb, which contains an adjusted quantity of air, and the interior of which communicates by an aperture with the liquid on which it floats; if the pressure on the surface of the water be increased, water passes into the bulb and compresses the contained air; the bulb as a whole becomes heavier and sinks (Descartes' Diver).

In a fish the air bladder acts during muscular relaxation as a float; during contraction of the muscular walls of the bladder the contained air is compressed, the mean density of the whole body is increased, and the fish sinks.*

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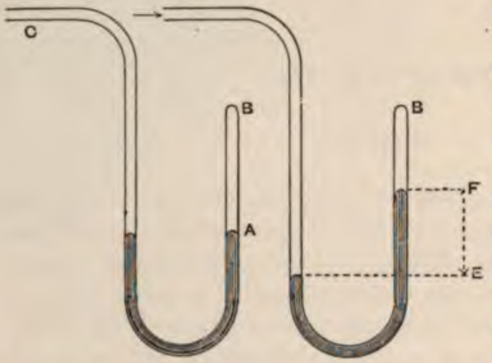
* The local contraction of one end of the air bladder causes the other end to act alone as a float, the head or tail being thus tilted up or down. The air bladder is in many cases too near the ventral aspect to sustain the fish in its normal position in the water; the action of the tail sustains the fish in its position of unstable equilibrium throughout the whole period of life.

Manometers can be constructed so as both to illustrate and to apply Boyle's law. If a tube bent and containing mercury as shown in Fig. 111, and enclosing a certain volume of air within the space AB, be exposed to an additional pressure acting through C, that additional pressure will be partly spent in sustaining the weight of the column EF of mercury raised in the tube, partly in maintaining a compression of the air AB within the space FB. By preliminary graduation such an instrument may be made to act as a manometer, and may be added to those of Fig. 105.

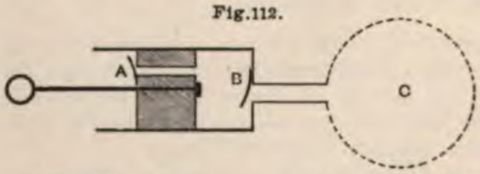
Fig.111.



Boyle's Law is somewhat departed from by oxygen, carbonic oxide, nitrogen, air, hydrogen, whose bulk at increasing pressures is greater than that law would indicate; while sulphurous acid, carbonic acid, and other easily condensible gases shrink in volume more rapidly when exposed to moderately-increasing pressures than the amount of pressure alone would lead us to expect; the latter gases presenting very curious aberrations when extremely high pressures—bringing the gas to the verge of liquefaction—or extremely low ones are applied.

The Tendency of Gases to Indefinite Expansion is utilised in the **Air-pump.** The primitive type of an air-pump is a cylinder, provided with a piston in which there is a valve A (Fig. 112), opening outwards. The cylinder itself is connected with C, the vessel to be exhausted, by a tube closed by the valve B, opening into the cylinder. Suppose the piston drawn out: the air within C and the connecting tube BC expands, thrusts aside the valve at B, and fills the whole space open to it—that is, the cylinder, the tube, and the vessel C. The piston returns: the valve B closes and the valve A opens, because the air between A and B is compressed; the air in AB is driven out through A. By repeating this process often enough the air in C

Fig.112.



is greatly reduced in quantity and becomes of correspondingly small density. This simple form of air-pump is liable to two objections:—(1) It is tedious to pull the piston against an external pressure which each time becomes more and nearly equal to the entire atmospheric pressure of 15 lbs. per square inch as the space between A and C becomes more nearly a vacuum; and (2) After a certain number of strokes the expansion of the air in C fails to lift the valve B. The latter objection may be obviated by causing the piston itself to lift the valve; the former is rendered less serious by connecting with the cavity C two such cylinders, and so arranging matters that when the one piston is being driven home the other is being drawn out; the whole being driven by a large and heavy wheel. Not only is a smaller force enabled by the leverage thus gained to resist a great pressure, but the inertia of the flywheel renders the action less painful, because more equable.

Sprengel's Air-pump in its simplest form consists of a long tube AB, Fig. 113, provided with a side-branch EF, which communicates with a vessel C, the vessel to be exhausted. At the upper end of the tube AB is D, a supply cistern of boiled mercury, which is allowed to fall down AB. As it passes EF the mercury entangles molecules of the expansive gas in EFC, and these are continuously removed by the falling stream and escape in bubbles at B.

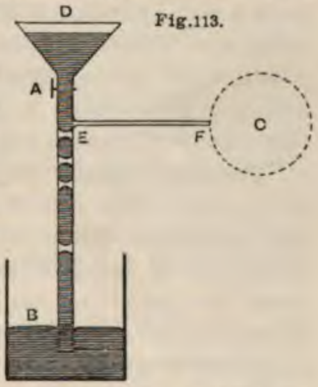


Fig. 113.

If the lower end of the tube AB be bent upwards, a vessel filled with mercury may be inverted over the upturned end, and as the gas issues at B it can take the place of the mercury in that vessel; the Sprengel-pump may thus be used as a means of transferring small quantities of gas from one vessel to another.

When the vacuum in C is tolerably complete, the mercury falls as a continuous mass, containing no bubbles.

The **Absorption** of Gases by solids is sometimes a true **solution,** as in the case of the alloy of metallic hydrogen[*] with palladium. This is produced by evolving hydrogen from a palladium electrode in the electrolysis of water (p. 610); or by heating a piece of palladium *in vacuo*, and allowing it to cool in an atmosphere of hydrogen; or even by heating it in a tube through which a current of hydrogen passes, and allowing it to cool in that gas. The same kind of colloid solution is exemplified by the alloys of iron and hydrogen, or of platinum and hydrogen; or, again, by the carbonic oxide, which (to the extent of 4·15 vols.) is retained on cooling by cast-iron, or the carbonic dioxide, of which a half-per-cent volume may be retained by indiarubber.

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[*] This seems (Graham, *Physical and Chemical Researches*) to be a white paramagnetic metal of density 1·95; diamagnetic (Blondlot).

Such absorption may, on the other hand, be due to **surface attraction** and condensation within the pores of the solid,—as in the case of animal charcoal, which can absorb so much oxygen or so much ammonia, that these gases must even be liquefied within its pores, or which can absorb both oxygen and oxidisable gases, and bring them into such close molecular relations that they combine, as in charcoal respirators; or in the case of platinum-black, which, if surrounded by a mixture of oxygen and hydrogen, absorbs both gases and brings their molecules into contact so close that they combine, and do so with evolution of heat so great as to cause the platinum to become incandescent and thus to ignite the remainder of the gas, as in Döbereiner's Hydrogen Lamp.

Chemical affinity may also promote the absorption of a gas by a solid. If a dish containing spirit of wine be suspended over quicklime within a confined space, the mixed vapours of alcohol and of water, which pass by evaporation into the space above the quicklime, are discriminated by it: the water is absorbed, the alcohol not; more water, but not more alcohol, is evaporated from the spirit of wine, and is again absorbed by the quicklime. The result is dehydration of the spirit, which may proceed to an extreme degree.

Where a gas is dissolved freely by a solid, that gas may freely traverse that solid. Thus hydrogen leaks freely through a white-hot palladium tube; so does carbonic oxide through glowing iron;* so do carbonic acid and marsh-gas and coal-gas in small quantities through indiarubber. The solids in which this effect is observable are as a rule colloid, or (like non-crystalline metals) resemble colloids, and they behave towards gas just as liquid films do.

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* Cast-iron stoves when red-hot allow carbonic oxide to escape into the air of a room; carbonic oxide in small quantities destroys the red blood corpuscles and produces anæmia.

A **solution** of a gas in a Liquid is a mixture of two liquids—the dissolving liquid itself and the liquefied gas. The more readily a gas can be liquefied, the more freely will it in general form such a mixture, and the greater will be its solubility.

According to Prof. Henry, carbonic dioxide, oxygen, nitrogen, and some other gases, are dissolved in the exact ratios of the pressures under which they are exposed to the surface of the liquid; at five atmospheres' pressure five times as much carbonic acid by weight can be dissolved in water as can be dissolved at one atmosphere. The *volume* of each gas dissolved by a given quantity of water at a given temperature is thus always the same. Henry's Law cannot be stated as a universal one with perfect numerical accuracy, though it is approximately adhered to by all gases.

When a Mixture of gases is exposed to a liquid, each gas is dissolved independently of the rest: each is dissolved (if Henry's law be obeyed) in proportion to the partial pressure exerted by it.

Thus, if water be exposed to air at the pressure of 76 cm. of mercury, the total pressure is made up of $\frac{20\cdot9}{100} \times 76 = 15\cdot884$ cm. pressure due to oxygen, and $\frac{79\cdot1}{100} \times 76 = 60\cdot116$ cm. pressure due to nitrogen. The Coefficient of Absorption (*i.e.*, the volume of gas dissolved by 1 vol. of water) of oxygen at 10° C. is ·03250; that of nitrogen is ·01607; both at the pressure of 76 cm. Hg. It happens that Henry's law applies to both these gases. Thus the volume of oxygen dissolved by 1 vol. of water is $\frac{15\cdot884}{76} \times \cdot03250 = \cdot0067925$ vol.; that of nitrogen is $\frac{60\cdot116}{76} \times \cdot01607 = \cdot0127114$ vol. Hence the air dissolved in water—that which subserves the respiration of fishes—contains oxygen and nitrogen in the ratio of ·0067925 to ·0127114, or, in percentages, 34·82 oxygen to 65·18 nitrogen.

The solubility of gases in liquids not only diminishes with diminished pressure, but also with increased temperature. The gases dissolved in a liquid may, if they form with it a simple solution,—as an aqueous solution of ammonia,—be entirely removed either by diminution of pressure or by increase of temperature. In some cases there is a chemical union between the gas and the liquid, or some constituent of it. Thus a solution of hydrochloric-acid gas, when heated, first loses some gaseous HCl, and then boils over as a whole: a solution of bicarbonate of soda, when the pressure is greatly reduced, somewhat suddenly loses half its carbonic acid: blood, when the pressure is gradually diminished, first loses the carbonic acid and the oxygen which it holds in simple solution, and then, at a very low pressure, those quantities of these gases which it holds in feeble chemical combination are suddenly given off.

A gas will traverse a liquid diaphragm with great rapidity if it be soluble in it: it is dissolved, diffuses, and emerges on the other side. A soap bubble or a wet bladder, containing hydrogen and surrounded by carbonic acid, absorbs the latter gas and enlarges in size, although, as we shall see, hydrogen runs with the greater speed through dry or indifferent membranes, and a dry bladder under similar circumstances would collapse.

It seems that there may be something of the nature of a solution of a gas in a gas. Oxygen evolved from chlorate of potash may contain no chlorine or any oxides of chlorine recognisable by any chemical test; yet if it be passed through a red-hot tube it will be found that chlorine can now be detected in it (Schützenberger). And further, a gas or vapour may dissolve a solid; boracic acid in steam; naphthalene in coal-gas rich in hydrocarbon-vapours.

Diffusion of Gases.—Two gases in vessels between which a free communication is established are found to mix freely, and if sufficient time be allowed the mixture will become uniform throughout. The rate of diffusion is somewhat rapid. Pure carbonic acid and air placed in communication will diffuse at such a rate that the air at a distance of half a metre will be found in seven minutes to contain one per cent of carbonic acid: hydrogen will similarly travel a third of a metre in one minute (Graham). The lighter the gas the more rapidly does it travel.

This process is molecular, and solid particles floating in either gas remain practically at rest; and thus mere diffusion is not sufficient for purposes of ventilation.

In the lungs, diffusion carries air out of the air-cells and oxygen into them; the oxygen tends to travel more rapidly inwards, and hence there is a small force tending to dilate the air-cells (Graham).

Effusion.—When gas is caused to flow through apertures, such as pinholes in membranes, the law of Torricelli is obeyed, and the velocity of outflow $v = \sqrt{2gH}$. Air rushing through such an aperture into a vacuum will do so as if the atmosphere consisted of a uniform layer of fluid, of uniform density $= \cdot 0012932$, and throughout which g is constant, all at the freezing temperature and at the pressure of 76 cm. of mercury; the layer having a depth H of 790020 cm. (nearly 5 miles). Then $v = \sqrt{2gH} = \sqrt{2 \times 981 \times 790{,}020} = 39320{\cdot}3$ cm. per second.

In different gases at the same pressure the height H will vary inversely as their densities; the gases will therefore pass through apertures with velocities inversely proportional to the square roots of their densities. Thus oxygen and hydrogen, whose densities are 16 : 1, will have effusion-velocities $\frac{1}{\sqrt{16}} : \frac{1}{\sqrt{1}}$; *i.e.*, 1 : 4 at the same temperature and pressure.

In any gas the velocity of outflow is not affected by changes of pressure. H is proportional to the pressure; it also varies inversely as the density; $H \propto p/\rho$. If the pressure be increased, Boyle's law shows that the density is increased in the same ratio; hence p/ρ is constant. Wherefore H is constant, and $v \; (= \sqrt{2gH})$, the effusion-velocity of each gas, is constant under all circumstances of pressure; and the normal rate of outflow of different gases at constant temperatures depends only on the nature of the gases. Under changes of temperature at constant volume, $v \propto \sqrt{p}$; $\therefore v \propto \sqrt{t^\circ} \, Abs.$ Under changes of temperature at constant pressure, $v \propto 1/\sqrt{\rho}$. Perturbations are, however, produced by variations in the viscosity of different gases at different temperatures; these cause slight departures from this law.

This phenomenon of outflow or effusion is one of masses, and in it gases act as fluids, practically continuous.

If a gas be driven under pressure through a substance which is porous, but whose pores are too small to allow the mass to traverse it without great resistance, the result is the **transpiration** of the gas, a slow flow under resistance. Transpiration may be studied by driving gases through long capillary tubes, or even through tubes which are not capillary, provided that their length so far (4000 : 1) exceeds their diameter that considerable resistance is offered to the onflow of the gas. It is found that in each case the gas moves with a velocity which is proportional to the pressure, but which is also found to vary inversely as the length, directly as the density of the gas (a singular result), and further, to depend on a constant, the Coefficient of Transpiration. $V = k . p\rho / l$. A film of gas adheres to the sides of the tube, and the gas flows in an axial stream in each channel.

The coefficient of transpiration peculiar to each gas is a very isolated factor, and does not seem to have any intelligible relation to the other properties of gases. The transpiration-coefficients of nitrogen, of nitric oxide, of carbonic acid, are double that of hydrogen: those of ether and of hydrogen are the same: those of oxygen and nitrogen are related to one another in the ratio 14 : 16, so that equal times are taken by equal masses of these gases to pass through long or capillary tubes.

If a gas be heated it will become lighter, and its transpiration-rate will be lessened: if the barometric pressure rise, it will be compressed and its transpiration-rate will be increased.

Membrane-Diffusion.—Gases placed on opposite sides of an indifferent porous membrane, and exposed neither to the influence of a difference of pressures nor to that of a difference of solubilities in the material of which the membrane is composed, will pass through it in virtue of their own molecular motion.

The velocity with which hydrogen and oxygen, separated by a partition of plaster-of-Paris, or graphite, or biscuitware, will traverse that partition is exceedingly small in comparison with the rate of effusion through a relatively large aperture into a vacuum; but it is found to be proportional to the mean velocity of the molecules in the gas.

We have already seen (p. 233) that the mean velocity of the particles of any gas is inversely proportional to the square root of the density of that gas; and hence the rate of diffusion of any gas through an indifferent membrane is inversely proportional to the square root of the density of that gas.

A dry bladder filled with hydrogen and surrounded by oxygen will partially collapse, for hydrogen leaves it four times as fast as oxygen enters it.

This difference of diffusion-rates may be made to effect a partial separation of gases. If a long porous tube be fitted so as to pass through a vacuum or a neutral gas, and if a mixture of gases be passed through the porous tube, the components of that mixture will escape through the walls of the porous tube in unequal proportions. If the vapour of chloride of ammonium be passed through such a tube, the hydrochloric acid (density $= 18{\cdot}25$ when $H = 1$) and ammonia (density $= 8{\cdot}5$) into which the chloride is dissociated by heat pass through in the ratio of $\frac{1}{\sqrt{18{\cdot}25}}$ to $\frac{1}{\sqrt{8{\cdot}5}}$: the ammonia therefore passes through in excess, and litmus paper placed in the neighbourhood of the porous tube will indicate an alkaline reaction.

A gas may pass through the pores of a solid by liquefaction in those pores: sulphurous acid may pass through charcoal and evaporate on the farther side.

Diffusion of Gases from Liquids.—If a layer of liquid charged with gas be placed upon one free from gas, the gas rapidly permeates the whole liquid. If the two layers be separated by a membrane wetted by both, the diffusion is rapid. If the two layers, thus separated by a thin membrane, be in a state of relative motion, the diffusion-rate may be accelerated if the velocities be not too great. If two streams so separated move in opposite directions they may completely exchange gases; for suppose two such streams to be charged, as they arrive at the opposite ends of a certain tract of vessel, with gases A and B: then throughout the whole of that part of their course during which they are contiguous, the A-charged stream passes and diffuses A into a stream which is at every point poorer in A than it itself is at the same point; and *vice versâ:* so that, if the course be long enough, the A-charged stream may lose all its A, and the B-charged stream all its B.

The Statics of Gases.—A gas always fills the whole space within which it is contained. There is no difference in respect of statical theorem between a gas and a liquid which also fills the whole space within which it is contained. Pascal's principle, that of the so-called Transmissibility of Pressure, that of the perpendicularity of the pressure exerted by a fluid upon its bounding surface—all these apply equally to all fluids: so do the principle of the Hydraulic Press and that known as Archimedes' Principle.

The last must be kept in mind when accuracy is required in weighing. A piece of brass of density 8 and weighing 1 kilo. *in vacuo* occupies 125 cub. cm. ($\frac{1}{8}$ the bulk of an equal mass of water). It apparently loses when weighed in air the weight of 125 cub. cm. of air; that is, $125 \times {\cdot}00129366$ grammes $= {\cdot}1617$ gramme. The substance to be weighed also loses weight, but if it displace more air than the counterpoising mass of brass does, it loses more than the brass does, and an inaccurately large quantity of it has to be used to counterpoise the metallic kilogramme.

Balloons and soap bubbles containing coal gas or hydrogen rise in the air; bulk for bulk they are lighter than air. The lighter they are, the more rapidly they ascend; and they can be loaded until they weigh bulk for bulk the same as the air in which they float. If a balloon with its contained gas weigh 100 lbs., and the bulk of air displaced by it weigh 120 lbs., the balloon will rise with an ascensional force equal to the weight of 20 lbs. applied to a mass of 100; its upward acceleration will be equal to $g \times \frac{20}{100} = \frac{1}{5}g$.

The pressure on the walls of a closed vessel containing gas is greater the lower the level at which it is measured: the law is exactly the same for gases as for liquids. The effect is seldom perceptible, because within vessels of ordinary size the mere weight of the gas adds little to the atmospheric or other pressure acting.

With vessels of ordinary dimensions a manometer applied laterally at any part will indicate the internal pressure; strictly speaking, in gases, as in liquids, it indicates only the pressure at the horizontal level of the orifice of communication between the manometer and the vessel.

Streams of Gas.—The statements made in the discussion of streams of liquid in Chap. XI. apply also to streams of gas. The Law of Continuity, Torricelli's Law, the distinction between Velocity-head and Pressure-head, the gradual disappearance of the latter, coupled with the simultaneous heating of the flowing fluid, the Lateral Pressure in a main pipe, and the propulsion of the fluid up piezometer tubes or through lateral orifices,—all these apply to gaseous as well as to liquid streams.

In calculations based on Torricelli's Law it is necessary to find H. H is the height of that column of the outflowing fluid which would, if acting alone, produce a pressure equal to that actually undergone by the fluid set in motion. If the gas issue from a vessel in which the pressure is such as to support a manometric column of, say, 24 cm. Hg in addition to the atmospheric pressure; and if the atmospheric pressure at the time, as shown by the barometer, be 76 cm. Hg: the total pressure on the gas is, for each square cm. of its bounding surface, equal to the weight of a column of mercury whose content is 100 $(= 76 + 24)$ cub. cm. This is equivalent to the weight of 1359·6 cub. cm. of water. If the density of the gas be $\frac{1}{800}$ that of water, the pressure is equal to the weight of $800 \times 1359{\cdot}6 = 1{,}087{,}680$ cub. cm. of that gas, the same gas as is driven out in a jet. This column, standing on a sq. cm. base, is 1,087,680 cm. high. Hence, for the gas in question, $H = 1{,}087{,}680$; and the velocity of that gas, rushing into a vacuum under a total pressure of 100 cm. Hg (as long as the pressure is maintained at that value), is $v = \sqrt{2gH} = \sqrt{2 \times 981 \times 1{,}087{,}680}$ in cms. per second; while into the atmosphere it would run with a velocity $\sqrt{2 \times 981 \times (800 \times 13{\cdot}596 \times 24)}$, due to the difference (24 cm. Hg) between the internal and external pressures.

Recoil.—When a stream of gas issues from a jet or burner, the reaction is equal to the action, and there is a tendency for the burner itself to move backwards. This tendency we see turned to account in certain revolving shop-window gas-illuminations.

Viscosity.—The viscosity of gases, which is due to diffusion, is on the whole similar in its results to that of liquids. We have already seen its influence in transpiration.

A stream of air, driven through air, soon comes to rest. If it have a great velocity it can cut its way through air to a greater distance than a slower stream can.

If a stream of air be introduced into a room through a funnel-shaped aperture, the broad mouth of the funnel being open to the external air, it will enter the room through the narrow orifice with great velocity, and will pass a considerable distance (being acutely felt as a draught), until at length the process of diffusion between it and the surrounding air relieves it of its momentum. If, on the other hand, the narrow end of the funnel be presented to the exterior air, the stream as it enters will (obeying the Law of Continuity) widen out in accordance with the shape of the funnel, and its velocity will be proportionately diminished; the result being that a considerable amount of air may, through ventilators of such a form, be introduced into a room without producing a perceptible draught.

All objects surrounded by air bear on their surface an adherent film of air which is almost dustless. When a body moves in air, this film rubs against contiguous layers of air, and the movement of the body is retarded by internal friction in the air. Haughton and Emerson Reynolds found that a granite ball suspended in the air, and swung pendulum-fashion, suffered, on each successive swing, a diminution of amplitude of $\frac{1}{6052}$ due to this cause.

The friction within a gas is independent of its density, but increases with its temperature—(Clerk Maxwell, *Phil. Trans.* 1866).

A body falling *in vacuo* is not retarded, and falls with an acceleration fully equal to g; falling through air, it is retarded because the viscosity of the air causes friction. A thick piece of gold and a piece of paper fall *in vacuo* at the same rate: through air the gold falls more rapidly, because it presents less surface in proportion to its weight: but even through air a piece of smooth paper and a piece of gold leaf presenting the same total surface and the same weight, or bearing the same proportion between their surfaces and weights, will fall side by side.

Falling water is retarded by the air; and conversely, air is dragged down by falling water. If a stream of water be made to fall through a closed cavity, the water will drag down with it a considerable volume of air; and if a lateral communication be established between this cavity and a vessel containing air, much of the air in that vessel may be extracted. If a stream of air or steam be maintained through a cavity, it is not only itself retarded, but the surrounding air is dragged with it, and the pressure in the cavity is diminished.

This action, due to viscosity, is independent of the general diminution of pressure experienced by fluids in motion. A vibrating tuning-fork brought near a suspended pith-ball seems to attract it; the air between the objects vibrates, the pressure is lessened, and the exterior atmospheric pressure urges the ball against the tuning-fork.

When the density of the vibrating fluid is ρ, at a point in the fluid where the greatest velocity of vibration is v, the diminution of pressure is $\frac{1}{2}\rho v^2$; provided that the cause of the vibration is the to-and-fro movement of solids moving within a finite space of the fluid (Sir Wm. Thomson).

Measurement of Flow.—The amount of flow of gas through pipes may be measured on the same principles as the amount of flow of liquids.

(*a*) The amount of gas actually passed may be collected and measured. It may be collected in a balanced bell-jar, inverted over water like a small gasometer (Hutchison's Spirometer), or in a very large and thin flexible caoutchouc bag (Boudin).

(*b*) It may be made to drive a registering train of wheelwork, like a gas-meter, as in Bonnet's pneumatometer.

(*c*) The principle of the hydrostatic pendulum or

(*d*) that of Pitot's tubes may be employed.

THE PRESSURE OF THE ATMOSPHERE.

Most of our experiments and observations are complicated or affected by the fact that we live at the bottom of an atmospheric ocean which exerts pressure upon every surface exposed to it, and which penetrates even into the recesses of everything porous, and there also exerts pressure, unless special appliances be made use of in order to remove it wholly or in part. We live at the bottom of such an atmosphere without inconvenience, just as deep-sea fishes live at the bottom of the ocean: as long as they are in their *habitat*, the internal pressure of the gases contained and dissolved in their organisms is equal to and is in equilibrium with the immense external pressure exerted by the surrounding water; but when they are brought towards the surface, the external pressure becoming greatly less, the gases contained in the swim-bladder and throughout the tissues undergo expansion, and the fish explode.

The pressure within our organisms cannot be less than the atmospheric pressure, that exerted by the atmosphere on the surface, 1,033,663 dynes per sq. cm., or a pressure equal to the weight of about 15 lbs. per sq. inch. If the internal pressure in any part become less than this, the fluids or the semi-fluid tissues or masses of the body must flow towards the region of diminished pressure. Hence a permanent vacuum within the body, total or partial, is impossible.

The abdominal walls are closely appressed against the viscera: the walls of these are pressed against one another as far as their contents will allow.

The lungs are pressed against the ribs by the atmospheric pressure acting down the trachea and bronchi, and they are thereby expanded when, but for this action, the expansion of the ribs would tend to form a vacuum between the pulmonary pleura and the parietes of the thorax. This expansion does not take place when the thorax is so opened by a wound that, on expansion of the ribs, air can pass through the wound into the pleural cavity, and can thereby equalise the internal and external pressures without the aid of pulmonary inflation.*

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* In such a case some of the air in that cavity can be expelled by an expiratory effort with closed glottis, and can be prevented from returning by a valve opening outwards.

The atmospheric pressure acts freely upon and through a mass of gas, if that mass be free to expand or contract, whatever be its temperature. The air in a room may be hot, and yet the atmospheric pressure, acting down the chimney and through all the chinks and orifices of the room, will be undiminished in amount and in effect.

A trap in a wash-hand basin in a room will not be unable to prevent gases from being forced into the room from the drains, simply because the air in the room is warm. It may be unable to do so if the pressure within the drains become excessive, or if the air in the room be rarefied by a strong draught up the chimney, especially where the fittings of the room are so air-tight that the external pressure cannot force air into the room except through the trap.

If any object containing gas or air be placed in a region of space from which the air has been wholly or in part extracted—such as the bell of an air-pump—the internal pressure may overpower the external, and the body will then tend to become inflated and may even burst.

A little indiarubber balloon, a bladder half filled with air, a shrivelled apple, a dish of soap suds, present under the air-pump a singular appearance of expansion. If a loaded piece of wood be put in a dish of water, and the whole placed under the air-pump, the wood will appear to effervesce; the air contained in its pores expands and forms bubbles. If soda-water already flat be subjected to similar treatment it will renew its effervescence.

This inflation is not due to any suction on the part of the air-pump, but is due to the expansion of the contained gas, which always tends to expand, but which can only do so when the resistance offered to its expansion on the part of the external pressure is diminished or removed. The gas expands until the internal pressure of the expanded gas is equal to the pressure of the rarefied air or gas; the latter, as we have already seen (Boyle's law), suffers diminution proportionably to the density.

If the pressure within an object or a cavity exceed or be made to exceed the external atmospheric pressure, there is, as in all such cases, a tendency to establish equilibrium by setting up a flow from the place of greater pressure to one of less. Thus, if a bladder containing gas and provided with a stopcock be loaded with a weight, and its stopcock opened, the atmospheric pressure tends to drive air into the bladder, but it is overpowered by the greater pressure within the bladder, and there is an outward flow set up, due to the difference between the internal and the external pressure.

A gasholder, consisting of an inverted bell floating on water, may be loaded so as to exercise any given expulsive pressure. Thus coal-gas driven out "at a pressure of 1 inch of water" is subject in the gasometer to an internal pressure = atm. pr. + "1 inch of water," and to an exterior pressure at the burners = atm. pr. only.

If air be blown into a flask partly filled with water, partly with air, and provided with a narrow open glass tube passed through the cork, and if the flask be suddenly inverted, water will rush out through the nozzle: the air has been compressed, and its pressure has become greater than the atm. pr.; this difference of pressures causes an outward flow, a jet of liquid.

In the dome of the fire-engine air is compressed in the same way: the inflow is intermittent, the outflow continuous, for the air never ceases to be compressed and exercises a continuous pressure.

If a gas-evolution flask containing, say, zinc and sulphuric acid, be fitted with an ordinary safety-funnel dipping into the liquid, the hydrogen evolved will pass out by the intended channel: the liquid of the flask will be observed to oscillate a little in the safety tube, which acts as a manometer indicating the internal pressure. If any obstruction offer, the gas accumulates in the flask, a difference is set up between the internal and the external pressure, and the liquid is forced up the safety tube. The safety tube should dip into the liquid only just so deeply that before the liquid forced up into the funnel can overflow, the level of the liquid in the flask shall have been so far depressed that nothing but gas can pass out through the safety tube.

If a cistern at a height be connected by a tube with a large flask containing air, in such a way that liquid may pass from the cistern into the flask, air is driven out of the flask: it may be driven out through a tube; this tube may be connected with any cavity through which it may be necessary to drive air. This is one form of Aspirator.

The same principle is applied in the plenum method of ventilation: a local excess of pressure is set up by forcing air into a building, and the air is left to find its own way out.

When the thoracic walls contract, air is driven out of the lungs, and blood out of the thoracic organs in general.

When the abdominal walls contract, a general-contents-pressure is set up, always at right angles to the general surface of the practically-fluid visceral mass, and opposed partially or completely by a uniform atmospheri pressure

When the external atmospheric pressure exceeds that within an object or cavity, air may be forced into it or it may be compressed, or if these effects be not possible, the existence of the atmospheric pressure generally becomes in some way strikingly manifest.

The Magdeburg Hemispheres, a couple of hemispheres fitting together so as to form a sphere, and ordinarily separable with ease, but when apposed, and the air extracted from between them, not to be separated without great force; the boy's leather Sucker, a piece of moistened leather closely applied to any object and pulled—any residual air still remaining being rarefied—the pressure of air between the sucker and the object becoming very small, and the sucker being thus firmly pressed by the weight of the atmosphere against the object on which it is placed;* the difficulty experienced in pulling the handle of a good Syringe when the nozzle is stopped up, or in the continued working of an Air-pump,—all these clearly point out the part played by atmospheric pressure.

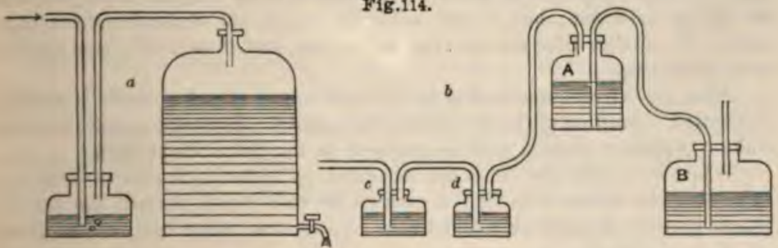
In the experiment previously described, in which gas escaped from the pores in a piece of wood kept under water and exposed to the action of the air-pump, it is only necessary to allow the atmospheric pressure again to act to see the water driven by that pressure into the pores of the wood, which thus becomes too heavy to float.

The atmospheric pressure is a prime agent in most of what we usually call the phenomena of **Suction.** A syringe has its nozzle inserted in water; the handle is drawn up: in the body of the syringe there would arise a partial vacuum were it not that the external atmospheric pressure overcomes the feeble internal pressure, and pushes the liquid through the nozzle into the body of the instrument.

If the syringe have a thin closed wooden nozzle, and if the vacuum in the syringe be relatively good, the atmospheric pressure can force water or mercury through the pores of the wood.

If there be no safety tube attached to a gas-evolution apparatus, and if the evolution of gas suddenly cease while the gas still continues to be absorbed by the liquid into which it is passed, we find the gas diminishing in amount, and the external atmospheric pressure forcing the absorbing liquid back into the gas-generating flask. If there be a safety tube, the very short column of liquid at its lower end is forced down, and air enters the flask until the internal pressure becomes equal to the external.

Fig.114.



Aspirators are generally constructed on this principle. Water flows from a large flask or can, Fig. 114 *a:* air must take its place: this air "is drawn," or rather is pushed by the atmospheric pressure, through a series of flasks which it must traverse on its way from the outer air to its place in the aspirating flask. With the arrangement *b* of Fig. 114, air can be forced either from *c* towards *d*, or in the reverse direction, according as A or B is accorded the higher relative position.

The vacuum method of ventilation is an exhaust-method: air is removed at a certain point, and finds its way from different parts of the building towards this point.

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* The air does not force its way between the sucker and the object pulled upon, for the intervening film of moisture is held in place by adhesion.

Filtration may be assisted by connecting the filter with a partial vacuum: the funnel is for this purpose fixed into a flask by a cork through which there also passes a tube leading to an aspirator of any kind, a Sprengel pump worked by water, and called a Bunsen pump, being frequently employed. The atmospheric pressure on the liquid in the funnel forces it through the filter into the partial vacuum.

Suction nipples and **bleeding cups** illustrate not suction but atmospheric pressure: the pressure within them is less than the external pressure; the part of the surface of the body exposed to their action suffers less pressure than the contiguous parts of the skin, which are acted upon by the full atmospheric pressure. The result is as if all parts of the surface except the area operated on were subjected to a powerful squeeze: the fluids are squeezed by the atmosphere towards the area subjected to least pressure.

When the thoracic walls expand, their soft parts are driven inwards, air is driven into the lungs, and blood is driven into the thorax from the parts of the body acted upon by the full atmospheric pressure; all this being the consequence of the so-called **negative pressure** (*i.e.*, pressure less than that of the atmosphere) in the thorax. The lungs act like a sphygmoscope (Fig. 105, S): they are dilated by internal pressure until their resistance to farther dilatation is equal to the dilating force. The less extensible they are or become, the sooner will this limit be reached: if their extensibility become so small that the limit of expansion would, if the ribs expanded to their full extent, be reached before the pleural cavity is filled, then the blood and the thoracic walls themselves are pressed inwards and the chest-walls lose the habit and the power of expansion. If while the chest is expanding, there be an orifice open in a large vein, the diminution of thoracic pressure allows the atmospheric pressure not only to drive venous blood towards the heart, but also to force air into the open vein, and thus into the circulation.

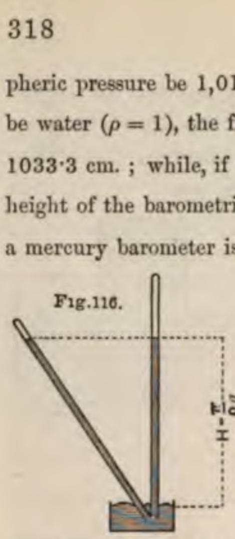
If a test tube be inserted in a larger test tube containing water, it will float. If the whole be inverted, surface-tension may for some time prevent the escape of water; but if any water do escape, the atmospheric pressure pushes the smaller tube up into the larger one, and thus causes it to appear to be sucked up.

After an extreme contraction of the abdominal muscles, there is elastic restitution of position of the abdominal walls, and the intra-abdominal pressure sinks. Apparent suction is thus exercised on the pelvic diaphragm.

When in a joint the bones are separated by extreme flexion or extreme extension, the tendency to form a vacuum between them permits the atmospheric pressure to press skin and tissue between the bones, and thus to form an external dimple.

Columns of liquid supported by the atmospheric pressure.—If a vessel filled with liquid be inverted with its mouth beneath the surface of liquid standing in a larger vessel, we see—provided that the inverted vessel does not exceed a certain height, about 33 feet in the case of water, about 30 inches in that of mercury—that the liquid does not fall out of the inverted vessel, but remains in position, supported by the atmospheric pressure. If in Fig. 115 the inverted vessel have a mouth whose area is a, and if the height of the column of fluid pheric pressure be 1,013,667·3 dynes per sq. cm., and if the liquid employed be water ($\rho = 1$), the free height of the column will be $\frac{\Pi}{\rho g} = \frac{1{,}013{,}667 \cdot 3}{981} =$ 1033·3 cm.; while, if the liquid employed be mercury ($\rho = 13 \cdot 596$), the free height of the barometric column will be $\frac{\Pi}{\rho g} = \frac{1{,}013{,}667 \cdot 3}{13 \cdot 596 \times 981} = 76$ cm. Hence a mercury barometer is much more convenient an instrument than a water barometer, for the height of the column in the latter is over 33 feet.

Fig. 116.



If the tube be tilted obliquely, its lower end being kept immersed, the liquid will move upwards in the tube: the vertical height remains unaltered (Fig. 116).

A common water-pump cannot act if it be so deep that during its action the atmospheric pressure would have to support a greater column than one of about 33 feet: a vacuum might be produced at the top of the cylinder of the pump, and yet no column whose height exceeded $\Pi/\rho g$ could possibly ascend in it. The Torricellian vacuum is utilised in the so-called mercury air-pump. A flask is filled with mercury: this flask is connected with a flexible tube also filled with mercury: this mercury is continuous with that in a cistern into which the flexible tube dips. The flask may be raised a certain height without the mercury leaving it, but if it be raised so high that the upper limit of its cavity comes to an elevation greater than $\Pi/\rho g$ above the surface of the mercury in the cistern, a Torricellian vacuum is formed by some of the mercury leaving the flask. The vacuum may be laterally connected with flasks filled with fluids, the gases contained in which are to be extracted for analysis. When the flask is raised and a vacuum formed in it, the liquids in the lateral flasks effervesce and the gases previously dissolved in them take their places in the mercury flask, which may be disconnected and removed for further research.

When the free height of the vessel is less than $\Pi/\rho g$, the column of liquid fills the vessel.

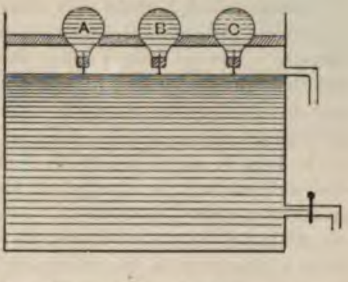
If a card be laid across the mouth of a tumbler completely filled with water, the whole can be inverted; the card will not drop off, and the water will not drop out of the tumbler: atmospheric pressure keeps the whole in place. It is important to observe that there is no tendency for the card to become bulged in any sense.

A pipette completely filled with liquid and closed by the thumb will not allow the contained liquid to escape, unless the lower orifice is so oblique or irregular as to permit successive portions of liquid to trickle away. If it be partly filled and closed by the thumb, the pressure of air in the upper part would neutralise the effect of the external atmospheric pressure, and the liquid would be free to fall were it not in the first place for the surface-tension at the lower orifice, which, if the orifice be very small, may be able to support a considerable column of liquid, and in the second for the rarefaction which is set up by the escape of some drops of liquid.

A gas-holder may contain a certain quantity of gas above and of water below, and even though an orifice be made in the walls of the vessel below the level of the water—provided that it be not too large—none of the gas will escape, for the atmospheric pressure keeps the whole in place.

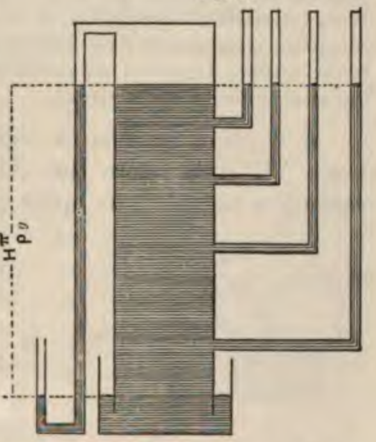
It is often of importance to keep water in a cistern at a constant level. The arrangement shown in Fig. 117 enables this to be done. The instant that the level of the liquid passes below that of the orifices of the nozzles of the flasks A, B, C, air enters these flasks, and water passes into the cistern. The aggregate delivering power of the flasks must not be less than that of the cistern itself.

Fig. 117.



When a column is supported by the atmospheric pressure, its own lateral pressure differs at different altitudes. This is illustrated by the indications of the lateral manometers of Fig. 118.

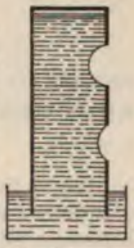
Fig. 118.



If the walls of the tube in which such a column is supported be rigid, these walls will, on account of differences between the internal pressures and the external atmospheric pressure, be subjected to stress: this stress varies from point to point according to the altitude.

If some parts of the walls be flexible, mercury will leave the column, and the tube will yield laterally as in Fig. 119; this it will do until the resistance to farther distortion offered by the walls is equal to that difference of pressure which tends to produce it.

Fig. 119.

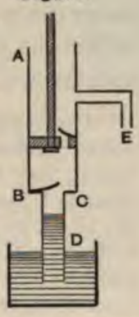


If the column be not barometric but closed, and if in the same way the containing vessel have local flexibilities, the upper flexible parts of it will yield inwards, the lower will bulge outwards; in each case equilibrium is established between the internal pressure, the atmospheric pressure, and the elasticity of the walls. If the whole walls be flexible, the whole mass becomes pyriform; here the atmospheric pressure produces no special effect in the determination of form, for it is equably yielded to.

If the upper part of the walls be rigid while the lower are flexible, the lower part will bulge, but the upper will be completely filled, provided that the whole column has a height not greater than $\Pi/\rho g$; if the height be greater, there will be a Torricellian vacuum produced. If the upper rigid part of such a vessel become flexible in whole or in part, it will collapse to some extent, and fluid will pass into the lower part of the column. The

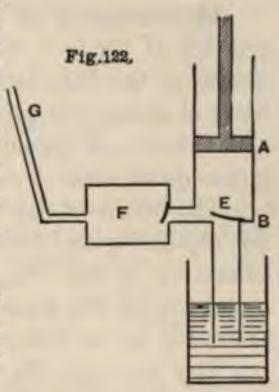
The Common Pump (Fig. 121).—By an upward stroke of the piston the air in the cylinder AB is expanded and rarefied. The atmospheric pressure drives up a column of liquid along DC. The piston is driven downwards, or else descends by its own weight; the valves now permit a certain quantity of air to escape to the upper side of the piston, but permit none to return to the column CD. At the next stroke the air in AB and CD is again rarefied, and more water rises in DC. This is repeated till the water rises into the cylinder AB, which it will do provided that the column CD is somewhat less than $\Pi/\rho g$ in height. The piston then scoops up the water in the lower part of the cylinder, always allowing it to pass to its upper surface, but never to return, and thus at each upward stroke of the pump water is lifted up and falls out at E.

Fig.121.



By the force-pump water may be raised to very great heights. Fig. 122 shows the arrangement of the valves. The piston is solid, and when it is pressed down the valve E is closed, while air or water is forced through the valve F against the pressure of air or water in the tube G, which tends to close that valve. In the Fire-engine there may be one or two such force-pumps which drive water into the dome.

Fig.122.



Digression on Valves.—There are three principal types of valves in use. Of these the first is the ordinary and very familiar clapper valve.

The second is the conical valve shown in Fig. 123. The pressure of the fluid in A may displace the valve: a spring returns it to its place when the relative pressure in A has become sufficiently diminished to permit it to do so.

Fig. 123.

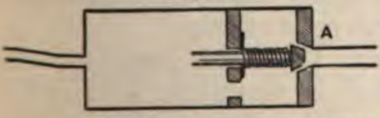
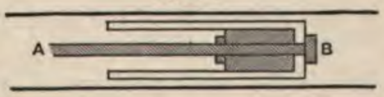


Fig. 124.



The third kind is that shown in section on Fig. 124. The piston AB is furnished with a cap of indiarubber, which is slightly smaller than the tube in which the piston moves. In the direction

A to B the piston can be freely moved through the liquid; but if the piston be moved in the contrary direction, the indiarubber cap flies open, and it exactly and equably fits the tube so that no water can pass it. A pouch is formed: the greater the pressure within the pouch, the closer the apposition between the indiarubber and the walls of the tube, and the better the action of the valve.

A somewhat similar form of valve is found in the heart. The semilunar valves (pulmonary and aortic) consist of pouches attached to the walls of the vessel; they lie loosely against the walls and allow the liquid to flow past them as it issues from the heart; but when a backward impulse is given to the blood, or the valves pushed forward against the blood, they are caught by the liquid, the pouches are distended, they touch one another and completely block up the lumen of the tube.

The other valves of the heart are clapper-valves, attached to the walls of the cavity of the heart, two or three in each situation, together attached to a complete circumference, acting together, slightly overlapping one another, and completely closing the lumen of the tube, and provided with tendinous and muscular arrangements which prevent their being driven too far towards the auricle when they are impelled backwards by a predominant ventricular pressure.

Measurement of Atmospheric Pressure.—The atmospheric pressure Π per unit of surface may be easily calculated if H, the height of the barometric column, be known, for $\Pi = H\rho g$. The habit of stating the pressure in terms of H,—as thus, "a pressure of 30 inches of mercury,"—is general, and if clearly understood is unobjectionable.

The height of the barometric column is subject to corrections for capillarity and for temperature; the latter involve the consideration of the less density of warm mercury, and of the expansion of the glass of the tube, which expansion involves an alteration in the correction for capillarity.

The aneroid barometer is essentially a hollow box of elastic metal in which there is rarefied air. Any given amount of external pressure produces a corresponding amount of compression of this box; a multiplying arrangement causes a lever to indicate, by its position in reference to the face of the dial, the amount of this compression. Careful preliminary graduation enables the absolute amount of external pressure corresponding to each indication of the instrument to be recorded.

The pressure $\Pi = H\rho g, =$ (say) 1,006,506 dynes per sq. cm., is the same pressure as would be exerted by a uniform atmosphere throughout which g was uniform, whose uniform density was $\frac{1}{770}$, and its uniform height $H = \frac{\Pi}{\rho g} = \frac{1,006,506}{\frac{1}{770} \times 981}$ $= 790,020$ cm. $= 7900{\cdot}2$ metres. If a barometer on the floor stand at 76 cm., the same barometer raised to the height of 1 metre should stand at a height of 76 cm., *less* $\frac{1}{10\cdot 4}$ mm., a perceptible diminution.

The pressure does not diminish regularly with the height, as it would in an ocean of incompressible fluid. The lower strata of the air are compressed, and therefore, to set up a given difference of pressure, a shorter vertical ascent among them is sufficient than is necessary among the higher strata.

Each stratum differs from the one below it in two respects:—(1) it has fewer strata above it; (2) it is therefore less compressed, and for equal mass has greater volume. If we imagine the whole atmosphere to be divided into 7900·2 strata, the lowest of them all, which bears the weight of 7899·2 strata, will be 1 metre thick; the next, which bears the superincumbent weight of 7898·2 strata, will have a thickness of 1 metre × $\frac{7899\cdot 2}{7898\cdot 2}$; the next stratum will have a thickness greater *than this* in the ratio $\frac{7898\cdot 2}{7897\cdot 2}$; *i.e.*, it will be $\left(1 \times \frac{7899\cdot 2}{7898\cdot 2}\right) \times \frac{7898\cdot 2}{7897\cdot 2} = \frac{7899\cdot 2}{7897\cdot 2}$; and the nth layer will be $\frac{7899\cdot 2}{7900\cdot 2 - n}$ metres thick.

Altitudes as indicated by the Barometer.—If x be the vertical height between two stations, H the height of the barometer at the lower station observed at temperature t, and $H_{,}$ the height of the barometer at the higher station at the temperature t', λ being the latitude; then

$$x = 18393 \,.\, (1 + \cdot 002837 \cos \lambda) \,.\, \log \frac{H}{H_{,}} \cdot \left(1 + 2\,\frac{t + t'}{1000}\right).$$

(Laplace's Formula).

Variations in the barometric pressure occur from moment to moment as the atmospheric ocean is disturbed by currents, driven in whirlpools, varied in thickness by superficial waves or locally varied in its superincumbent mass by expansion (due to heat) and lateral overflow. When any spot has a low pressure, there is a tendency for the surrounding air to rush in from all sides towards that spot, the centre of depression; the greater the difference of pressure between two places—*i.e.*, the steeper the barometric gradient—the greater will be the tendency to an inflow of air towards the centre of depression. This tendency is so modified by the rotation of the earth from west to east (in a direction opposed to the apparent movement of the sun) that the flow does not take place directly towards the centre, but round it in a circular storm or cyclone, whose direction is in the northern hemisphere opposed to, in the southern the same as that of, the hands of a watch (Dove's Law of Storms). The wind whirls round the centre and also towards it; air ascends in the centre: it expands and becomes cooled; moisture condenses; rain falls. "Put your back to the wind and the barometer is lower towards your left hand (northern hemisphere)."—(Buys-Ballot.)

Correction for pressure.—Variations in the barometric pressure render it necessary in measuring quantities of gas by volume to make a correction for pressure, and to reduce the gas to standard pressure—*i.e.*, to state what the volume would have been had the atmospheric pressure at the time of measurement been 76 cm. of mercury. Boyle's law teaches us that the volume varies inversely as the pressure. If therefore the pressure on gas measured as x cub. cm. at 76·1 cm. had been, not 76·1 but 76·0 cm., the volume of that gas would have been greater under the less pressure in the ratio of 76·1 to 76·0. The general rule is, that a volume of gas measured at the pressure h cm. of mercury must be multiplied by $h/76$ in order to reduce it to the standard pressure.

Standard Atmospheric Pressure.—In many modern books, instead of a pressure of 76 cm. mercury, or 1033·296 cm. water, or 1,013,663·376 dynes per sq. cm., the standard atmospheric pressure is taken as 1,000,000 dynes, or one megadyne, per sq. cm.

Gases passed into the Torricellian Vacuum.—If a bubble of gas be passed into the Torricellian vacuum, it will expand so as to fill it; further, it will exert pressure on the top of the column of mercury—it will therefore depress that column; the extent to which it depresses the column measures the pressure which it exerts upon the mercury: conversely, that depression measures the pressure of the mercury upon it, and therefore indicates the pressure under which it itself assumes its actual volume.

Let a barometer tube whose cross area is $\frac{1}{4}$ sq. in., and whose free internal height is 34 inches, have standing in it a column of 30 inches of mercury. Pass a cubic inch of air (measured under a pressure of 30 inches) through the mercury into the four-inch-long Torricellian vacuum. It would exactly fill that vacuum, exerting a pressure of 30 inches on the top of the mercury. This is impossible. The gas expands; it depresses the mercury through x inches: it is then subjected to a pressure of x inches of mercury as compared with the atmospheric pressure of 30 inches under which it was measured. Its volume is now accordingly increased to 1 cub. in. $\times\ 30/x$. The length of tube occupied by this volume is $(30/x) \times 4 = 120/x$ inches; of these, 4 inches were already taken up by the vacuum. The actual depression is therefore $(120/x) - 4$; but this depression is x itself. Hence $x = (120/x) - 4$; or $x = 9{\cdot}15$; and the mercury will stand in the tube at a height of 20·85 inches.

CHAPTER XIII.

HEAT.

Heat is a form of Energy. It would, perhaps, indeed be more correct to say that we designate under the one name Heat two totally distinct forms of Energy. The one of these is the energy of a wave-motion in the Ether, passing from a hot body to surrounding objects across the intervening space, as from the sun to our earth, or from a hot fire to the colder objects upon which it shines: this we call Radiant Heat. The other form is that of a confused oscillatory disturbance of the particles of a body: in virtue of this molecular movement a body may appear to our cutaneous sense of heat (a sense quite distinct from that of touch) to be more or less hot or warm; or in the converse case it may, on account of the small amount of this movement, appear to be relatively cool or cold. The latter form of heat may be called Sensible Heat, or Heat simply, and of it we shall proceed to treat in this chapter. It is the only form of heat for the perception of which we have special sense-organs. We do not directly perceive the undulations of radiant heat by our senses: when the sun shines on us heat-waves strike the skin, throw it into vibrations, and the sensible heat of the skin, not the radiant heat of space, affects the appropriate nerve-ends. When we touch a hot body it communicates its oscillations to the nervous system: when we approach a hot body we become indirectly sensible of the radiant undulations into which it is throwing the surrounding ether. Thus we may state that our sense of heat is our power of perception of the confusedly-vibrating condition of a body; and that the more pronounced this condition of agitation, the hotter will a body appear. A hotter body may be readily supposed—and rightly so if we confine our attention to bodies formed of the same substance—to have in it a greater amount of Heat than a colder one. And a hotter body can become cold, a colder body can become warm: heat can be supplied to bodies, or they can be deprived of it; heat can be gained or lost by material bodies. The primitive interpretation of this was that Heat was a substance, a fluid, the so-called Caloric, invisible, imponderable; that a piece of hot iron was a kind of temporary union of cold iron with this subtle imponderable fluid. When a piece of metal was rubbed it became warm: the reason assigned was that Caloric was squeezed out of it, like water out of a sponge. But this material theory of heat became untenable when it was shown that there was absolutely no limit to the amount of sensible heat which might be so produced by the friction of a trifling amount of metal; the amount of water that might be boiled, for example, by heat produced in this way depended only on the mechanical power available (Rumford). The heat evolved by friction—as, for instance, in metal boring or turning—is practically limitless. Even two masses of ice, caused to rub against one another, melt (Davy)—a fact which leads the material theory of heat into helpless confusion. Water was admitted to be ice *plus* caloric; if, then, ice with its caloric rubbed or squeezed out of it and lost—that is to say, ice *minus* caloric—become water, how can the theory stand? Plainly Heat is not material: it *is* the Energy imparted to the system—it is equal to the work done upon it; and we find that Heat and the other forms of Energy are reciprocally convertible.

When a body is sensibly hot its particles are in an active state of motion. The particles strike one another and rebound; the more rapidly they do so, the greater is the mean velocity of the particles, and the greater is the kinetic energy of the whole mass; but it is impossible that the energy of the molecules should be entirely due to such a movement of Translation. They are not material points, and they have—if not in solids or in liquids, yet certainly in gases—six degrees of freedom; when they strike each other they not only rebound but they also spin; to the energy of translation must be added one of Rotation. Further, the molecules are made up of atoms: atoms are not stationary in the molecule, but may be so violently agitated as to leave it altogether, and thus to give rise to the phenomena of chemical decomposition by heat; part of the energy of a heated body is due to intramolecular Atomic Oscillations. Lastly, the ether entangled in a molecule is also set in vibration, and absorbs some energy which appears as kinetic energy of Ether-Vibrations. The sum of these is found, by the agreement of experimental results with calculations based on the hypothesis that such is the law, to be proportional on the average—an average not perceptibly departed from for any appreciable interval of time—to the kinetic energy of translation alone.

Heat is not Motion, for it is neither Change of Position, nor yet Momentum; it is the Energy of Motion. Double the quantity of molecular Motion, and you quadruple the molecular kinetic Energy, that is, the Heat.

The convertibility or identity of heat with energy is independent of the inner mechanism of the moving molecules which possess it; and it is confirmed by instances from all sides.

The Energy of work which is apparently wasted in friction becomes Heat: the heating of a locomotive brake, the ignition of a lucifer match, the heat evolved during the mechanical operations of metal boring or turning, the heat found in a body which has received a sudden blow or a sudden distortion, or suddenly yielded to pressure,—all these prove the proposition.

If work be done in driving a paddle in water, no work being done other than that of churning the water, when the operation is over the work appears to have been wasted and to have disappeared; but the energy is not destroyed; it exists in the water in the form of heat. If 772·55 foot-pounds of work (measured at sea-level and latitude of Greenwich, Joule) be expended in churning a pound of water, the temperature of that water will be raised by 1° F., from 60° F. to 61° F.; a similar rise of 1° C. in a kilogramme of water will be effected by the expenditure of 423·985 kilogrammetres or 41,593,010,000 ergs of work. Hence the water at the base of Niagara Falls ought (setting aside the effect of evaporation and of cooling or heating by the air) to be about $\frac{1}{5}$° F. higher in temperature than at the top, for the vertical fall is 161 feet. Hence also the sailor's maxim that the sea is warmed by a storm.

When in a steam-engine at work the steam at its entrance to the cylinder from the boiler is compared with that which goes to the condenser, it is found that the latter is colder. The difference of heat is found to be equivalent to the work which the engine has done; and if the engine do no work, then the energy which has not been converted into work remains as heat in the outgoing steam, and the engine may become heated (Hirn).

When a quantity of gas or of liquid is forced through a tube, as in Fig. 109, the potential energy of the system before the flow is started is greater than the kinetic energy of the outflowing stream. If the resistance be so great that the velocity of outflow is practically null, the whole of the work done on the fluid is spent in heating it.[*] The work done is equivalent to the heat produced. We are now able to state the First Law of Thermodynamics. **Heat, being a form of Energy, can be measured in ergs**, in foot-poundals, or in foot-pounds.

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[*] This must be done at a pressure corresponding to a certain definite head H of the same fluid. The fluid is found to rise in temperature by x° C.; a head of H/x cm. would cause it to rise by 1° C.; a vertical free fall of H/x cm. would cause it, if abruptly stopped, to rise in temperature by 1° C.; the amount of energy corresponding to such a fall would be $(H/x).mg$ ergs; this energy in the form of heat $(H/x).mg$ ergs, would heat a mass m of the fluid through 1° C.; $(H/x).g$ ergs would heat one gramme of the fluid through 1° C.; $(H/x).(g/k)$ ergs (k being the *specific heat* of the fluid, p. 339) would heat one gramme of *water* through 1° C.

This law is usually stated in a somewhat different form. An arbitrary unit of heat is chosen, and designated a *calorie:* this is the amount of heat which is required to raise the temperature of one gramme of water from 0° C. to 1° C. This quantity of heat is found to be 41,593,010 ergs. This last number, 41,593,010 ergs is the "Mechanical Equivalent of Heat," or "Joule's Equivalent:"* it should perhaps be called the Dynamical Value of the Conventional Unit of heat, the calorie. The first law is, then, that one calorie (*ca*) is equal to 41,593,010 ergs or 41·59301 megergs.

Another unit of heat has been proposed, the Electromagnetic Unit, or 10,000,000 ergs; this is the amount of heat developed in one second in an electrical circuit whose resistance is one Ohm when a current passes whose intensity is one Ampère. (See p. 592.)

Heat is energy, and it is the lowest form of energy. It may be said to have no organisation, but to depend on undirected and blind activity of molecules, which dash hither and thither. When in any action energy is liberated which is not guided by the environment into any specialised form, it manifests itself as heat; and when energy is spent in doing work, the equivalent of which appears in no other form, it then appears as heat. This statement is widely applicable and important.

Work done upon a dynamoelectric machine whose circuit is complete appears in the first place as the energy of an electric current: if no exterior work be done, the system as a whole becomes heated.

A voltaic cell can do exterior work: if it do none, the current being allowed to circulate uselessly, the whole of the energy liberated during the chemical combination appears as heat in the circuit.

Heat being a form of energy, many propositions relating to it are merely special cases of propositions relating to energy.

If a certain number of bodies be arranged in a system A whose potential energy—depending on the arrangement of the bodies in the system—is P_A; if the same bodies can be arranged in other systems B, C, D, whose respective potential energies (less than that of the former) are P_B, P_C, P_D, etc.: then the transformation of the more highly-stressed system A into a less-stressed system B, if this be brought about by a rearrangement of its constituent bodies, involves a liberation of energy equal to $P_A - P_B$. If in this case the system A be converted into the system B without doing any exterior work, the whole of the energy liberated appears in the form of heat; and, numerically expressed, the heat thus liberated is equal to the work W which would have had to be done upon the system B in order to convert it into the system A, if that converse operation had been effected; that is, the heat so liberated is equal to $W = P_A - P_B$.

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* Joule's Equivalent in its original form was a number (772) which denoted the number of foot-pounds of work found to be equivalent to the heat necessary to raise 1 lb. of water through 1° F.

A gramme of hydrogen and eight grammes of oxygen form a system (system A) which after explosion may be converted into nine grammes of water-vapour of the same volume (system B) at a temperature of 136°·5 C. The former is converted into the latter without doing any exterior work. Much energy is liberated in the form of heat, and though the absolute values of P_A and P_B are unknown, their difference is found (see Calorimetry) to be an amount of energy equivalent to 28,580 *ca*.

If the products be cooled down to steam at 100° C., the total amount of heat liberated is equal to 28,738 *ca*, or 1,195300,000000 ergs; if to water at 0° C., it is equal to 34,462 *ca*, or 1,433,378,310000 ergs, or 1,433378·31 megergs. The potential energy which such a mixture loses when its particles clash together and combine is the energy of chemical separation. A mixture of explosive gases may be made to yield up some of this energy in the form of work, as in the modern gas-engine; if no work be done, and if there be no other transformation, the whole of it must appear in the form of heat.

Chemical combination is thus often attended with the evolution of heat. One gramme of carbon burned in oxygen yields 8,080 *ca* or 336,071,520800 ergs; 1 gramme of carbonic oxide yields 2,403 *ca* (2,431 *ca*, Andrews); 1 gramme of marsh-gas, 13,063 *ca*; 1 gramme of dry albumen, 4,998 *ca*; urea, 2,206 *ca*; fat, 9,096 *ca*; starch, 3,901·2 *ca*, or 162,262,650,612 ergs per gramme (Frankland).

When copper or antimony is dropped into chlorine it takes fire, and a chloride is formed: heat is evolved.

In some instances the converse is true; work has to be done upon separate elements in order to force them directly or indirectly to combine: and when their compound decomposes, heat is evolved. Carbon and sulphur will only combine when they are kept hot by an external source of heat: they must be forced to combine. Nitrous oxide (N_2O) evolves heat when it is decomposed into nitrogen and oxygen; and hydrogen dioxide (H_2O_2) evolves heat when it is decomposed by contact with platinum.

A change from the condition B to the condition A (which possesses more potential energy) cannot be effected unless there be energy added *ab externo*, or else unless some of the kinetic energy of the body, if it have any, assume the potential form; in the latter case the body may lose sensible heat, and may become cold.

When a chemical decomposition is effected by heat, if heat had been evolved during the formation of the compound, heat must be continuously tions, and to effect the reverse operation by another series, such that the one series of changes would evolve more energy than the converse one consumed, and the result would be a repeated restoration of the *status quo*, associated with a perpetual supply of energy, available for useful work, and created out of nothing.

The energy absorbed by a system during a given change of state is exactly equal to that which is liberated when the change is reversed.

It is assumed in this statement that no exterior work is done through the instrumentality of the change of state.

The potential energy of every system of bodies always tends to diminish as far as possible. Every system which possesses potential energy thus tends to lose it; its potential energy tends to become kinetic, and, if it assume no other form, to take the unspecialised form of heat. In any system which undergoes spontaneous transformation, the transformation generally tends, unless prevented, to take such a course that the heat evolved by it shall be a maximum.* If there be any such transformation possible, which would be accompanied by the evolution of heat, that is a necessary transformation, and sooner or later it will take place, directly or indirectly, the potential energy of the system being added to the unavailable heat of the universe.

In many cases a single change of state may be analysed into several others. The heat-value of the total change is equal to the sum of the heat-values of the separate component changes.

Thus when a piece of sodium is put into water the following changes occur simultaneously:—(1) decomposition of water into free atoms of hydrogen and oxygen; (2) coalescence of atoms of hydrogen to form molecules; (3) reduction of hydrogen to the gaseous state; (4) exterior work done by the hydrogen escaping against atmospheric pressure; (5) combination of sodium with oxygen and hydrogen atoms to form sodium hydrate; (6) solution of sodium hydrate in water. Each of these changes has its own heat-value, positive or negative, according as it involves the evolution or the absorption of a certain amount of energy. On the whole, potential energy is lost and heat is liberated.

The combustion of 8 grms. oxygen with 1 grm. H. yields 34,462 *ca* heat. The same quantity of the same elements combining in the nascent state yields 54,623 *ca*. Hence the heat evolved during the combustion of one gramme of hydrogen is the resultant of an absorption of energy (20,161 *ca*) due to the break-up of the gaseous molecules into atoms, and an evolution

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* Berthelot's *Mécanique Chimique* contains the very interesting and important results of sixteen years' experiments on this aspect of chemical physics. M. Berthelot has thrown great light on the subject, and he gives very complete references to the papers of Thomsen and others who have studied chemical reactions from this point of view.

(54,623 *ca*) due to the combination of these atoms in the formation of water-molecules and condensation into liquid water at 0° C. The balance of the account shows energy to be liberated as heat.

When a gas is dissolved in water there are two effects:—(*a*) liquefaction of gas with evolution of heat; (*b*) satisfaction of chemical affinity between the water and the gas, with the evolution of still more heat. When NH_3-gas is dissolved in water, there is no evolution of heat corresponding to any union of NH_3 and H_2O to form NH_4HO.

When a solid is dissolved in water the liquefaction of the solid causes the absorption of heat (as in freezing mixtures), while the satisfaction of mutual chemical affinity causes its evolution. When glacial acetic acid is dissolved in water, the absorption of heat caused by imparting greater fluidity to the acetic acid overpowers the evolution of heat due to chemical union.

When two or more changes of state occur concurrently, it may be that some of these changes are accompanied by the liberation, some by the absorption of heat, and that these changes exactly compensate each other; the result being that on the whole there is neither absorption nor liberation of heat.

For example, a gas, while expanding (from whatever cause), against the atmospheric pressure, does work in lifting the atmosphere; if it increase in volume alone, without undergoing any change in its temperature, energy must be supplied to it in order to enable it to do this work; if it diminish in temperature without suffering any change in its volume, it must necessarily lose heat; if, on the other hand, it undergo both these changes—increase in volume and diminution of temperature—concurrently, it is possible that these two changes may be so adjusted that the body, while it undergoes the double change, neither loses heat nor acquires energy from without. Such expansion is called (Rankine) **adiabatic** expansion—expansion during which the substance neither gains nor loses heat by conduction or radiation to or from surrounding objects, and in the course of which, as it expands, it cools down. This is a kind of operation which could only be perfectly realised in practice if the expansion were infinitely rapid; but any gas suddenly expanded is thus chilled. Conversely, adiabatic contraction of volume is associated with increase of temperature.

We have hitherto regarded any change of state, simple or complex, as a possible antecedent cause of the liberation or of the disappearance of heat. We shall now change our standpoint, and consider the effects (including change of state or of condition) produced *by* the increase of heat in a body or by its withdrawal.

EFFECTS OF HEAT.

The principal effects of an increase of heat in a body may be the following:—

A. **Internal Work.**

- *a.* Increase of the kinetic energy of the molecules of the body—an increase of the sensible heat of the body; *i.e.*, an increase of *temperature*.
- *b.* *Intermolecular* work—work done by or against molecular forces—change of volume, change of cohesion, change of elasticity, etc.
- *c.* *Intramolecular* work—work done within each several molecule—production of intramolecular vibrations.
- *d.* *Chemical* work, intermolecular and intramolecular.

B. **External Work.**—Work done by or on a body as it expands or diminishes in bulk.

These effects are not necessarily all produced by the action of heat upon any substance.

There may, as in the following example, be no external work done when a body is heated; the whole energy imparted to the body being spent upon the internal accumulation of energy in the form of heat. Water at 3°·4 C. if heated to 4°·4 C. first contracts and then returns to its original dimensions. On the whole there is in this case no external work done. Neither is there any work done in giving the particles a new position in opposition to the intermolecular forces,* nor is there any chemical effect. The whole heat imparted may thus be held to be spent in raising the temperature by 1° C.

When a bar of iron is heated in a vacuum there are two effects: (1) increase of temperature; (2) expansion of the iron, which represents work done against the molecular forces. When the same bar is heated in air, there is added a third effect, viz., the thrusting aside of the surrounding air by the expanding bar, in consequence of which exterior work is done during expansion. In a bar of iron the exterior work done in this way is very small, and the interior work done predominates so largely that the exterior work may for many purposes be neglected.

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* This statement is only approximately true, for there are physical differences—of viscosity and the like—between water at 3°·4 C. and water at 4°·4 C. The temperature of the maximum density is lowered 0°·0177 C. per atmo. pressure (Tait).

When a mass of gas is heated, the work which is done in expanding the gas itself is appreciably null, for this is one of the characteristics of gases; if there be any work done during the expansion, it is all exterior. The effects in this case are two: (1) the increase of temperature; (2) exterior work done in overcoming the exterior (atmospheric or other) pressure.

When water above 3°·9 C. is heated it expands. The effects are—(1) increase of temperature; (2) work done in separating the molecules; (3) a small amount of work done *against* the external pressure.

When water between 0° and 3°·9 C. is heated it contracts. The effects are—(1) increase of temperature; (2) intermolecular work; (3) a small amount of work done *by* the external pressure.

When a piece of caoutchouc is heated it contracts; when pulled it expands and assumes the dimensions proper to a lower temperature, intermolecular energy is set free, and the caoutchouc becomes warm. A piece of metal suddenly extended becomes cool.

When ice at 0° C. is heated the whole energy imparted to it is expended in producing the following results:—(1) Fusion, with contraction of volume (intermolecular work—work spent in producing a new arrangement of the molecules); (2) A slight amount of work done *by* the exterior pressure on the body. The latter may be for most purposes neglected; if we do so we may say that all the heat supplied to the ice is spent in doing the interior work of liquefaction, and that none of it is spent in producing an increase of temperature. When, therefore, a piece of ice is heated it melts, but it does not rise in temperature until it has been wholly melted. The water produced has a temperature of 0° C., and it does not begin to rise in temperature until the ice has entirely disappeared: when this has occurred, the continued action of heat causes the water to rise in temperature.

A gramme of ice at 0° C. absorbs 80·025 (Bunsen) *ca* of heat, and becomes a gramme of water at 0° C. Conversely, a gramme of water at 0° C. must continue to lose heat until it has parted with 80·025 *ca* before it can become a gramme of ice at 0° C.; whence we observe that water does not freeze throughout at the instant of the thermometer's touching the freezing point.

It was obvious that a gramme of water differed from one of ice in somehow possessing 80·025 *ca* of heat; but this was not sensible to the thermometer; hence the heat so possessed by the water was said to be hidden or **Latent Heat.** We now know that it is not Heat of any kind; it is latent or potential Energy; work must be done against molecular forces in order to convert ice into water: water somehow differs from ice at the same temperature in possessing more potential energy.

Direct increase of the kinetic energy of the particles of a heated gas is demonstrated by the Radiometer.

If a surface be heated, a molecule of gas striking against it is heated; it leaves the hot surface with a velocity greater than that with which it had approached it. If the surface be fixed, the gas in front of it is driven away from it by the bombardment of the molecules which have touched the hot surface, and on their return strike their fellow-molecules; in front of the hot surface the gas is therefore under a greater pressure than it would have been had the surface been cold. If the hot surface be not fixed, this increase of pressure has—reaction being equal and contrary to action—a tendency to drive that surface backwards.

If the hot surface be the front aspect of a disc, the back of which is by some means kept colder than the front, and if this disc be suspended in a gas, the heat of the front surface increases the pressure towards the front, and the gas flows round to the back of the disc. Thereafter the disc is struck on the hotter surface by fewer molecules with greater velocities, on the colder surface by a greater number of molecules with lesser velocities; there is thus compensation; the result is that the disc is equally pressed upon in front and on the back; it does not move.

Let us now suppose that the particles recoiling from the heated surface do not meet other molecules, but impinge on the walls of the vessel. A layer of particles in such a condition is called a Crookes's layer.

This will occur in two cases—(1) when the gas is so rarefied that the mean free path of the molecules exceeds the distance between the hot surface and the walls of the vessel; (2) when, whatever the density of the gas, the opposite wall is so near the hot surface that the distance between them is less than the actual mean free path of the molecules. These conditions, which are substantially identical, may concur: there may be both rarefaction of the gas and approximation of the opposed surfaces.

In such a case there is no flow of gas from the hotter surface towards the colder one: each molecule which strikes the hotter surface and rebounds with a greater speed adds independently to the recoil which the hotter surface suffers, and if the hotter surface be movable, it is driven backwards. If it be not movable, the particles which rebound from it strike the opposite wall of the containing vessel, and that wall has a tendency to move forward.

There is yet another case: if the rarefaction of the gas be extreme, the particles which strike the heated surface are few in number or none at all, there is little or no recoil, and there is no movement set up when the rarefaction is carried too far.

The disc of which we speak—a disc of which one face is kept hotter than the other—may be a disc covered on the one side with some heat-absorbent material such as lampblack, the other face being whitened. When radiant heat or light falls upon the disc, even in an equally-lighted field, the blackened side becomes hotter. If such a disc be suspended vertically by two threads, it will diverge slightly from the perpendicular. Such a disc may be attached to the end of a counterpoised rod, the whole being suspended by two threads: the effect of heat or light is to twist the suspending threads to a certain extent. If the suspensory arrangement be replaced by a pivoting one, we have the **Radiometer.** A globe of glass, in which a vacuum is made, carries a vertical needle axially fixed, on the summit of which is poised a rotating vane consisting of light rods, to the extremities of which discs are affixed, each similarly blackened on one side. Such an instrument placed in light, even a uniformly-lighted field, has the black sides of its discs more heated than the unblackened sides, and if the radiance be of sufficient energy the vane rotates. Moonlight is too weak to produce this effect: a candle will make a sensitive radiometer rotate; a paraffin lamp without a globe will at close quarters make the vane fly round so fast as to be invisible.

If a radiometer be floated in water, and if the vane be so constructed—one of its spokes being a magnet—that a powerful magnet in the neighbourhood can hold it motionless, when the radiometer is exposed to light the bulb itself will rotate in the water in which it floats.

The radiometer is a machine in which heat (generally derived from the transformation of light into heat) is directly converted into the energy of work.

The less the distance between the discs and the walls of the bulb, the greater will be the effect, and the faster will the vane rotate, provided that the rarefaction is less complete than that which gives the greatest effect. Too complete a rarefaction is not an advantage, for it leaves an insufficient supply of working molecules.

When the distance between the disc and the opposite wall is excessively small, the vacuum need not be very good; indeed the effect of repulsion may be made manifest even in the open air.

When a drop of water is placed upon a very hot iron it assumes the so-called **Spheroidal State**; it does not wet the hot iron, but gathers itself into a drop, which rapidly evaporates and alters the local conditions of its surface-tension so as to present an appearance of varying scroll-work on its surface, while the drop oscillates so as to present the form of rosette and other patterns, these being due to the formation of nodes and vibrating loops. The drop may be of very considerable dimensions—several ounces in weight. It does not touch the iron; there is an intervening layer of aqueous vapour on which it floats; through the space intervening between the drop and the hot solid the light of a candle may be seen. This layer of aqueous vapour is a "Crookes's layer;" particles strike the heated surface, rebound, and strike the liquid, thus maintaining a clear space between the metal and the drop. Ether and small drops of bromine float in the same way on the surface of hot water. A lump of carbonate of ammonia thrown into a red-hot platinum crucible assumes the spheroidal state superficially, but does not melt. The hand can be safely immersed in melted metal if it be not too dry, and if the immersion be effected with a certain degree of prompt deliberation; a Crookes's layer of water-vapour intervenes between the hand and the metal.

When liquid sulphurous acid is dropped into a white-hot platinum crucible it sinks greatly in temperature on account of its rapid evaporation and its slow reception of heat across the Crookes's layer; if a little water be added to it the water freezes. Ice can thus be produced in a white-hot platinum crucible. A similar Crookes's layer is formed if a quantity of solid carbonic dioxide be *lightly* placed on the tongue; the extreme cold ($-80°$ C.) is not felt.

When the hot solid-body cools down, the Crookes's layer disappears, the liquid suddenly comes in contact with the solid still relatively hot, and the liquid explodes in vapour. This occurs in the case of water and iron at about 180° C.

Melted copper can be cast under water in a canvas mould; and, singularly, it often remains fluid so long and cools down so far in that condition that there is no explosion.

Increase of Temperature.—We have freely made use of the term temperature because it is a term in common use, and not likely, so far as we have used it, to lead to ambiguity. We have still to defer the consideration of thermometry; but we must now consider increase of temperature as directly due to increase of the molecular kinetic energy of a body. When we double the Molecular Kinetic Energy of a hot body we double its Temperature.

Observe that it is not asserted that we double the temperature when we double the total energy of a body: some may disappear in doing work, and take the form of the so-called latent heat.

This implies that there must be some point of Absolute Zero of Temperature, independent of the conventions of Fahrenheit, Celsius, and others, afterwards to be explained—a point of Absolute Cold, beyond which no cooling is conceivable.

We have already seen that in a perfect gas—one in which there is no complication due to inter-molecular forces—the pressure is proportional to the molecular kinetic energy of a given mass, occupying a given volume; the temperature is, or may by definition be held to be, also proportional to this kinetic energy; it follows that the Temperature is proportional to the Pressure when the volume occupied by a given mass remains unchanged. It is found that in all gases the pressure diminishes by about $\frac{1}{273}$ for each centigrade degree of cooling, the temperature of 0° C. being the starting point, and the volume being maintained constant. If a gas could be cooled down in this way to -273° C. (a feat unachieved), it would have no pressure and therefore no temperature, for it would have no kinetic energy, no heat. The Absolute Zero of temperature is therefore -273° C. (or more accurately $-273^\circ{\cdot}72$ C.), and the Absolute Temperature of a body whose temperature, as measured by the centigrade thermometer (see p. 370), is x° C., is $(273+x)^\circ$ Abs.; thus the boiling point of water, 100° C., is 373° Abs.

Specific Heat.—The heat-energy of a molecule of hydrogen is equal to that of a molecule of oxygen at the same temperature; but the latter weighs sixteen times as much as the former, and a mass of hydrogen contains sixteen times as many molecules as an *equal mass* of oxygen under similar physical conditions. Hence a given mass of hydrogen at a given temperature possesses sixteen times as much heat-energy as an equal mass of oxygen at the same temperature.

To produce a given rise in the temperature of a mass of hydrogen we must supply sixteen times as much heat as we would find necessary to produce an equal rise of temperature in an *equal mass* of oxygen; the Specific Heat or thermal capacity of hydrogen is sixteen times that of oxygen.

In general, the lighter the molecules of which a substance is made up, the more numerous must they be in a given mass, and the higher the specific heat of the substance, *i.e.*, the more heat must be expended upon it in producing a given rise of temperature.

For reference and comparison a standard of specific heat is necessary; the specific heat of water is chosen as the standard. One calorie will raise the temperature of one gramme of water from 0° C. to 1° C.; the specific heat of water is unity. The relative specific heat of any other substance is, accordingly, the fractional number of calories of heat required to raise one gramme of the substance from 0° C. to 1° C.

The law here indicated—that the specific heat of an element varies inversely as its atomic weight—is based on the assumption that a mass of a heated substance behaves like a group of isolated molecules which have no action on one another. It is not surprising, accordingly, to find, when heat supplied to a body is spent not only in raising the temperature of a body, but also in doing internal and external work, that the law is only approximately obeyed. Still the approximate obedience is sufficiently striking to have caused Dulong and Petit to enounce it as a law, and as such it bears their name. It has been utilised as one means among others of ascertaining the atomic weight of different elements.

For the formula "$\text{sp. heat} \propto \frac{1}{\text{at. wt.}}$" we may substitute $\text{sp. heat} = \frac{\mathit{const.}}{\text{at. wt.}}$; or $\text{sp. heat} \times \text{at. wt.} = \mathit{const.}$ This constant product, which bears the name of Atomic Heat, is about 6·4; the metals, phosphorus, sulphur, may be said to form a group in which it varies from 5·86 to 6·93. Divergences from the average value are most marked in the case of solid bodies. In the case of carbon, silicon, and boron at ordinary temperatures the product is small, being about 3·3; but at higher temperatures the specific heat of these substances increases so that the product rises to about 5·5.

The Molecular Heat of a compound is approximately equal to the sum of the atomic heats of its component elements: this rule applies with tolerable accuracy to gaseous compounds formed without condensation; solid and liquid compounds, and even gaseous compounds whose formation from their elements is accompanied by condensation,—compounds which in some sense approximate to the liquid state,—depart from it to a marked degree.

This product—the **atomic heat** of elements, the **molecular**

heat of compounds—has the following physical meaning. Of any substance whose atomic or molecular weight we know we may take a number of grammes numerically equal to the atomic or molecular weight; for example, 35·5 grammes of chlorine, 16 grammes of marsh gas; we may call such a quantity the gramme-atom or the gramme-molecule of the substance. The Atomic Heat or the Molecular Heat of a substance is the number of calories of heat necessary to raise the temperature of a gramme-atom or of a gramme-molecule of the substance through 1° C. The atomic heat of elementary substances is approximately the same—another form of Dulong and Petit's law.

The specific heat of a substance determines the temperature which it will assume when a definite quantity of heat is supplied to it or liberated in it.

Thus when 1 grm. of hydrogen and 8 of oxygen are exploded together, but not allowed to expand in volume, 28,580 *ca* of heat are liberated. If we could assume the action to be instantaneous, we might assume that none of the heat is lost. The 28,580 *ca* would then be divided among 9 grammes of water-vapour whose sp. heat at constant volume is 0·37; the temperature attained would be $\frac{28580}{9\times0\cdot37}=8883^\circ$ C. above the temperature (136°·5 C.) proper to a volume equal to the original volume of the mixture. This case is instructive as showing the influence of dissociation; for when a temperature of 3000° C. is actually attained, further combination becomes impossible, and the action is arrested, but not wholly, for it is gradually completed *pari passu* with the loss of heat by conduction or by radiation. If, however, the exploding mixture be allowed to expand, doing external work, the temperature of 3000° may never be attained, and the action may suffer no such check.

Where a substance while being heated is not allowed to expand, there is probably no internal work done; neither is there any external work done; all the heat supplied is applied in raising the temperature. The specific heat in this case is specially known as the **specific heat at constant volume.** If, however, the substance be allowed to expand while it is being heated, an external pressure being maintained, both external and internal work are done, and in order to effect a given increase of temperature more heat-energy is required than in the former case. The **specific heat** of any particular substance **at constant pressure** is therefore greater than that at constant volume, and it is found in gases to exceed it in the proportion of 1·4058 : 1.

The ratio $\left(\frac{1\cdot4058}{1}=k\right)$ of the specific heat at constant pressure to that at constant volume may be found in two ways.

I. Theoretical considerations lead to the conclusion (see Baynes's *Thermodynamics*, p. 134) that if a gas suddenly exchange its pressure p, its density ρ, and its absolute temperature t, for others $p_{,}$, $\rho_{,}$, $t_{,}$, the ratio of its specific heats being k—

$$k = \frac{\log \frac{p}{p_{,}}}{\log \frac{\rho}{\rho_{,}}} = \frac{\log \frac{t}{t_{,}}}{\log \frac{\rho}{\rho_{,}}} + 1 = \frac{\log \frac{p}{p_{,}}}{\log \frac{pt_{,}}{p_{,}t}}.$$

Whence if two of the changes p to $p_{,}$, ρ to $\rho_{,}$, t to $t_{,}$, can be found, the value of k may be calculated. The experimental adiabatism necessary is very difficult to ensure; yet Röntgen has performed the following series of operations upon known quantities of gas and determined the value $k = 1{\cdot}4053$.

1. Gas in a reservoir at a pressure p exceeding the atmospheric, at density ρ, and temperature t° Abs.

2. Open a stopcock: air rushes out of the reservoir till the pressure p falls to Π, the atmospheric pressure.

3. *Immediately* close the stopcock. The air within the reservoir is at pressure Π, but has been cooled by doing external work during expansion; it comes to the same temperature as surrounding objects—that is, again t° Abs.: it now has the pressure $p_{,}$ and the density $\rho_{,}$, which can be found at leisure, and the above formulæ applied.

II. From the velocity of sound in air. This is 33,200 cm. per sec. Newton's law of the velocity of propagation of waves is that $v = \sqrt{K/\rho}$. The coefficient of elasticity K is equal numerically to the pressure in a gas *if the temperature be constant*; $\therefore v = \sqrt{p/\rho} = \sqrt{\Pi/\rho}$ if the pressure be the atmospheric. For air $\rho = 0{\cdot}012{,}932$ grms. per cub. cm. at 0° C. and 76 cm. bar. pr. at Paris; $\Pi = 1{,}013{,}660$ dynes per sq. cm. Hence, according to Newton's law, $v = \sqrt{\frac{1{,}013{,}660}{0{\cdot}012{,}932}} = 27997$ cm. per sec.; but in fact it is found to be 33,200 cm. There is here to all appearance a material divergence from Newton's law; but it is corrected when we observe that the assumption that the temperature is constant is unfounded; that a travelling wave of sound subjects the air to adiabatic compression—adiabatic * because the heat has not time to become diffused; that the elasticity of air so compressed is greater than that of air maintained at a constant temperature; that the ratio of these two elasticities of a gas is otherwise known to be the same as the ratio of their specific heats at const. pr. and at const. vol.; and therefore that the coefficient of elasticity in the formula should have been, not K the elasticity at const. temp., but $k \times K$. Whence $v = \sqrt{kK/\rho}$; $33{,}200 = 27{,}997 \sqrt{k}$; $k = 1{\cdot}40622$.

The mean value of k is thus $1{\cdot}4058$.

In a **perfect gas**—one whose molecules did not act upon one another—the specific heat at constant volume would be quite independent of the temperature or of the pressure.

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* If the heat produced had time to become diffused, or if, as might be the case in excessively slow vibrations or rare gases, the gas had time to flow round the vibrating object, so that it could not become compressed or evolve heat, the speed of propagation would tend to approximate to the value $\sqrt{K/\rho}$.

In air the specific heat is sensibly, though not perfectly, constant at all temperatures between −30° C. and +225° C., and at pressures from 1 to 10 atmospheres. We shall see that this justifies us in relying upon the indications of the air thermometer.

In a perfect gas the pressure at constant volume and the volume under constant pressure would both vary directly as the temperature.

The general law is $pv \propto t$, or $pv = \mathrm{R}t$, where R is a constant; this constant is otherwise known to be numerically equal to the difference between the two specific heats of the particular gas. Therefore $pv = \mathrm{R}t = (kc - c)t = 0{\cdot}4058\,ct$, where c is the sp. heat at const. volume. Hence $c = pv/0{\cdot}4058t$; and in this expression t may be measured in arbitrary units, say centigrade degrees; then the value of c, the number of calories required to heat one gramme through one arbitrary unit of temperature, varies with the unit so employed.

In atmospheric air, when $p = \Pi = 1{,}013{,}660$ dynes per sq. cm., the volume of one gramme is $v = 773{\cdot}2833$ cub. cm. at 0°C. or 273°·72 Abs. Then $c = 1{,}013660 \times 773{\cdot}2833 \div (0{\cdot}4058 \times 273{\cdot}72) = 0{\cdot}1719$ *ca* per C.° per gramme. The observed value of c is 0·1684 *ca* per C.° per gramme.

When a gas is compressed it becomes heated—that is, provided that external pressure have produced the compression, and added energy to the gas by doing work upon it.

When a gas is allowed to expand it becomes cool—that is, provided it expand against external pressure and sacrifice energy by doing external work.

The work done upon or by the gas appears or is lost as heat. The rise of temperature may be calculated, on the express assumption that there is no internal work done—an assumption approximately but not perfectly true (Joule)—by dividing the whole work done on the gas (measured in terms of calories of heat) by the mass and by the specific heat of the gas.

Saturated vapour behaves in this regard in a peculiar manner. If work be done upon saturated steam at any temperature below 789°·8 Abs. (516°·8 C.), the heat evolved causes the vapour to become a superheated vapour, and heat must be parted with in order to allow the steam to remain saturated. Conversely, if saturated steam below 516°·8 C. be allowed to expand, doing external work while no heat is supplied to it, it loses energy, loses latent heat, and is partly condensed; and it does not fall in temperature as much as it would do if it were a perfect gas, expanding to the same extent, for the liquefaction of the vapour liberates heat. Thus an expanding saturated vapour, such as steam, liberates more energy and can do more work than an expanding gas. Above 516°·8 C. a sudden adiabatic expansion of saturated steam would, on the other hand, produce evaporation of water in contact with it; and compression would produce condensation.

The above facts are comprised in the equation—S being the specific heat of saturated steam, t the abs. temperature—

$$S = \left(42{,}136{,}000 - \frac{33{,}280{,}000{,}000}{t}\right) \text{ in ergs ; or}$$

$$S = \left\{\left(42{,}136{,}000 - \frac{33{,}280{,}000{,}000}{t}\right)\frac{1}{41{,}593{,}010}\right\} \textit{ca} \text{ per gramme.}$$

Below 789·8° Abs. S is negative ; above that temperature it is positive.

The vapour of bisulphide of carbon acts at ordinary temperatures like that of water below 516°·8 C. ; that of ether, on the other hand, is rendered cloudy by compression even at ordinary temperatures.

The specific heat of substances is not perfectly constant at all temperatures: whence the necessity of the qualification "from 0° to 1° C." This want of constancy is, among gases, most remarkable in those which are most condensible; but among solids and liquids the variations of specific heat are still more remarkable, and indicate differences in the amount of internal work associated with changes of temperature at different temperatures, this internal work being done in effecting changes in the density, the intermolecular stresses, the allotropic form, and so on.

The specific heat of a body may be expressed by the fraction—

$$\frac{\text{Increment of heat supplied (measured in calories) to unit-mass}}{\text{Increment of temperature produced}},$$

where both the increments are very small: if an amount of heat δH produce a change of temperature δt, the specific heat is $\delta H/\delta t$; and this is one of the **Thermal capacities** of a body, of which six may be distinguished.

1. Specific thermal capacity per unit-increase of temperature at constant pressure; the amount of heat required to raise the temperature of unit-mass by 1° C., the pressure being constant. This is called the Specific Heat at constant pressure, K.
2. Specific thermal capacity per unit-increase of pressure effected, the temperature being constant. This has no special name.

3. Specific thermal capacity per unit-increase of pressure effected by heat, at constant volume. This has no other name.
4. Specific thermal capacity per unit-increase of volume, the pressure being constant.

5. Specific thermal capacity per unit-increase of temperature at constant volume (the Specific Heat at constant volume, k).
6. Specific thermal capacity per unit-increase of volume, the temperature being constant. This is called the Latent Heat of Expansion, l.

Under No. 5, heat $= k$ units supplied to unit-mass, produces a unit-increase of temperature ; a rise of temperature $\dot{t}$ is produced by heat $= k\dot{t}$ units. Under No. 6, in the same way, heat $= l\dot{v}$ produces an increase of volume $\dot{v}$. We commit no sensible error if we suppose that when the temperature and volume both vary, the amount of heat which must be supplied to a unit-mass of substance is found by simple addition, and is equal to $k\dot{t} + l\dot{v}$.

Internal Work.—If any substance were a perfect gas, heat imparted to it would to no extent be spent in doing internal work against intermolecular forces. There is, however, no such perfect gas, as we shall now show.

If our physical gases were perfect gases we would find—

1. That the amount of heat evolved on compressing a gas would be exactly equal (when measured in ergs) to the work done in compressing the gas.

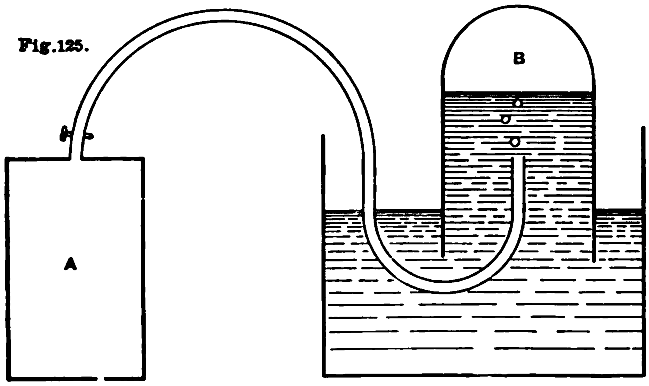
If the original pressure and volume be $p_\circ$, $v_\circ$, and the new volume be $v_{,}$, the work done is $p_\circ v_\circ \log (v_{,}/v_\circ)$: a conclusion deduced from the two equations—

$$(1) \text{ work done} = \int_{v_\circ}^{v_{,}} p\,dv\,; \text{ and } (2)\ p_\circ v_\circ = p_{,} v_{,} \text{ (Boyle's law).}$$

2. That when a gas expands doing external work, the gas loses energy; and that a perfect gas would in this way lose heat exactly equal in amount to the external work done, and would accordingly sink in temperature.

A vessel, A, of compressed air (Fig. 125) is provided with an exit tube furnished with a stopcock: the extremity of the exit tube dips under water in a bell jar B. The stopcock is opened; air flows out; it replaces the water in the bell jar: in so doing it forces water down against the atmospheric pressure: it thus does work; the air remaining in A becomes cold (Joule).

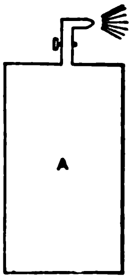
Fig. 125.



A similar vessel of compressed air (Fig. 126); the extremity of the exit tube communicates with the open air. The stopcock is opened; air flows out; it thrusts aside the air immediately surrounding the orifice; the air within A thus does work against the atmospheric pressure: the air remaining in A becomes cold.

3. That if a stream of a perfect gas were checked, the whole kinetic energy lost by the gas would appear as heat in it.

Fig. 126.



The heating effect of checking a stream of gas may be readily shown

(Verdet) by pinching a rapidly-issuing jet of air between the finger and thumb, or by partly blocking it with the finger-tip.

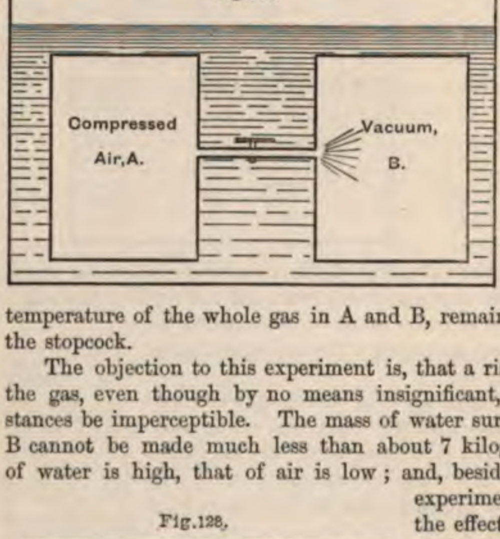
In the same way a jet of high-pressure steam when liberated into the free air suddenly expands and partly condenses into scalding droplets ; then a little way farther on, by reason of friction against the air and of intermolecular friction, it is deprived of its momentum, and is heated so far as to become superheated or gaseous steam ; still farther on it again becomes opaque.

If the vessel A of Figs. 125 and 126 be connected with another in which a vacuum has been produced, the air in A loses energy and is cooled. The part of the gas which first arrives in B is heated by compression exercised by the part which arrives afterwards ; the latter is also heated by having its motion checked : the temperature in B thus becomes higher than the original temperature.

4. That expansion of a perfect gas would not, if no external work were done, affect its temperature : for no internal and no external work being done, the amount of kinetic energy possessed by the gas would remain unaltered, and the temperature would be unchanged. There is no gas whose temperature remains unaffected under such circumstances; therefore there is no perfect gas.

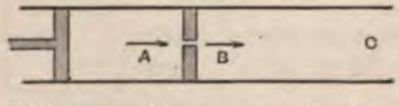
The apparatus of Fig. 127 being immersed in a large vessel of water, the stopcock is opened ; the air in A is cooled, that in B is warmed ; the amount of heat-energy gained by B is equal to that lost by A : the water surrounding A and B (which must be stirred) is not on the whole perceptibly cooled or warmed. This experiment, made by Joule, was believed to show that air did behave approximately as a perfect gas ; for the temperature of the water, and therefore the average temperature of the whole gas in A and B, remains unchanged after opening the stopcock.

Fig.127.



The objection to this experiment is, that a rise or fall of temperature in the gas, even though by no means insignificant, would under such circumstances be imperceptible. The mass of water surrounding the vessels A and B cannot be made much less than about 7 kilogrammes : the specific heat of water is high, that of air is low ; and, besides, it is desirable that the experiment be continuous, and that the effects, if there be any, be accumulated.

Fig.128.



Hence a new form of the experiment was devised by Joule and Thomson. A tube obstructed by a diaphragm with a narrow orifice takes the place of the vessels A and B. Air is forced from A, Fig. 128, towards B. The pressure within A is greater than that within B; the gas which passes into B ultimately becomes simply the same gas with a larger volume: it cannot become cooler by reason merely of its thrusting the exterior air at C out of the tube, for it simply acts as a buffer between the air in A and the exterior air at C, and the exterior work which it does is equal to that done upon it. If the air were a perfect gas, the temperature at B would be the same as that at A. It is found in such apparatus to vary from spot to spot on account of eddies; these must be got rid of. This is done by substituting for the diaphragm with the single opening a porous plug of graphite or of cotton wool. It is then found that air is not a perfect gas; the temperature in B is a little lower than that in A. Energy has been consumed in doing internal work—probably in separating the particles of the gas—to the extent, when the pressures in A and B both differ little from the atmospheric pressure, of about $\frac{1}{477}$ of the whole work spent upon the gas in forcing it through the plug. The proportion of the total energy spent in doing internal work varies from substance to substance, and from condition to condition. In carbonic dioxide, at a pressure varying little either in A or B from the atmospheric pressure, it amounts to about $\frac{1}{77}$; in air at a pressure in A of 19 atmospheres, it amounts to as much as $\frac{1}{63}$ of the whole.

In the case of hydrogen, curiously, there is a slight *increase* of temperature: the expanded gas has *more* kinetic energy than the unexpanded gas: energy is liberated when hydrogen expands; its particles seem to repel one another.

At equal temperatures, therefore, compressed air contains less intrinsic intermolecular *potential* energy than an equal mass of rarer air; compressed hydrogen the reverse.

In gases the amount of heat which disappears during expansion in doing internal work is generally small in proportion to the external work done against the atmospheric pressure: in solids and liquids the internal work done is relatively much greater.

When a substance is heated and rises in temperature without being allowed to expand, so much heat is absorbed during a given rise of temperature; when expansion is permitted, an additional supply of heat is required. The **Latent Heat of Expansion** of a substance may thus be found by difference.

When temperature varies, the volume being constant, the heat, H, supplied to a unit-mass is equal to $k\dot{t}$; when the pressure is kept constant and expansion is allowed, the heat supplied, $\mathrm{H}_{,}$, is equal to $k\dot{t}+l\dot{v}$; whence $l=(\mathrm{H}_{,}-\mathrm{H})/\dot{v}$. $\mathrm{H}_{,}$ is equal to the specific heat at constant pressure, whence $l=(\mathrm{K}-k)\dot{t}/\dot{v}$.

The latent heat of expansion of gases is tolerably easy to find, for it is possible to heat a gas without permitting it to expand; but the estimation of that of solids and of liquids passes beyond the bounds of our experimental powers. To ascertain that of water between 0° C. and 100° C., for instance, it would be necessary to compare the amounts of heat required to heat a certain mass of water from 0° to 100° C. when it is free to expand and when it is prevented from expanding: but the latter investigation would require the application of a pressure of 8772 atmospheres.

In the same way wrought-iron heated through $15\frac{1}{4}°$ Fahr. exercises a pressure of 1 ton per square inch.

The latent heat of expansion of a substance is, as a numerical coefficient, the number of units (calories) of heat required to effect unit-expansion in a gramme of that substance—that is, to double its volume—and which disappears in doing that work.

The work of expansion is always associated with that of raising the temperature; only in idea can we form an abstract conception of the amount of heat required to effect a certain expansion while the temperature is supposed to remain unchanged. Temperature and volume vary simultaneously, and the physical constant known as the **coefficient of expansion** states numerically the relation between these associated effects of heat.

A substance whose volume is $v_\circ$ at a temperature $t_\circ$ assumes a volume $v_,$ at the temperature $t_,$; the change of temperature is $t_, - t_\circ$; the proportionate change of volume is $(v_, - v_\circ)/v_\circ$; the quotient $(v_, - v_\circ)/\overline{v_\circ (t_, - t_\circ)}$ is the coefficient of expansion. If $t_, - t_\circ = 1°$, the coefficient of expansion is $(v_, - v_\circ)/v_\circ$.

The coefficient of expansion of any substance is the ratio between the *increase of volume* which it undergoes when its temperature is raised by 1° C. and its *original* volume.

If a cube of volume $v_\circ$ assume volume $v_,$, its side $\sqrt[3]{v_\circ}$ becomes $\sqrt[3]{v_,}$; its coefficient of linear expansion is therefore $\frac{\sqrt[3]{v_,} - \sqrt[3]{v_\circ}}{\sqrt[3]{v_\circ}}$, or approximately $\frac{1}{3}(v_, - v_\circ)/v_\circ$ Thus, if a body measuring a cubic foot on being heated 1° assume a volume of 1·0003 cub. ft., the side of the cube (1 foot) has become nearly 1·0001 linear foot.

Since we have in general to deal with expansions proportionately very small, we may say that the coefficient of linear expansion—the proportionate increase in length, breadth, or thickness per degree centigrade—is equal to one-third the coefficient of cubical expansion.

If l be the coefficient of linear expansion of a body whose length at $t_\circ$ is $L_\circ$, the length of the body at the temperature $t_,$ is $L_1 = L_\circ + L_\circ l (t_1 - t_\circ)$ or $L_\circ (1 + l (t_, - t_\circ))$. In this equation there are five terms, any four of which being known, the fifth can be found.

In some cases—many crystalline bodies—the coefficients of linear dilatation are not equal in all directions. Crystals have three axes, in the directions of which the coefficients of expansion $(l_{,}, l_{,,}, l_{,,,})$ are not always equal to one another; thus the angles of crystals are often modified by changes of temperature. Substances belonging to the regular system have the coefficients equal in the three axial directions, and they preserve similarity of figure when heated; dimetric crystals have two axial coefficients equal, the third different; trimetric crystals have all three coefficients unequal. In general the cubical coefficient $= l_{,} + l_{,,} + l_{,,,}$.

Take plates of gypsum, cut parallel to the prismatic axis: cement them together so that the direction of the axis of one plate forms a right angle with that of the other. Heat till the cement is melted; allow to cool. The unequal contraction in cooling will warp the whole (Fresnel). In the case of this substance a contraction in one direction is associated with expansion in two others.

Indiarubber and iodide of lead, iodide of lead and silver (PbI_2, AgI), iodide of silver up to 156°·5 C., and garnets, as well as water between 0° and 3°·9 C., contract when heated: their coefficient is negative. In some substances (zinc and iron) the coefficient of expansion slowly alters with lapse of time.

When a hollow body such as a flask or thermometer-bulb is heated, it expands almost exactly as if it were solid: a glass tube expands as if it were a glass rod. It follows that when a hollow body is heated, its internal cavity increases in volume in the same proportion as it would have done if it had been occupied by a solid the same as that which surrounds it.

Examples of Expansion by Heat.—Bodies which, when cold, exactly fill certain apertures, will not enter them when they are warmed. Railway rails are not laid in exact contact; allowance must be made for their summer expansion and winter contraction. In designing lattice-girders for bridges, the same necessity must be taken into account. Railway-distance signals are controlled by rods, which differ considerably in length at night and by day; provision must be made for tightening them up, or the reverse. If the neck of a stoppered bottle clasp the stopper too tightly, it may be loosened by causing the neck to expand while the stopper does not do so; this may be effected by winding a string round the neck and pulling it backwards and forwards so as to produce heat by friction; the neck is heated before the stopper itself is affected. Glass suddenly heated expands superficially while the inside is still cool: under the stress set up the glass may break; hence the thinner a flask the less risk there is of its cracking when it is heated. A cart-wheel tire is fitted on when it is hot; when it cools down it contracts and holds the rim, spokes, and hub firmly together: if it be originally too small it may break itself by its own contractile tension. The lead on a roof expands by day and contracts at night; gravity aids the one and checks the other tendency; the lead creeps down. The same theory has been applied to glacier movement.

Applications of Expansion.—The Compensation-pendulum is a pendulum constant in length, whatever be the temperature. A simple bar of metal would, by its variations in length, produce oscillations irregularly unequal, the clock going slow in summer, fast in winter. In order to correct this, the bob of the pendulum is suspended from a framework of bars of iron and brass so arranged that expansion of the bars of iron tends to depress the bob: that of the bars of brass tends to raise it; by proper adjustment these effects compensate one another.

The bob itself is sometimes made of a tube containing quicksilver: the expansion of the suspending bar tends to lower the centre of gravity of the pendulum: that of the mercury tends to raise it; a proper adjustment of the quantity of mercury in the bob produces sensibly accurate compensation.

Sometimes the rod of a pendulum bears a transverse bar, which is loaded at each end with heavy masses. This transverse bar consists of strips of different metals; in weather warmer than the average the lower strips expand most, distort the bar, raise the heavy masses, and thus raise the centre of gravity of the whole pendulum: in colder weather the reverse effect is obtained, for the lower strips contract most.

Measurement of Coefficients of Expansion.—In solids the coefficient of linear expansion is found by direct observation. A bar is heated to a known temperature; its original length and temperature are known. The elongation of the bar may be measured by a traversing bar with micrometer, or by the method of Fig. 5, or by the expansion of the bar in a tube pushing out a piece of porcelain, which can move outwards but cannot return. The first-mentioned method is by far the least liable to error, especially when the distance between two distinctive points on the bar is observed at two given temperatures.

$$\frac{l_{,}-l_0}{l_{,}} \cdot \frac{1}{t_{,}-t_0} \text{ is the coefficient of linear dilatation.}$$

The coefficient of cubical expansion may be found by multiplying the coefficient of linear expansion by 3; or, better, by finding the different specific densities of the solid at different temperatures.

The mass (= weight $/g$) remaining the same, ρ and $\rho_{,}$ being the densities, the volumes v_0 ($= m/\rho$) and $v_{,}$ ($= m/\rho_{,}$) are easily found; and $(t_{,} - t_0)$ being the difference of temperatures, $\left(\frac{v_{,} - v_0}{v_0} \cdot \frac{1}{t_{,} - t_0}\right)$, the coefficient of cubical expansion, can be computed.

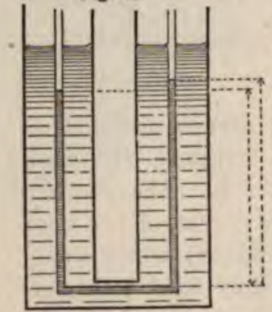
If a solid be heated in a flask with a narrow orifice, completely filled with mercury, the mercury expands, the flask expands, and so does the solid immersed in it. The absolute expansion of the mercury is previously known, that of the glass vessel must be known, and the amount of mercury which would fall out of the flask if the flask were completely filled with mercury and heated to the same degree is already known; when the solid is immersed in the mercury, a different quantity of mercury escapes from the flask when heated; the difference is due to the difference of dilatation between mercury and the immersed solid: the coefficient of expansion of the immersed solid can thus be calculated.

In liquids the expansion may be found by observation of the apparent increase of bulk undergone by a liquid contained in a flask. The width of the neck may be ascertained by the addition of known weights of mercury: an apparent rise of the liquid in the neck may be interpreted as corresponding to so many cubic cm. apparent increase of bulk. But it is important to bear in mind that the cavity of the flask also expands, and that the real expansion of the liquid is the sum of the expansion of the cavity of the flask, and the apparent expansion of the liquid in the neck. If the liquid have the same cubical coefficient as the glass, there would be neither a rise nor a fall in the neck; if it have a less rate of expansion than glass it will sink in the neck, and will then apparently contract; only when it has a greater coefficient of expansion than the glass will it rise in the neck, and thus under such circumstances manifestly appear to expand.

When a thin glass-flask filled with water is suddenly heated it expands before the water contained in it has had time to become heated, and the liquid in the first place appears to shrink into the flask. Then the liquid becomes heated and rises in the neck of the flask.

The expansion of a liquid may also be found by observing its density at different temperatures. This may be done by means of separate observations. It may also be done by the observation of the simultaneous heights of a hotter and a colder column of the same liquid, which balance one another in a U-tube. The heights are reciprocally proportional to the densities, and thus it is easy to find the coefficient of expansion per degree centigrade. Fig. 129 shows that each limb of the U-tube is maintained at a constant temperature by surrounding baths (of water, mercury, oil, etc.) whose temperatures are known. The heights of the columns may be measured by means of a cathetometer. The absolute expansion of mercury is by this method found to be, per degree centigrade (between $-36°$ C. and $100°$ C.), almost exactly $\frac{1}{5555}$ of its total bulk at 0° C.; above 100° C. it increases rapidly with the temperature. The total amount of expansion is thus not quite proportional to the rise of temperature.

Fig.129.



In gases the coefficient of expansion is nearly uniform, about $\frac{1}{273}$ for every degree centigrade. Not quite uniform; for all gases are not necessarily in the same physical condition merely because they are at the same temperature, for some may be near, others far from, their point of condensation; and the volume of gases is not exactly proportioned to their absolute temperature.

The coefficient of expansion in gases may be determined by direct observation, the volume being allowed to vary, while the pressure is maintained constant during a given change of temperature; or inferentially by observation of the increase in pressure exercised by a gas when its volume is kept constant during a given change of temperature, coupled with the assumption that Boyle's law is perfectly obeyed, and that the volume and the pressure bear an exact inverse ratio to one another. The latter method, as we shall see, is more valuable in thermometry than in the determination of the actual coefficient of expansion of a gas.

If we assume Boyle's law and Charles's law to be both true, we have the equation $\frac{pv}{t} = const.$ If the same quantity of gas change in pressure, volume, or temperature, again $\frac{p_, v_,}{t_,} = const.$ Hence $\frac{pv}{t} = \frac{p_, v_,}{t_,}$. This enables us to solve, to a first approximation, such problems as the following :—

Fifteen litres of air at 0° C. and 761 mm. bar. pr. are heated to 10° C.

while the barometer sinks to 759 mm.: what volume does the air assume?

$$\frac{pv}{t} = \frac{p_{,}v_{,}}{t_{,}};\ \frac{761 \times 15}{273} = \frac{759 \times v_{,}}{283}.$$

$$\text{Whence } v_{,} = \left(\frac{761}{759} \times \frac{283}{273} \times 15\right) \text{litres.}$$

Again, 15 litres of air at 0° C. (273° Abs.) and 762 mm. Hg. pressure are, when they are heated to an unknown temperature and exposed to a pressure of 1000 mm. Hg., doubled in volume: what is the unknown temperature?

$$\frac{pv}{t} = \frac{p_{,}v_{,}}{t_{,}};\ \frac{762 \times 15}{273} = \frac{1000 \times 30}{t_{,}};$$

$$t_{,} = \frac{1000}{762} \cdot \frac{30}{15} \cdot 273 = 716^{\circ}{\cdot}5 \text{ Abs.} = 443^{\circ}{\cdot}5 \text{ C.}$$

We may combine with these equations the two following propositions:—

1. The specific density of a gas is numerically equal to half its molecular weight.

2. One gramme of hydrogen measures 11·1646 litres at 0° C. and 760 mm. bar. pr.

Problem.

Fourteen litres of carbonic acid are measured at 10° C. and 759 mm. pressure: what is their mass?

First reduce the 14 litres to the volume which they would occupy at 0° C. and 760 mm. bar. pr.—*i.e.*,

$$\left(14 \times \frac{273}{283} \times \frac{759}{760}\right) \text{ litres.}$$

Each litre of carbonic acid at 0° C. and 760 mm. weighs $\frac{1}{11{\cdot}1646} \times \frac{44}{2}$ grammes. The whole weighs

$$\left(14 \times \frac{273}{283} \times \frac{759}{760} \times \frac{1}{11{\cdot}1646} \times \frac{44}{2}\right) \text{ grammes.}$$

Problem.

What bulk is occupied by 20 grammes of ammonia gas at 15° C. and 740 mm. bar. pr.?

One gramme of hydrogen occupies at 0° C. and 760 mm. a bulk of 11·1646 litres; at 15° C. and 740 mm. it would have a volume of $(11{\cdot}1646 \times \frac{288}{273} \times \frac{760}{740})$ litres; but ammonia gas has a sp. density $= \frac{17}{2}$; hence 20 grammes of ammonia occupy a bulk

$$\left(20 \times \frac{2}{17} \times 11{\cdot}1646 \times \frac{288}{273} \times \frac{760}{740}\right) \text{ litres.}$$

It may be left as an exercise to the student to find what corrections should be applied to reduce the apparent weight of a substance weighed in air at a given temperature to the real weight at a standard temperature, say 0° C., the coefficients of expansion of air, of the counterpoising weights, and of the substance weighed, being supposed to be known.

Fusion.—Heat sometimes operates liquefaction of solid bodies. The temperatures at which fusion is effected differ widely: the fusing point of solid alcohol (− 130°·5 C.), that of mercury (− 40° C.), and that of platinum (about 1775° C.) which can only be fused by the oxyhydrogen blowpipe or the electric arc, may be taken as examples.

In general there is expansion during fusion; in such event there may be a small amount of work done against external pressure. If the external pressure be increased the amount of heat-energy that must be supplied in order to effect this external work in addition to the internal work of fusion is proportionately increased. The temperature of fusion is thus in most cases raised by increase of pressure. In the cases of water, antimony, cast-iron, and many rocks, the freezing point is lowered by pressure, because these substances expand when they freeze. Tables of melting points therefore denote the melting points of substances at the atmospheric pressure.

We may here state the reasoning by which it was predicted * that an increase of pressure would be found to lower the melting point of ice; though some of the steps will not be understood until after we have considered Carnot's cycle of operations and his "perfect engine."

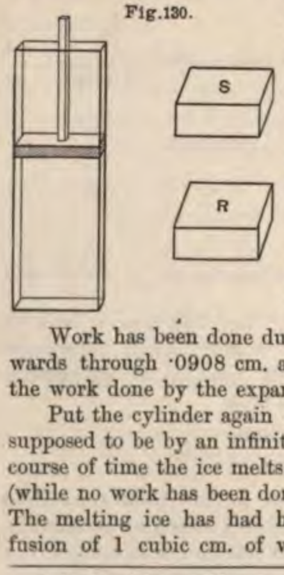


Fig.130.

A cylinder with a square base, 1 cm. square, contains one gramme of water—*i.e.*, 1 cubic cm. S is a *source* of heat at 0° C. (which must be situated within a sufficient space entirely devoid of air); R is a refrigerator situated within a region where the atmospheric or other pressure is equal to p dynes per sq. cm., and maintained at a constant temperature $-\tau°$ C., very slightly below 0° C.

1. The cylinder is kept upon the source S until the water assumes the temperature 0° C. We now have 1 cub. cm. water at 0° C.

2. Put the cylinder on the refrigerator R; keep it there until the water is wholly frozen to ice at 0° C. We now have 1·0908 cub. cm. ice at 0° C. (the sp. density of ice being ·91674, Bunsen).

Work has been done during expansion; the piston has been thrust upwards through ·0908 cm. against an external pressure p dynes per sq. cm.; the work done by the expanding substance is 0·0908 p ergs.

Put the cylinder again on the source: the temperature of the source is supposed to be by an infinitely small amount higher than that of the ice. In course of time the ice melts; now we again have 1 cub. cm. of water at 0° C. (while no work has been done upon the melting ice by any exterior pressure). The melting ice has had heat imparted to it equal to the latent heat of fusion of 1 cubic cm. of water—that is, 80·025 *ca* = 3,328,480,625 ergs.

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* By Prof. James Thomson; experimentally confirmed by his brother, Prof. Sir William Thomson.

This amount of heat has been absorbed from the source at 0° C.; heat has been lost to the refrigerator at $-\tau°$ C. The piston returns to its normal position, as we have seen, and the whole contrivance, perfectly imaginary, will act as a "perfect engine," with ice or water as its working substance, provided that τ has a certain value to be deduced from the equation—

$$\frac{\text{Work done}}{\text{Heat absorbed}} = \frac{\text{difference of temp. between source and refrigerator}}{\text{absol. temperature of source}} = \frac{\tau}{273}.$$

$$\frac{0{\cdot}0908\,p}{3{,}328{,}480{,}625} = \frac{\tau}{273}.$$

$$\tau = \frac{273 \times 0{\cdot}0908}{3{,}328{,}480{,}625}\,p.$$

When the external pressure p changes by $\Pi = 1{,}013{,}663$ dynes per sq. cm.—that is, when it changes by an amount equal to one atmosphere—τ changes by ·0074° C. This means that such an engine is reversible, and its operation is theoretically perfect when the freezing operation is conducted at a temperature lower than 0° C. by an amount equal to ·0074° C. for every additional atmosphere-pressure suffered by the freezing water. If the freezing could occur at a higher temperature than this, there would be production of work by the expanding ice accompanied by a withdrawal of heat from the source insufficient to account for it, and the perpetual motion would become possible.

When a piece of ice is placed in contact with another, both being at 0° C., a very slight pressure will, by lowering the melting point, cause a certain quantity of ice at the point of contact to melt. When the pressure is relieved the mass solidifies and becomes continuous ice.

Ice is not without a small degree of viscosity, and very cold ice can slowly flow down a slope of 1 in 4 under a pressure equal to the weight of 300 feet of ice-cliff (Moseley and Browne); but at temperatures between 0° C. and about $-\frac{1}{2}°$ C. it can be driven through narrow passages by the above process of Regelation, for when crushed the fragments are relieved of pressure and reunite, again to be crushed and forced onwards. To the small viscosity of ice and to the process of crushing or regelation as well as to creeping (p. 350), is to be ascribed the flow of glaciers.

Sometimes the fusion-point of a mixture is below that of its ingredients. A mixture of common salt with about $2\frac{1}{2}$ parts of crushed ice melts at about $-18°$ C. or 0° Fahr.: above this temperature it is liquid; and when ice and salt are mixed, the result is very cold liquid brine.

When the pavements in snowy weather are cleared by means of salt, the brine thus formed being at a temperature of 0° Fahr., or at "thirty-two degrees of frost," penetrates the shoe-leather and chills the feet of pedestrians, while it refuses to dry, the salt being hygroscopic—that is, having a great affinity for water.

Boiling or ebullition is a rapid process of reduction of a liquid to vapour. Evaporation is thus distinguished from ebullition; in evaporation particles fly from the surface and mingle with the particles of gas or vapour already existing in the neighbourhood of the surface of the liquid, and drive or repel only a certain proportion of them away from the surface: in boiling, the particles which fly from the surface bombard the surrounding particles so hotly as to drive them all from the neighbourhood of the surface of the boiling liquid, and to take their place. Thus the vapour of a boiling liquid has to exert a pressure which is just a little greater than the atmospheric, or, in general, the exterior pressure, whatever that may happen to be; the vapour of an evaporating liquid exerts a pressure which is only a fractional part of the atmospheric or exterior pressure.

Besides, the process of evaporation is restricted to the exterior free surface: that of boiling occurs both at this surface and at the internal surface of bubbles in the interior of the liquid.

A liquid may be heated to a temperature above its boiling point, and if there be no bubbles formed, no point at which the action may preferably start, the whole liquid may become overstressed, like a Rupert's drop, and when it does give way and form vapour, it may do so explosively. This kind of explosive boiling may be observed when water void of air is heated, or when drops of water are suspended in a mixture of light and heavy oils of the same specific density as water and then heated, or when water is heated in a glass vessel, especially if it have been carefully cleaned with sulphuric acid. In the last case the surface of the vessel is very uniform, and there is no sharp point or roughness at which a bubble may commence: thus the temperature rises above the boiling point until it is brought down by a sudden outburst of vapour, and bumping ensues. There is less of this in a smooth-metal vessel than in a glass one; still less in a rough-metal vessel; still less where jagged pieces of platinum or stone have been immersed in the liquid to be boiled. The process of boiling depends to a great degree for its regularity on the presence of air-bubbles: we may sometimes see that water when long boiled ceases to evolve bubbles, and evaporates only at the surface with an occasional outburst of steam.

A bubble of air or vapour produced in the interior of a hot liquid is increased in size by molecules escaping into it from the surrounding liquid; if the temperature of these molecules, their energy, their velocity, their pressure, be such that they can expand the bubble against the surrounding pressure, the bubble enlarges and rises. If we artificially produce bubbles in the interior of a heated liquid, as when we electrolyse hot water, the liquid boils very rapidly at the electrodes, where gaseous oxygen and hydrogen are being given off.

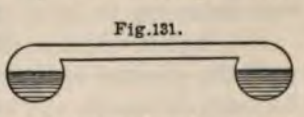
The boiling point at different pressures.—The greater the external pressure to be overcome, the greater must be the energy, and therefore the greater the temperature, of the rising vapour. The temperature of ebullition and the external pressure are not directly proportional to one another, but are found experimentally, and recorded in tables such as those great tables of Regnault's, to be found in his *Relation des Expériences.*

At mountain heights the atmospheric pressure is less and the boiling point is lower; thus at Quito, at a height of 9540 feet, water boils at 90°·1 C.

If a flask containing water, boiling at 100° C., be corked and set aside until it has cooled, say, to 90° C., and if the upper part of the flask, the part containing the vapour of water, be suddenly cooled by cold water dashed upon it, the vapour in it will be partly condensed, and a partial vacuum will be formed: the water will find itself at a temperature of 90° C. under a pressure of less than 525·45 mm. of mercury, and it will again begin to boil: the water is thus seemingly induced to boil by the application of cold to the flask containing it.

If a cryophorus tube, Fig. 131, of which both bulbs are half filled with water, have one bulb immersed in a freezing mixture, the vapour in the cold bulb is condensed; the vapour in the tube is pushed into the cold bulb by the uncompensated pressure of particles rising from the liquid in the warmer bulb; this process is continuous; work is continuously done in maintaining the flow of vapour, which is as continuously condensed; the liquid in the warmer bulb continuously evolves vapour, and does so so rapidly, the pressure being small, as to boil; it continuously does work, but receives no energy; it cools and ultimately freezes, even while evaporating.

Fig. 131.



Boiling and evaporation may thus involve not only the giving of momentum to particles of the liquid, but also external work done against resistances; and during **evaporation** there may be **cooling** due not only (1) to the latent heat of evaporation absorbed in producing change of state, but also (2) to the external work which is done by the evaporating body—work which generally takes the form of thrusting aside the external air.

Examples of cooling due to evaporation are:—The cooling of the skin by perspiration or by a draught of air, even though the air be warmer than the skin ; a dog cooling himself by panting with his tongue exposed ; a porous water-cooler or *alcarraza*, the evaporation at the surface of which cools the contained water ; the practice in some hot countries of cooling a room by throwing water over the floor ; the cooling undergone by a liquid which is being rapidly evaporated, as, for example, the rapid cooling of sulphurous anhydride or of ammonia, which is effected in the course of the process of artificial ice-making by the rapid evaporation of the liquefied gases under a powerful air-pump ; the cooling of a jet of liquefied carbonic acid when allowed to escape into the air, so that the substance is in part solidified.

Ethylene (olefiant gas) may be liquefied by cold and pressure ; on being rapidly evaporated under the air-pump it becomes so cold that air, greatly compressed, can be liquefied by it. This liquefied air, when allowed to evaporate freely, produces temperatures apparently below $-210°$ C. (Olszewski).

The latent heat of evaporation of steam is $\lambda = (33011{,}504000 - 33{,}200000\ \theta)$ ergs per gramme, where the temperature of ebullition is $\theta°$ Abs. At $994°{\cdot}32$ Abs. or $720°{\cdot}6$ C., $\lambda = 0$, and this temperature is for steam the Critical Temperature, beyond which there is no change of state when liquid water becomes water-vapour.

Saturation-Pressure.—In the case of every vapour we find that for each particular temperature there is a maximum density ; if we compress the vapour beyond this density, a portion of it will be liquefied. If we allow it to expand, then—provided that the temperature is kept constant, and that the vapour is kept in contact with its own liquid—a portion of the liquid will be evaporated; thus the density is maintained constant and the vapour is kept saturated. Each volatile liquid has its own saturation-pressure for each temperature, this being the pressure necessary to bring the vapour to its maximum density.

A vapour which is not saturated may by compression, exerted until the pressure of the vapour is equal to the saturation-pressure, be made saturated, and by further pressure will be caused partly to condense.

The saturation-pressure of any vapour at any temperature is the same as the pressure at which the corresponding liquid boils at that temperature.

Even in contact with ice, water-vapour has a saturation-pressure, and evaporation will go on until this pressure is attained. A strong wind blowing over a snowfield may remove much of the snow by true evaporation without liquefaction.

Saturated steam in contact with ice at $\theta°$ C. has a pressure $p = \{107{\cdot}2 + (6255 \times 1{\cdot}080^{\theta})\}$ dynes per sq. cm.—(Regnault.)

As a general rule each component of a mixture of gases exercises its own pressure, and is not affected by the others which accompany it. Yet this rule is not absolute ; for if we heat in a flask a certain quantity of air alone, we find that it exerts a certain pressure; a certain quantity of water-vapour introduced alone into a vacuum would exert a certain pressure; but when both the water-vapour and the air are introduced into the same vessel, the joint pressure falls somewhat short of the sum of the several pressures, and thus it is shown that there is an attractive action between water and air.

Vapours at variable pressures and temperatures generally obey Boyle's law with tolerable regularity until the pressure comes up to about $\frac{8}{10}$ of the saturation-pressure, and that whether they be alone or mingled with air or other vapours.

Measurement of Vapour Density at different temperatures.

a. By measurement of the pressure exercised by the vapour of liquid at a series of known temperatures.

This is effected by the arrangement sketched in Fig. 132. The mean temperature of boiling is indicated by four thermometers, two in the liquid, two in the vapour: the vapour is condensed in A and returned to the flask: the pressure is measured by a manometer.

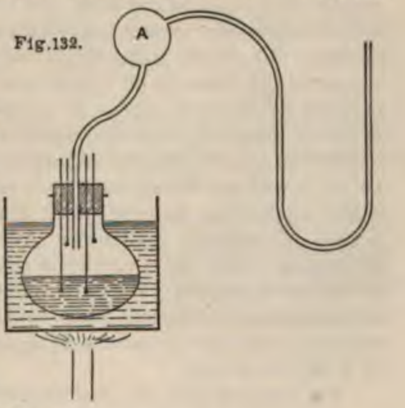


Fig. 132.

The use of this method depends on a tacit assumption that Boyle's law is obeyed throughout all ranges of temperature; but this method is not applicable except at low temperatures and low pressures; for at high pressures the vapour assumes abnormally small volumes as it approaches its saturation-pressure.

β. By measurement of the volume occupied by a known weight of fluid, or by measurement of the weight of vapour which can occupy a known volume.

The first of these methods is that of Gay Lussac. A tube filled with mercury is inverted like a Torricellian barometer in a vessel of mercury, and has a Torricellian vacuum at its upper part; the whole is immersed in a bath of liquid kept at a definite temperature. A little bulb containing a known quantity of the liquid to be vaporised is passed up into the tube; being heated it bursts; the vapour occupies a certain volume of the tube; the mercury stands at a certain height in the tube. The mercury stands at a different height in an ordinary barometer; the difference of readings indicates the pressure exercised by the vapour. Its weight is known, its volume, and its temperature. A series of observations is made at different bath-temperatures. It is difficult to ensure that all the substance is liquefied.

The second method is that of Dumas. A bulb with a long-drawn neck is filled with liquid and immersed in a heated bath. The liquid in the bulb violently rushes out in the vaporous state through the narrow neck; this ceases and equilibrium is set up; the bulb is filled with vapour at the temperature of the bath. The end of the neck is then sealed by a blowpipe-flame; the whole is removed, cooled, weighed. This gives the weight of bulb + vapour; already the weight of the bulb, its volume, the bath temperature, are known; the density of the vapour occupying the bulb at the temperature of the bath can be thus found. At high temperatures bulbs of porcelain or iron, and baths of mercury-vapour, sulphur-vapour, or zinc-vapour, may be used (Deville and Troost).

The density of saturated vapour.—Fairbairn and Tate found the density of saturated steam by introducing into a recipient of known capacity and devoid of air a known quantity of water, and by measuring the temperature at which the whole of the water was evaporated.

The measurement of the pressure of unsaturated vapour, if it present itself alone, is simply the measurement of gaseous pressure, and calls for no further remark.

The measurement of the pressure exercised by an unsaturated vapour which forms one of the components of a mixture is in one case—that of Aqueous Vapour in the Air—a matter of importance. A numerical example will illustrate this. If water be exposed to a pressure of 9·16 mm. of mercury (= 0·01205 atmos.), it will boil at 10° C.; if water-vapour of such density (supposed constant) that it exerts a pressure of 0·01205 atmos. be exposed to a temperature *above* 10° C., it will be unsaturated; *at* 10° C. it will be saturated; at any temperature *below* 10° C. it will be in part condensed. 10° C. is, then, the condensation-temperature for aqueous vapour of this pressure of ·01205 atmos., just as the latter is the saturation-pressure for aqueous vapour at a temperature of 10° C. If, now, we take moist air containing aqueous vapour and air in the proportion of 0·01205 to 0·98795, at the ordinary atmospheric pressure: at any temperature above 10° C. it will not deposit moisture; at 10° C. it will begin to do so. 10° C. is the condensing temperature or **Dewpoint** for air containing this proportion of moisture. To other proportions of moisture other dewpoints correspond; these can be found in any table of the boiling points of water at different pressures. Hence, if we can find the temperature at which air containing aqueous vapour begins to deposit moisture, we can by reference to such tables find the proportion of aqueous vapour in the air. This is effected by a Hygrometer.

The essential part of a hygrometer is a glass—or, better, a smooth silver—surface, which can be cooled down until the moisture of the air begins to deposit as a film upon it, and whose temperature at the instant of the dimming of its brightness can be accurately ascertained. The surface may be fashioned into a bulb: this bulb may contain ether; the bulb may be cooled by blowing through and thus rapidly evaporating the ether; the temperature at the instant of dimming of the surface can be read off on a thermometer whose lower end is dipped in the evaporating ether. The whole may be allowed spontaneously to become warmer; as it does so the film disappears: the temperature at which this occurs is noted. The film is again caused to appear and disappear; by dint of repetition a mean point between the highest temperature of appearance of the film and the lowest temperature of its disappearance is obtained, which is the Dewpoint required.

Another method for ascertaining the dewpoint—one for doing so by a single observation—is the following:—If a thermometer bulb be by any means kept cool by evaporation—being covered with a wet piece of linen which dips in water, or the like—the bulb is cooled; the extent of cooling depends on the rapidity of evaporation: the rapidity of evaporation depends on the Humidity of the air—that is, on the ratio between the amount of aqueous vapour actually present in the air, and that which would be present at the temperature of the air if the air were saturated with moisture. The less the humidity of the air, the greater will be the evaporation, and the greater will be the difference between the readings of a thermometer kept cool in this way and those of a thermometer subjected to normal circumstances. Tables have been constructed in which, for each reading of the "dry bulb" and of the "wet bulb," the corresponding percentage of aqueous vapour in the air is recorded.

Dew.—When, on a clear night, the earth, stones, plants, etc., become cool by free radiation, their temperature may sink below the condensation-temperature proper to the particular percentage of aqueous vapour in the air. When the temperature thus sinks below the dewpoint, the moisture of the air is partly deposited in the form of dew; and the more highly charged with moisture the air had become during the day, the earlier and the heavier is the deposit of dew at night.

Changes of Elasticity.—Among other changes of state effected by heat it is important to note the change produced in the elasticity of a body.

If a gas be kept at the same temperature, its elasticity—its resistance to a small compression—is numerically equal to the pressure which it exerts.

If a gas be compressed in such wise as to lose no heat-energy, it becomes heated, exercises more pressure, and offers more resistance to compression, and is therefore more elastic; its elasticity under adiabatic compression is greater than that at constant temperature in the ratio of 1·4058 to 1.

TRANSFORMATIONS OF HEAT.

Transformation of Work into Heat may be effected directly by the agency of friction, or indirectly by the transformation of kinetic energy into the energies of noise, light, electrical condition, which are in their turn converted into heat. Even the conversion, apparently direct, by the agency of friction may be due in the first place to the generation of local electrical currents or conditions, the energy of which is afterwards converted into heat.

Transformation of Heat into Work.—From our previous discussions of the Indicator-Diagram we understand that the work done by any substance during expansion can be represented by the area $pp'v'vp$ (Fig. 133), where Op, Op' represent the original and final pressures, Ov and Ov' the original and final volumes. The work is positive, done *by* the expanding substance (steam, air, etc.) if the expansion be positive, from Ov to Ov'; negative, done *upon* it if the expansion be negative, as from Ov' to a less value Ov.

Fig. 133.

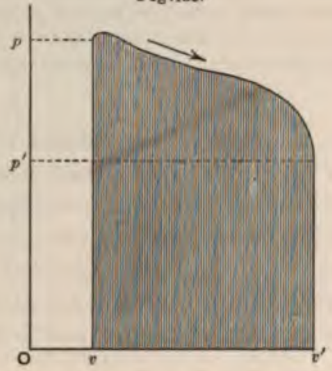
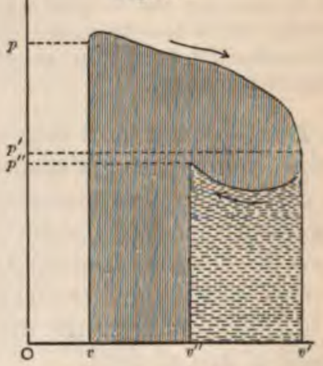


Fig. 134.

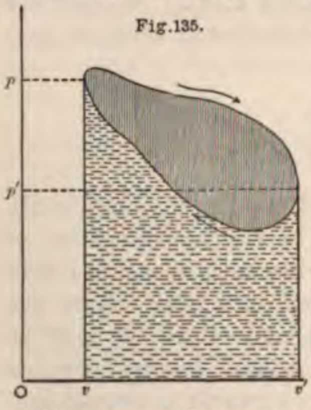


Where work is done both by and upon the working substance, as in Fig. 134, the negative work $p'p''v''v'p'$ being subtracted from the positive work $pp'v'vp$, there is left an area $pp'p''v''v$, which represents the work done.

If the curve $pp'p''$ be complicated, the total work done may be found by dissecting the figure; any complex operation may be resolved into a number of simple ones of which each produces its own effect; the work done is found by a process of summation of positive and negative areas.

When the working substance returns to its original volume and pressure as in Fig. 135, the shaded area again indicates the amount of work done by the working substance, just as if in Fig. 134 the line $p''v''$ had been made to coincide with pv. The work is positive if the change of pressure and of volume have been effected in the direction of the arrows; negative if effected in the contrary sense. Such an operation is a **Cycle**.

Fig. 135.



The advantage of studying the amount of work done by a working substance operating in a cycle is that we are not called upon to take any internal work into account. The body returns at the end of the operation to its primitive condition, and there is no balance of work done either by or against internal forces.

Into the consideration of a cycle we introduce an assumption that it is possible for a working substance to return to the same condition as regards pressure and volume at the original temperature; this might not have been true as regards any actual substance, though it is theoretically true as regards perfect gases; it is, however, actually true as regards physical gases, for the elasticity of gases is perfect.

We must choose some particular kind of cycle for our ideal operations; that to be explained is the one best adapted for the study of the relations between work and heat, and was devised in its primitive form by Sadi Carnot; it is hence known as **Carnot's cycle.**

If a gas expand at constant temperature, we know by Boyle's law that the pressure and the volume vary inversely; this law can be expressed graphically by an equilateral hyperbola, for in that curve $xy =$ const. The pressures and volumes at different temperatures are represented by points on different hyperbolas. Imagine the curves of Fig. 136 to represent portions of the hyperbolas corresponding to temperatures $\theta_1°$ Abs. and $\theta_2°$ Abs. for a given mass of substance. This substance, at the temperature θ_2 and pressure p_1, will have the volume v: to the pressure p_2 at the same temperature corresponds volume v_2; if the temperature be θ_1 and pressure p_1, the volume will be not v, but $v_{,}$, a point on the higher hyperbola, on the line—the so-called **Isothermal** line—corresponding to the higher temperature $\theta_1°$ Abs.

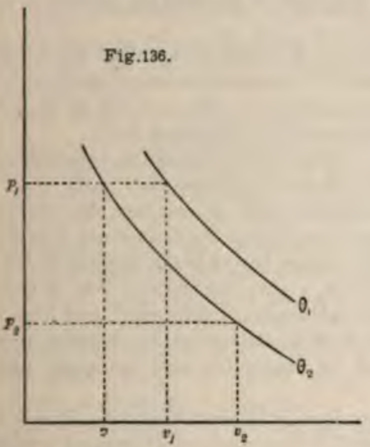


Fig. 136.

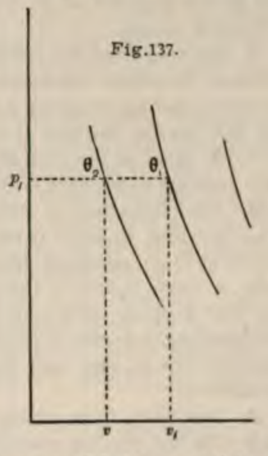


Fig. 137.

Expansion of a gas involves a more rapid fall of pressure when it is effected adiabatically than when effected at constant temperature, for the gas cools down: the **Adiabatic** lines, which express the relations between pressure and volume when heat is neither supplied nor allowed to escape, slope more steeply than the isothermal lines for the same substance. Fig. 137 represents these lines, and shows the relations between the pressures and volumes of a substance starting from conditions p_1, v_1, θ_1, and p_1, v, θ_2, which correspond to those of the previous figure.

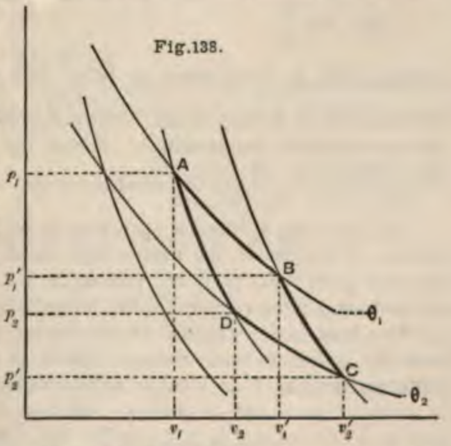


Fig. 138.

Let us now superpose the two figures 136 and 137, and we obtain Fig. 138, and are now prepared to understand Carnot's cycle in its modern form.

The steps of Carnot's cycle—

1. Starting with our working substance at the condition p_1, v_1, θ_1° Abs. (point A), we allow it to expand at the temperature θ_1°, this temperature being maintained constant. Running through successive pressures and volumes represented by successive points on the isothermal line θ_1, it assumes a pressure, say, p_1' and volume v_1' (point B).

Work is done equal to $p_1v_1 \log (v_1'/v_1)$ = the area AB$v_1'v_1$. This work is done at the expense of heat-energy supplied to the working substance from an external source.

2. Starting from the condition p_1', v_1', θ_1 (point B), we allow the working substance to expand adiabatically, until it assumes the temperature θ_2° and the corresponding condition p_2', v_2', θ_2° (point C). BC is a part of the adiabatic line passing through B and cutting the θ_2 isothermal in C.

Work equal to the area BC$v_2'v_1'$ is done by the expanding substance, but at the expense of its own heat-energy, for no heat is supplied to it.

3. The substance is now compressed until it assumes the condition p_2, v_2, θ_2—that is, until it runs from C so far up the isothermal line θ_2 as to meet at D an adiabatic line, which passes through the original point A. Work is done equal to the area CD$v_2v_2' = p_2v_2 \log (v_2'/v_2)$: but it is done *upon* the working substance, for that substance is compressed, and heat to a corresponding amount is lost by the working substance, for it passes to surrounding objects, and may be wasted by conduction and radiation into all the universe.

4. The body, from which no more heat is allowed to escape, is now supposed to be still further compressed until it has regained its original condition p_1, v_1, θ_1°. Work is done on the working substance thus compressed, but appears as heat in the substance, not as external work either positive or negative, and the temperature rises, for no heat is allowed to escape.

The whole energy supplied to the working substance from the source is $p_1v_1 \log (v_1'/v_1)$; that wasted is $p_2v_2 \log (v_2'/v_2)$; that utilised is $\dfrac{p_1v_1 \log \dfrac{v_1'}{v_1} - p_2v_2 \log \dfrac{v_2'}{v_2}}{p_1v_1 \log \dfrac{v_1'}{v_1}}$ of the whole. This can be proved (Verdet, *Thermodynamique*, p. 127) equal to $\dfrac{p_1v_1 - p_2v_2}{p_1v_1}$ of the whole, for $\dfrac{v_2'}{v_2} = \dfrac{v_1'}{v_1}$ when the area ABCD is very small. But $p_1v_1 = R\theta_1$; $p_2v_2 = R\theta_2$, where θ_1 and θ_2 are the respective temperatures. Hence the proportion of energy utilised is $\dfrac{R\theta_1 - R\theta_2}{R\theta_1}$, or $\dfrac{\theta_1 - \theta_2}{\theta_1}$ of the whole.

The working substance operating in such a cycle acts as a distributor of energy; it divides H, the heat-energy supplied to it from the Source of heat, into two parts: one part, H_1, passing to the Condenser, is lost by conduction and radiation; the remainder, W, is usefully converted into external Work.

The heat H is supplied at the higher temperature θ_1; the quantity of heat H_1 is lost to surrounding objects at the lower temperature θ_2; the **Efficiency** of such an ideal arrangement is the ratio

$$\frac{\text{Heat utilised}}{\text{Heat supplied}} = \frac{H - H_1}{H} = \frac{W}{H} = \frac{\theta_1 - \theta_2}{\theta_1}.$$

Thus, so far as Carnot's cycle is concerned, even though we could find a working substance and construct a machine which could carry the cycle out in practice, yet there would be a great waste of heat-energy, unavoidable unless we had a condenser at a temperature of absolute zero. If the temperature of the boiler of an ideal engine competent to work out Carnot's cycle were 120° C. (393° Abs.), and that of the condenser 0° C. (273° Abs.), the work done by such an engine could not exceed $\frac{393 - 273}{393}$, or about 30·6 per cent of the whole energy supplied as heat.

The cycle above considered is **reversible**; each step in it can be retraced —or could be retraced if we could construct an engine capable of working without waste of energy in noise, friction, excessive conduction and radiation of heat, and the like—work being done not by but upon the engine as it is driven backwards.

The effect of reversing such a cycle would be that work W being done upon the engine, the quantity H_1 of heat would be taken from the condenser, and the quantity H of heat would be communicated to the source.

Any engine must operate through periodic cycles if its action is to be continuous: and in every engine there is an absolutely necessary waste of energy arising from the necessity of restoring the engine to its primitive position in order that its piston may repeat its effective thrusts.

Carnot's ideal "perfect" engine is one which, with a working substance capable of returning to its primitive condition, will work out the *reversible* cycle above described, and thus attain the efficiency above indicated: an engine which wastes no energy otherwise than by restoring the primitive condition of its working substance.

The perfection of a perfect engine depends not on the nature of the working substance, but on the reversibility of the cycle which it operates, and the efficiency of such a reversible engine depends only on the temperatures between which it works. **Carnot's Principle**, as enounced by himself is—the motive power of heat is independent of the material agents employed to realise it; its quantity is determined solely by the temperatures between which the "transport of Caloric"[*] is effected.

The efficiency $\frac{W}{H} = \phi\,(t,\ t - \dot{t})$, where $t = \theta_1$ and $\dot{t} = \theta_1 - \theta_2$.

$$= f(t,\ t) - f'\,(t,\ \dot{t}).$$
$$= 0 - \psi\,(t)\,\dot{t}.$$

The efficiency depends upon $\dot{t}$, the difference of temperatures between the source and condenser, and upon $\psi\ (t)$, a function of t which is called **Carnot's function,** C.

We have also seen that efficiency $= \frac{\text{difference of temperatures}}{\text{temperature of source}} = \frac{\dot{t}}{t}$.

Hence $C = \frac{1}{t}$; and Carnot's function is the reciprocal of the absolute temperature.

The Efficiency of a Carnot's Reversible Engine is greater than that of any other engine. If it were possible to devise a more efficient engine it might be employed with the expenditure of a certain amount of heat to drive a reversible engine backwards; the source and the condenser of the Carnot's engine might be the same as those of the better engine: the Carnot's engine would be occupied in restoring to the source the heat taken from it by the better engine; on the whole, a surplus of work would during each cycle be done by the conjoined mechanism—a surplus not accounted for by heat lost by any body—a creation of energy.

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[*] An expression implying, as in his day, the material theory of heat.

If the better engine were employed in driving a larger Carnot's engine backwards, there might be no surplus, no external work done ; but a greater amount of heat would be conveyed to the source by the reversed Carnot than would be taken from it by the more efficient but smaller engine, and the whole heat of the universe might be, step by step, induced to travel through the condenser into the source of the conjoined mechanism—a conclusion evidently absurd.

That this conclusion is absurd, or at any rate contrary to experience, so long as we cannot deal like Clerk Maxwell's Demon (p. 50) with single molecules, it is the aim of the **Second Law of Thermodynamics** to state :—Heat cannot of itself pass from a colder body to a hotter one, nor can it be made so to pass by any inanimate material mechanism : and no mechanism can be driven by a *simple* cooling of any material object below the temperature of surrounding objects.

The word simple, or some equivalent word, is necessary in the above statement of the second law for the following reason :—A quantity of compressed gas *can* do external work, and in so doing cool itself below the temperature of surrounding objects ; but its cooling is not a simple loss of heat-energy ; there is a concurrent change of condition of the gas, a change which cannot be reversed without the expenditure of heat exceeding in amount the heat converted into work by the expanding gas.

This being admitted, we may reason backwards and arrive at the ratio of efficiency $\frac{\theta_1 - \theta_2}{\theta_1}$ in a reversible engine as a direct corollary of the proposition ; and the statement of that ratio of efficiency in a reversible engine is also known as the Second Law of Thermodynamics.

This Protean law assumes another form, apparently different from but essentially identical with both the preceding. Temperature being assumed proportional to the total heat-energy, the total amount of heat-energy H supplied at the higher temperature θ_1 is proportional to θ_1 ; $H = c\theta_1$; $H/\theta_1 = c$. Similarly H_1, the heat lost to the condenser at the lower temperature θ_2, is $H_1 = c\theta_2$; $H_1/\theta_2 = c$. Hence $H/\theta_1 = H_1/\theta_2$; and from this we may not only derive the former equation $\frac{H - H_1}{H} = \frac{\theta_1 - \theta_2}{\theta_1}$, but also the equation $\frac{H}{\theta_1} - \frac{H_1}{\theta_2} = 0$; an equation which, in the most general case, takes a form applicable to the most complex reversible cycle, namely, $\Sigma \frac{H}{\theta} = 0$, or $\int \frac{dH}{\theta} = 0$ (Thomson) ; an expression very convenient for mathematical purposes, but difficult to translate into words. In a perfect, a reversible cycle, the Entropy,* the sum of the equivalences of all the transformations is zero (Clausius). In a non-reversible process the sum of the transformations is positive, and since all processes are non-reversible, the sum of the entropies in the universe tends to a maximum. According to Rankine's mode of expression, substantially identical with the preceding, the second law is: If the absolute temperature of a uniformly-hot substance be divided into any number of equal parts, the effect of each of those parts in causing work to be performed is equal. This implies that the absolute temperature is proportional to the total heat-energy, and so merges into the preceding form of the second law.

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* Clausius has applied the term Entropy to the expression $\Sigma(H/\theta)$; and it will not be difficult to see that where that sum is positive, more heat is given to the engine by the source than is given when that sum = 0, the work done, W, remaining unchanged ; and this excess is wasted by passing through the condenser to the external universe.

Lastly, Carnot's principle itself is often called the Second Law of Thermodynamics.

We have already studied the direct transformation of heat into work in the radiometer. In the steam-engine the heat of the steam may be in part converted into work; the piston is bombarded by the particles of the steam, and if the resistance to its onward movement be not excessive, it is thrust forward by the joint impact of the particles which impinge on it, their several components of motion parallel to the piston-rod being effective in this respect.

Even under the most favourable circumstances which can be conceived, heat cannot be wholly converted into work by any form of continuously-acting mechanism. The efficiency of the ideal perfect engine—small though that efficiency be—is never approached in practice; and the efficiency of the human body considered as a machine—one-fifth of the total energy supplied to it being capable of utilisation—is remarkable when we consider the narrow limits within which it operates.

Work can thus be wholly converted into heat, but heat can never be wholly converted into work; whence a universal tendency to the Degradation of Energy into Heat, the lowest of its forms.

MEASUREMENT OF HEAT.

Temperature we have now seen to be, when measured from an absolute zero—a zero of absolute cold—(1) proportional to the absolute amount of molecular kinetic energy, and (2) to be the reciprocal of Carnot's function.

What is meant by equal degrees of heat? Why is the difference between 0° C. and 1° C. supposed to be equal to that between 100° C. and 101° C.?—In a perfect gas equal differences of temperature correspond to equal increments of energy.

In a diagram containing a system of adiabatic and isothermal lines, the isothermal lines must be so drawn as to cut off equal areas between the adiabatic lines.

Absolute zero would correspond to total absence of heat-energy.

If we had a perfect gas at command we might measure temperature by its means in either of two ways:—

(1) We might observe its **pressure** at constant volume: equal increments of pressure correspond to equal increments of temperature.

(2) We might observe its varying **volume** at constant pressure: the volume is proportional to the absolute temperature, and equal increments of volume approximately correspond to equal increments of temperature.

The former is the more accurate method.

We have no perfect gases to experiment upon: air, etc. are not perfect gases. Yet we may perform either of the above operations on a quantity of air confined in a flask, and thus construct an air thermometer. The former method—that of observation of pressure—is here doubly preferable to the latter—that of observation of expansion—because in the former there is no waste of energy in doing either internal or external work, and the increase of pressure is appreciably the same as that of a perfect gas. The indications of an air thermometer used in this way may hence be assumed as an approximate standard of comparison.

For the corrections necessary, see the table in Tait's *Heat*, p. 340.

By the air thermometer we find that for a fall of 1° C. (from 1° to 0° C.) on the mercurial thermometer, the pressure sinks in the ratio of 274 to 273; hence the temperature sinks in the same ratio, absolute zero is $-273°$ C., and Carnot's function has the numerical value of $\frac{1}{273}$ for a temperature of 0° C., and of $\frac{1}{273+t}$ for a temperature of $t°$ C.

Two bodies are said to be at different temperatures when the one has a tendency to lose heat to the other; to have the same temperature when there is no such tendency: and bodies are at the same temperature when they have the same heat-energy *per molecule*, not per unit of weight.

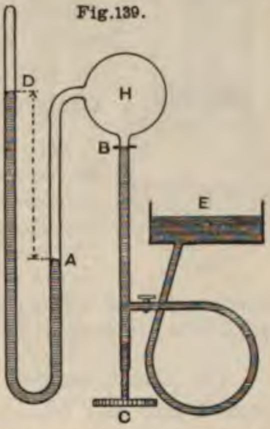
Differences of temperature may be roughly perceived by the hand; the sense of temperature can even be cultivated like that of musical pitch so as to arrive at approximate accuracy without actual recurrence to a standard of known temperature.

Any of the effects of heat may be used for detecting the presence of heat and for constructing a thermoscope. Arbitrary graduation of any thermoscope will enable it to be used as a thermometer.

Bréguet's metallic thermometer is a spiral strip composed of three metal strips soldered together by their broad surfaces: the different rates of expansion cause the spiral to roll or unroll according to the variations of temperature, and thereby to move a pointer.

The **air thermometer**, one of whose forms is shown in Fig. 139, is principally used as a standard of reference. AD is a manometer, in which above D there is a Torricellian vacuum: H is an air chamber, E an auxiliary cistern of mercury. As far as the mercury A is depressed below a certain mark, so far is the level of mercury at B raised by raising the mercury cistern E, closing the stopcock, and effecting a fine adjustment by means of the screw C. The volume of the gas between B and A is thus made constant, and the column of mercury AD measures the pressure of the gas in H.

Fig. 139.



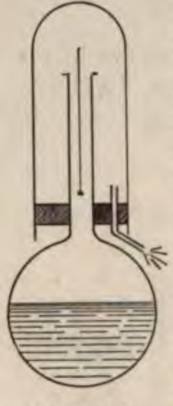
The same thermometer may, by an adjustment of the height of the column AD, be used as a constant-pressure-and-variable-volume air thermometer.

The air thermometer presents the disadvantage of being extremely unwieldy; but it has the advantage that the comparatively small expansion of the glass produces little effect in causing any difference between the apparent and the real expansion of the air, or in vitiating the adjustment to constant volume.

The ordinary **mercurial thermometer** is a familiar object. Its simplest form would be that of a flask with a long neck. If the neck were open, the mercury would be in danger of accidental loss and of evaporation; the neck must therefore be closed. If it were simply closed, the air contained in the neck would at high temperatures be compressed; the bulb would burst; hence a vacuum must be produced in the upper part of the neck. This vacuum is produced by closing the tube while mercury is boiling within it; on cooling the mercury contracts and retracts, leaving a space containing only a certain quantity of the vapour of mercury.

As to the graduation of the mercurial thermometer, this might be effected by comparison with an air thermometer, a troublesome process, resulting in degrees true but unequal in size; or by taking advantage (Renaldini) of the fact that the "freezing point" of water—or, better, the melting point of ice—and the "boiling point" of water—or, better, the temperature of steam at the pressure of 760 mm. Hg.—are constant temperatures, and may be taken as fixed points; that the height assumed by the mercurial column at these two temperatures may be marked on the tube; and that the tube between these two marks may (Newton) be mechanically graduated by equal division into degrees—a method certainly convenient, but only approximately correct.

Fig. 140.



The boiling point of water is estimated by inserting the thermometer in an atmosphere of steam surrounded by a steam-jacket (Fig. 140), intended (Berthelot) to check irregular condensation. The pressure must be the standard, 760 mm. Hg. The "freezing point" must be determined by the position assumed by the mercury when the water which trickles off melting ice flows in a stream over the mercury bulb, the whole being surrounded by a jacket of melting ice.

On the Centigrade thermometer (Linnaeus) the "freezing point" and the boiling point are respectively 0° and 100° C.; on the Fahrenheit scale they are 32° F. and 212° F.; 0° F. being the lowest temperature attained by Fahrenheit (*Phil. Trans.* 1724) by means of a mixture of ice, water, and salt or sal-ammoniac.

Fahrenheit did not use the boiling point of water as a standard, but imagined his zero to be an absolute zero, and then made or intended to make the freezing point of water to stand at one-third between this absolute cold and the temperature of the human body, which for convenience he called 96°.

Water has been used as the expanding substance in thermometers; it is objectionable on account of its point of maximum density. Alcohol is used at very low temperatures, because it is not readily frozen. Mercury, which is very advantageous on account of its low specific heat and its ready response, was brought prominently into notice by the astronomer Halley.

The sensitiveness of thermometers—the power of revealing minute variations of temperature—is increased by narrowing the tube or by enlarging the bulb. A large bulb is, however, inconvenient; because it is difficult of insertion in apertures—a fault which may be remedied by giving the bulb a cylindrical form; because it may alter materially the temperature of the object whose temperature is to be ascertained; because it slowly equalises its temperature with that of the object. A narrow tube is inconvenient because a narrow thread of mercury is difficult to see; this may be remedied by using a tube of flat elliptical section, and by enamelling the back of it.

The main causes of error in the use of a thermometer are, that the graduation alters, the "zero rises," or a thermometer inserted in melting ice comes in course of time apparently to indicate a temperature somewhat above 0° C. or 32° F., this effect being probably due to a slow yielding of the bulb to atmospheric pressure; and further, that it is not always possible to ensure that the whole of the mercury is at the same temperature.

In testing a thermometer it is important to see that the "freezing point" and the "boiling point" are accurately indicated by it, or that it agrees with a thermometer in this respect correct; and that the bore of the tube is uniform, so that a little detached portion of the thread of mercury may occupy an equal length in all parts of it.

For accurate comparison of thermometers they should be immersed together in a cooling fluid rather than in one which is being heated (Fourier); the temperature indicated by a thermometer in a cooling fluid is always a little higher than that of the fluid.

For practical details connected with testing thermometers see Gscheidlen, *Physiologische Methodik*, p. 76.

For observations of the temperature of the skin it is well (Colin) not to cover the bulb with flannels, or to leave the thermometer in such circumstances for too long a time, for the skin assumes the temperature of the interior; rather should quickly-acting thermometers be used. Apply a thermometer quickly, fresh from the pocket or the hand: keep it closely in contact with the skin; avoid blowing on the bulb; put a little cupola of paper or cotton over the bulb, but not in contact with it.

Special forms of Mercury Thermometers.—In the Maximum thermometer, above the column of mercury, a small bubble of air is introduced; above this a little thread of mercury. When the temperature rises, the air is compressed, the thread is pushed upwards; when the temperature falls back, the thread of mercury does not return.

The Minimum thermometer is usually a spirit thermometer with a little broad-headed piece of wire loosely fitting in the spirit. It is adjusted with its head touching the surface of the thermometric liquid. When the liquid contracts, surface-tension drags the wire with it; when the temperature rises, the liquid passes the wire without forcing it upwards: the position of the end of the wire nearest the free surface indicates the lowest level to which the surface had sunk, and therefore the lowest level which had been attained since the last observation.

In Metastatic thermometers any part of the mercury may be removed from the column and shaken aside into an apical cavity, or restored in whole or in part to the main thread; the thermometer, a very delicate one, being thus competent to read to very small fractions of a degree at any part of the scale chosen at will. See Gscheidlen, p. 84. The principle of overflow—

liquid being caused to expand and overflow, or vapour (iodine, mercury) being boiled out of a heated flask, what remains being weighed when cooled—is utilised in the construction of some pyrometers.

As for the measurement of temperature by electricity, that will be explained on page 575.

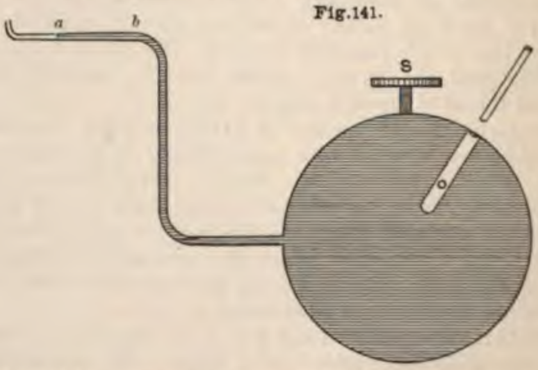
Calorimetry or the quantitative measurement of heat. —The Calorie (*Ca*) is the amount of heat required to raise the temperature of 1 kilo. of water (or $1/x$ kilos. of any substance whose specific heat is x) from 0° C. to 1° C. The calorie or small calorie (*ca*) is the amount of heat similarly required to heat one gramme to the same extent. Convenience rules the choice between these units.

1. The Method of Mixtures.—This may be illustrated by a numerical example. How many calories of heat does a gramme of mercury absorb when it is heated from 0° C. to 1° C.? —a question identical with an enquiry as to the specific heat of mercury.

One gramme of mercury at 100° C. and one of water at 0° C. are mixed: the result is a uniform temperature of 3°·194 C. The water has gained 3·194 calories; the mercury has lost the same. Mercury on losing 3·194 *ca* per gramme is cooled through 96°·8 C.; cooling through 1° C. involves a loss of ·033 *ca*. The specific heat of mercury is thus ·033, and the amount of heat contained in the mass of mercury mixed with the water was $(373 \times \cdot 033)$ *ca*, 373 being its absolute temperature and its mass being unity.

A modification of this method is that of Fig. 141. A globe filled with mercury: the free surface of the mercury at *a*; the

Fig. 141.



screw *s*, which alters the position of the surface *a* so as to bring it to the zero of a scale marked on the horizontal tube *ab*; the hot substance, introduced into a depression at *o*, heats the mercury, expands it, and causes the capillary surface *a* to assume a new position.

Otherwise, instead of observing the direct expansion of the fluid heated, its temperature may be taken by a thermometer.

Dulong's water calorimeter is of this kind: a copper chamber containing a living animal supplied with air by afferent and efferent pipes: round this a water-jacket, the water in which assumes a certain observed temperature. The number of calories taken up by the water and by the copper or other vessels (considered as equivalent to x times their mass in water, x being their specific heat) is found, and that amount thus measured is the amount of heat given out by the enclosed animal.

2. Latent heat methods.—The amount of heat of a hot body may be measured by the amount of ice melted by it—this being ascertained roughly (Lavoisier and Laplace) by the amount of water which trickles from ice amid which the hot body is thrust; or, better (Sir J. Herschel and Bunsen), by observing the actual decrease in volume of a mixed mass of ice and water when some of the ice is melted; or by the amount of liquid—water, ether, acetic aldehyde—which the heat of a hot body (or living animal—Rosenthal, *Arch. f. Anat. und Physiol.*, 1878) can evaporate.

TRANSFERENCE OF HEAT.

When two masses or parts of the same mass are in contact, the molecular agitation of each is in part communicated to the other: if they be equally hot, each receives as much heat as it gives up: if they be not equally hot, that which has the more molecular energy loses more than it receives, while the other, the colder, gains more heat than it loses. The flow in one direction thus overpowers that in the other, and, on the whole, heat is transferred from the hotter mass to the colder.

We regard in general only the difference, and not the common part; the surplus which flows from the hotter and not the compensated and non-apparent flow from the colder body.

This tendency is universal. Heat always tends to pass on the whole from hotter to colder bodies, and if these be in contact, the transference is effected by **conduction**; whence all bodies possess some degree of **conductivity** or power of transferring heat through their substance.

When two points in a substance are at temperatures constantly differing by $\delta\tau$, and are at a distance d, a flow of heat is set up between them. The amount of heat which passes from the one point to the other in time t is proportional (1) to the length of time during which the flow proceeds; (2) to $\delta\tau$, the difference of temperatures; and (3) it is inversely proportional to d, the distance between the points.

Otherwise, the flow $f \propto t.\delta\tau/d$, or $f = \mathrm{K}.t.\delta\tau/d$. Here K is a coefficient, the Coefficient of Conductivity, and $\delta\tau/d$ represents the Fall of Temperature per unit of distance, the Temperature-gradient.

The **coefficient of conductivity** varies from substance to substance, being greatest in the metals; some substances permit a rapid, some only a slow transfer of heat; compare a horn spoon and a silver one inserted in a hot liquid.

If the one point be maintained at the temperature T and the other at the temperature $\mathrm{T}_{,}$, intermediate points have temperatures which from point to point sink uniformly with the distance from the hotter point. There is thus set up a condition of **Steady Flow** of Heat.

Across a plate of thickness d whose sides are maintained at an *actual* and constant difference $(\mathrm{T}-\mathrm{T}_{,})$, the flow of heat per unit of area will in time t be $\mathrm{K}.t.(\mathrm{T}-\mathrm{T}_{,})/d$; across area A the flow will be $\mathrm{KA}.t.(\mathrm{T}-\mathrm{T}_{,})/d$.

This is $t.\mathrm{KA} \times$ the temperature-gradient.

If a bar be heated at one extremity, the amount of heat which will arrive at a sectional area a given distance along the bar will depend upon the thickness of the bar and its proportional surface. A thin iron wire may be melted at one end and not have its temperature raised by 1° C. at a distance of 6 feet; so much heat is lost on the way, being spent in warming the surrounding air and in keeping up radiation from the surface. For the same reason, the most volatile oil may be burned in a lamp with a sufficiently long wick-tube.

In bars of different thickness the distances from the heated extremity at which the same temperature can be kept up by heating the extremity of the bars to the same temperature are to one another as the square roots of the thicknesses; and in bars of the same thicknesses but of different lengths the flow of heat into the bar varies as the square root of the cube of the length.

A hot point in space conceived to be maintained permanently hot will be the centre of a flow of heat symmetrical in all directions. The points in the surrounding space which are at the same temperature may be connected and found to lie on concentric spheres, or spherical **Isothermal Surfaces.**

The heat travels by the shortest path from one surface to another, by **Lines of Propagation,** or Lines of Flow, at right angles to both; and there is on the whole no lateral propagation over an isothermal surface. The whole system of surfaces and lines closely resembles a system of equipotential surfaces and lines of force. The difference of temperature per unit of distance along the lines of propagation decreases with the distance, being proportional to ($1/\text{radius}^2$). The greater the curvature of a hot body, the greater will be its loss of heat by conduction. Hence an ellipsoidal body maintained at a uniform temperature loses most heat where the curvature is greatest—a proposition closely resembling one in the theory of electricity.

We must distinguish a Flow of Heat from a Flow of Temperature. The latter depends on the specific heat; and if we compare the passage of heat through two substances similarly heated, we find that even though the one substance have a greater conductivity than the other, yet, if its specific heat be greater in a still greater proportion, a given temperature may take a longer time, travelling in the better conductor, to reach a point at a given distance from the source of heat, than it does in the worse conductor.

Where a substance is not physically similar in all directions, as in the case of crystals, the conductivity may be unequal in three directions. Thus, a plate cut out of any crystal belonging to the biaxial system, and covered with a film of wax, will, if heated by a hot wire passed through its centre, so conduct the heat that the wax melts not in a uniform circle—as in glass or a crystal of the regular system it will do—but in an ellipse.

Some solids are extremely bad conductors of heat. Down is perhaps the worst of all conductors; hare's fur, sand, asbestos, are examples of substances within which warm objects may be placed and remain without losing their heat to any material degree for some time. Flannel, cork, etc., appear warm when they are touched by the bare skin, because they carry away by conduction less heat than the air had been removing before these materials had been touched. Wood is in the radial direction a bad conductor: this has a certain effect in preserving the tree in life.

The actual amount of the loss of heat suffered by a cooling body depends directly on the effective cooling surface: whence the natural tendency in warm weather to lie at full length, in winter to roll the body up into small compass.

The conductivity of the skin as a whole is greatly diminished by a layer of fatty tissue. The muscles are exceedingly bad conductors.

When a hot body is surrounded by one or more concentric jackets with layers of air between them, the loss of heat is remarkably diminished. A single layer of linen diminishes the loss of heat from the human body by about two-thirds; a double layer effects a much greater economy of heat, and so forth. The practice in cold countries of using double windows proceeds on this principle, and hence also the hygienic advice to multiply the number of light garments in cold weather rather than their weight.

The conductivity of liquids is as a rule greater than that of gases, which in the form of true conduction—as distinguished from convection—is very small.

It is impossible to keep the hands in water at 52° C., while it is quite possible, as observed by Banks, to remain for five minutes in air near the boiling point of water.

When a hot body is placed in air it sets up a number of **Convection currents.** Air becomes heated and rises, carrying away the heat of the hot body: colder air takes its place.

Newton's law of cooling in a current of air is, that at each instant the amount of heat lost varies as the difference of temperature between the solid and the air. This law seems to be adhered to within narrow limits.

In an undisturbed atmosphere the law of cooling by convection is, that the velocity of cooling is proportional to $p^a t^{1 \cdot 233}$, where a is a constant ($\cdot 45$ for air), p the pressure, and t the excess of temperature (Dulong and Petit).

In hydrogen the process of cooling is very rapid.

The carbonic acid, etc., of the atmosphere are mixed thoroughly and equably, not by diffusion, which would take several hundred thousand years to accomplish the task, but by convection currents.

Convection currents, as they pass colder or warmer strata of air, exchange molecules with them by diffusion; the temperature of the whole mass thus rapidly becomes uniform.

Convection currents may be demonstrated by throwing some coloured powder into cold water and proceeding to heat the liquid over a lamp; by looking at distant objects through the heated gases which arise from a heated boiler or wall: the rise of smoke itself is an example of solid particles borne upward by convection currents—particles which, when the ascending air has become cool, again fall, and may aid in producing fogs by the condensation of water around them.

Though two bodies be not in contact with one another, they may yet exchange heat across the intervening space, and the hotter body, giving out more heat than it receives, is said to **radiate heat** to the colder body. This transfer of heat is effected by means of the ether of space, and we shall, in the meantime, defer the consideration of the transfer of heat by radiation until we can take a general view of waves in the Ether.

Dulong and Petit found that between 0° C. and 200° C., the amount of radiation is proportional to $(1 \cdot 0077^t - 1)$, where t is the excess of temperature above the surrounding enclosure.

Transport of Heat from place to place may be effected by storing up work-energy in springs which, on being released, set a mechanism at work which evolves heat by friction; or by storing up heat as "latent heat," or raising the temperature of a substance whose specific heat is high. The former method is not effective, because so large a number of units of work correspond to so small an amount of heat; the latter are exemplified in heating by hot water or by steam. A hot-water bottle contains several calories of heat, according to its size and its temperature; these can be liberated by conduction at any desired situation. Steam at 100° C., when condensed, liberates at the point of condensation 546 calories of heat for every gramme of water con densed, and can still, in the form of hot water, surrender more heat to surrounding objects whose temperatures are below 100° C.

CHAPTER XIV.

ON SOUND.

THE word Sound is used in four different senses :—

1. The physiological sensation perceived by means of the ear.

2. The complex harmonic motion of sounding bodies—the Fourier-motion, the periodic or vibratory motion of elastic masses whose vibration is the physical cause of sound.

3. The disturbances of the air which affect the ear. "Sounds," says Newton (*Princip.* ii. Prop. L, Prob. xii. Schol.), "since they arise in tremulous bodies, are no other than waves (*pulsus*) propagated in the air."

4. The energy of a sounding body. "Heat converted into Sound," etc. It is better in this sense to say explicitly, "the Energy of Sound."

A sounding body is a vibrating body.

Cause a tuning-fork to sound in the usual way—by striking it on the knee or drawing a violin-bow across it, or by forcing a steel rod between its prongs and drawing it through the point of the fork. Apply the point of the vibrating tuning-fork to the lips, to the surface of water, to a piece of glass. Bring a vibrating tuning-fork under a light splinter of wood lying upon two points of support ; on contact the light body will be hurled upwards. Cautiously bring a vibrating tuning-fork or bell into contact with a pith-ball suspended by a thread.

Pluck one of the strings of a violin : look at it as it vibrates : touch it. Look at a harmonium or concertina reed while it is in action.

Observe the distinct tremor caused by a large organ pipe while sounding, or even by a large drum.

Relatively deep, grave sounds are produced by slower vibrations; higher, shriller sounds by more rapid vibrations.

Take a long strip of iron—say a strip 4 feet long ; fix it in a vice ; pull it aside and let it go ; it will oscillate transversely at a rate such that the oscillations can be counted ; remove it, and refix it so that only 2 feet of it are now free to move ;—it will now oscillate four times as frequently : 1 foot free—sixteen times as frequently as at first ; 6 inches free—sixty-four times as frequently, and so on. The oscillations now become so rapid, the number of them in a second (*i.e.*, their *frequency*) becomes so great, that they can no longer be counted directly; now we hear a sound; the shorter the vibrating part, the more rapid become the vibrations, the shriller the sound.

The transmission of sound from a vibrating body to the ear involves, as a rule, the formation of sound-waves in the air.

This may be rendered impossible, *e.g.*, where the sounding body—a bell suspended or placed upon wadding within the bell of an air-pump from which the air is exhausted—has no contact with air, and therefore no means of transferring its own vibration to air; in such a case the ear perceives no sound, even though the bell be struck, for there are no air-waves set up.

But it may be impossible for another reason. Air will not oscillate in waves such as can be propagated to a distance, unless there be some well-marked compression or rarefaction produced at the centre of disturbance. Take as extreme instances of sound produced by well-marked compressions or rarefactions the effect of the discharge of a cannon, which abruptly adds a mass of gas to the already-present atmosphere, and thereby produces great and sudden compression; or the rarefaction produced by the sudden collapse of a weak boiler when the steam contained in it has cooled down. Thus a vibrating body, before it can act as a sounding body, must produce alternate compressions and rarefactions in the air, and these must be well marked. If, however, the vibrating body be so small that at each oscillation the surrounding air has time to flow round it, there is at every oscillation a local rearrangement—a local flow and reflow—of the air, but the air at a little distance is almost wholly unaffected by this. The same result follows if the medium surrounding the vibrating body be rare—*e.g.*, hydrogen—or rarefied—*e.g.*, rarefied air; then, on account of the small inertia of the medium, it is easily induced to flow round the vibrating body; in such cases there is but little wave-motion caused at any distance, and thus there is but little sound produced.

A string stretched between two points of a rigid and massive framework produces surprisingly little sound when caused to vibrate: it does not act upon the air otherwise than by setting up local flow and reflow. If the same string be stretched over bridges upon a sounding-board, the string gives at each oscillation an impulse to the sounding-board which causes it to yield slightly; and thus the string causes the sounding-board to vibrate. But though the amplitude of its vibration is small, the sounding-board is broad, and the air cannot, by flowing round its edge, evade compression and rarefaction; the air is, accordingly, alternately compressed and rarefied, and thus a system of waves is effectively set up in it. Thus the loudness of the sound produced by a string may, by the use of a sounding-board, be multiplied many thousandfold. A similar experiment may be performed with a vibrating tuning-fork suspended in the air by a string, and the same fork vibrating while its shank is pressed against the panel of a door.

In these cases the energy of vibration of the string or tuning-fork is very much more rapidly dissipated, while the large-surfaced sounding-board is enabled to produce an intenser or louder sound than is produced when the string or the fork vibrates alone; and the vibration sooner comes to an end.

The speaking-trumpet is in part an application of the same principle. Instead of a comparatively small surface, the oral aperture, being the source of sound, the much broader aperture of the trumpet is practically converted into the source, and the broad sound-waves thence issuing are only slightly weakened at their origin by lateral flow.

As a general rule it is therefore advisable, when sound is to be heard at a distance, to make the sources of sound of the largest size convenient. Smallness of size may, however, be compensated by quickness of vibration.

Thus the chirp of certain insects is produced by such extremely rapid movements—as many as 12,000 to-and-fro vibrations per second—that the air is alternately compressed and rarefied on each side of the wings or in the neighbourhood of the stridulating organs, without having time to flow round them.

Characteristics of Sounds.—The Fourier-motions which may produce sounds differ amongst themselves in their

(*a.*) **Frequency** — the number per second of the slowest component-oscillations.

An oscillation is a complete oscillation, once to-and-fro. The frequency of a seconds pendulum is $\frac{1}{2}$; in one second it performs half a complete oscillation. In French works we find that a "*vibration simple*" is half a complete oscillation, a swing over from one side to the other; and a seconds pendulum is held to effect such "*vibrations simples*" at the rate of one per second. The reason for the apparently more artificial mode of defining an oscillation here used will be seen on considering the meaning of *period* in S.H.M.; a complete oscillation restores the oscillating body to its starting point.

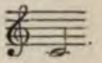
(*b.*) They differ as to their **Energy**. Proportioned to the energy are the Intensity and the Square of the Amplitude.

(*c.*) They also differ as to the **Relative Amplitudes** of their **Components.**

Of these three particulars, the first, the frequency, depends on the vibrating body itself, its form, its material, etc., and upon its tension, but is very slightly affected by its viscosity; the second depends entirely on external causes; the third depends partly on the form, the tension, the rigidity, etc., of the vibrating body, partly on the manner in which it is set in motion.

By variations in these particulars an infinite variety of Fourier-motions may be produced in vibrating or sounding bodies; and as a natural consequence we might expect to find, as we do find, an infinite variety of musical sounds actually occurring in nature.

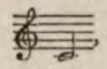
Musical sounds may differ from one another in three corresponding respects, viz.—**Pitch, Loudness,** and **Quality** or Character.

Pitch.—The pitch of a clear musical sound depends on the Frequency of the Fundamental Vibration of the sounding body. Suppose a string to vibrate harmonically, and its component vibrations to occur 261, 522, 783, 1044, etc. times per second: then that string would have a fundamental vibration whose frequency is 261 per second; and a sound of this fundamental frequency is recognised by our musical sense as the note .

The **loudness** of a sound increases with the amplitude of oscillation of the vibrating body; if two strings, otherwise similar and similarly circumstanced, oscillate through ranges of $\frac{1}{8}$ and $\frac{1}{4}$ inch respectively, the latter has twice the amplitude and tends to produce four times as much sound as the former: the loudness or intensity of sound being among sounds of the same pitch proportional to the energy of vibration, and therefore to the square of the amplitude. Mark, however, that the relative loudness of different sounds as perceived by the ear is not to be measured by their physical intensity or the square of the amplitude of the vibrations at their source, for the ear is not necessarily, and is not in fact, equally sensitive to sound of every pitch.

Viscosity of a sounding body, while it scarcely affects the pitch, aids in causing the amplitude of the vibration, and therefore the loudness of the sound produced, gradually to dwindle away.

As to their **Quality** or **Character,** we find among sounds an infinite variety. We can distinguish a sound produced by a violin from one of the same pitch and loudness produced by a clarionet, a flute, or a pianoforte; we can distinguish the sound of a viola from that of a violin; one violin from another; one player from another on the same violin; one person's voice from that of another; the voice of the same person in different moods or states of health. The basis of all this variety lies in the endless differences that may exist between Fourier-motions which, though they agree as to the frequency of their fundamental or slowest component and as to the total energy involved in their movement, do not necessarily coincide in the relative amplitudes of their component harmonic motions.

But if, as this theory indicates, an extended series of component vibrations go to make up the aggregate vibration of a sounding body, ought we not in the sound produced by a sounding body to hear a series of tones corresponding to the series of vibrational components? If a string produce the note , corresponding to a fundamental vibration whose frequency is 261 per second, ought we not at the same time to hear other sounds corresponding to 522, to 783, to 1044, etc. vibrations per second? The reply is that we do actually hear such tones; but we do not attend to them, and for practical purposes we are therefore deaf to them. We are accustomed to interpret a sound produced by a single sounding-body—the voice of a person, for example—as a single sound; from earliest infancy we unconsciously train ourselves to listen only to the fundamental tone of any single note: and the presence of the other tones of the really-compound sound produced by a single vibrating-body has the apparent result of determining the Character of that tone to which alone we consciously listen. In many cases, when we listen for the higher component sounds, knowing what to listen for, we can hear them, even with the unaided ear: after practice the ear acquires the power of recognising the presence of these **harmonics** with great readiness—a power which may easily become oppressive to its possessor. The special training which confers this power differs only in degree from that which enables one to discriminate the different notes which make up a chord, sounded in harmony; for to the untrained ear even a chord, if it be well in tune, seems to be a single mass of sound.

Noise.—If all the keys of a piano within the compass of one or two octaves be simultaneously struck, the result is a confused jangle, a Noise. Here we have the Superposition of Fourier-motions resulting in an apparently irregular disturbance of the air. This may go still further; the Fourier-motions, which are superposed on one another, may have no relation of frequency and little or no individual persistence. The more markedly this is the case, the less musical will be the sound produced, and the more markedly will it bear the character of noise. The general hum of a town is made up of sounds and cries, each of which, taken singly, may perhaps not be unmusical; but because they are not related to one another by any simple numerical ratio of frequency, they together produce the disagreeable effect of a noise. Noises, then, such as the sound of steam escaping from a boiler, wind rushing through trees, the clatter of falling objects, and so forth, may be considered to be produced by the superposition of a number of distinct musical sounds. Some of these may predominate in intensity and in persistence; and thus a noise may have a distinguishable pitch. We may recognise differences in pitch between the noises produced by drawing the thumb-nail at various speeds over the cover of a book bound in cloth, by blowing across the mouth of keys or tubes or flasks of various sizes, by letting boards of various sizes fall on a wooden floor, by blowing through glass tubes on which bulbs of various sizes have been blown, and so forth. Even where the original disturbance is in the highest degree irregular, as where bricks are pitched out of a cart, the elasticity of the bricks, small though it be, affects the pitch of the noise produced, for the noise produced by soft porous bricks is graver than that produced by hard glazed-bricks of the same size.

If we listen to a continuous noise with the aid of a resonator (p. 395) tuned to some particular tone, we can often recognise the presence of that tone as a component of the noise; the resonator will, if it be present as a component, sound it forth—continuously if it be continuously present; intermittently if it occur at intervals only.

Even a single vibrating-body may, when struck, produce a noise. A bell is not, with ease, so cast as to be perfectly uniform; when struck it tends, if not quite uniform, to divide into unequal sectors, each of which pulsates at its own rate; the physical result is a number of simultaneous vibrations bearing no simple relation to one another, and the physiological result is a mixed sensation, a jangle, a kind of noise.

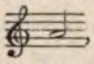
Thus sounds originate in Fourier-motions; a musical note in a single Fourier-motion; a noise in a number of simultaneous Fourier-motions whose fundamental frequencies bear to one another no simple numerical relation; and, as we shall afterwards see, the sensation of harmony in a number of simultaneous Fourier-motions whose fundamental frequencies have a simple numerical relation to one another.

The simplest possible sound would be one produced by a vibration in which the Fourier-motion was represented by one component; such a sound would be a **pure Tone.**

The **pitch** of the sound or note produced by a vibrating body is the pitch of the gravest component, the fundamental Tone; and it may be specified in two ways:—

(1.) **Physically,** by stating the number of vibrations per second which correspond to that fundamental tone;

(2.) **Musically,** by referring the tone to its place in an arbitrary scale of pitch in conventional use among musicians.

To find the frequency of vibration corresponding to any given note:—As the note in question let us take, for the sake of example, that produced by an ordinary "A" tuning-fork. A card or strip of metal is placed so as to touch at one end the cogs of a little cog-wheel, while the other end is firmly fixed; the wheel is rotated slowly—each cog makes one click; more rapidly—the clicks blend into a hum; still more rapidly—the hum rises in pitch, and the faster the rotation the shriller becomes the sound; at a certain rate of rotation the sound is neither graver nor shriller than that produced by the tuning-fork; this rate of rotation is such that the card is struck 435 times per second; 435 impulses per second given to the card, and by the card to the air, produce the sound , "$a'=435$." Higher sounds are due to more rapid, lower sounds to slower, vibrations than this. This arrangement is known as Savart's Wheel.

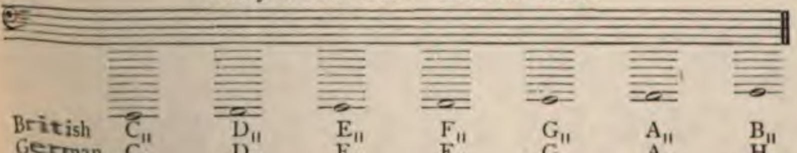
Another contrivance, devised to the same end, is the Syren. A rotating disc is pierced by holes arranged equidistantly in a circle, whose centre is in the axis of rotation of the disc. A tube brings a current of air to a spot near the disc so situated that in some positions of the disc the air can blow clear through one or other of the holes, while in others the current of air is almost cut off by the disc itself. Rotate the disc; the current of air is alternately cut off by the disc and allowed to blow through it. If there be 87 holes in the circle of holes, and if the disc rotate five times per second, there are then produced 435 puffs of air per second, and the note "a'" is heard: its quality is, however, decidedly inferior, for the principal sound heard is the noise made by the current of air when it strikes the disc. If the current be divided by 87 pipes, so as to blow through the 87 holes simultaneously, and to be simultaneously cut off from them all, the sound is very much clearer and louder than when there is only a single stream of air blowing through one hole at a time. Instead of 87 pipes issuing from a wind-chest, we may employ a wind-chest capped by a fixed disc containing 87 holes, arranged in a circle like that of the rotating disc: the rotating disc rotates in the immediate vicinity of the fixed one: simultaneously the air rushes through all the apertures of the rotating disc, simultaneously it is cut off from them all. The number 87 is in practice never used; some such number as 24 or 48 is chosen. Connected with the rotating disc is some form of mechanism for recording the number of rotations effected by it in a given time; the rotating disc is caused to rotate at such a speed as causes the desired sound to be produced: the number of apertures in the disc, multiplied by the number of rotations per second, gives the number of impulses per second imparted to the air, and thus determines the frequency of the tone in question. The syren works under water as well as it does in air.

The experiment already described on page 378 also gives roughly the means of finding the frequency of any given note. The thin strip of metal is, by successive trial, carefully withdrawn into the vice, until its free part gives, when set in vibration, a sound of precisely the same pitch as the tone whose frequency is to be determined. Say that this length is 1 inch; and also that if 30 inches of the strip be free, it executes 29 complete oscillations per minute. The number of oscillations varies inversely as the square of the length; whence $(1 \text{ inch})^2 : (30 \text{ inches})^2 : 29 : x$, or $x = 26{,}100$ vibrations per minute, 435 per second.

Still another method of determination of the frequency of vibration of sound of a given pitch is graphically to record the actual vibrations of the sounding body. A tuning-fork has a little feather-barb attached by cement to one of its prongs: the extremity of the barb is brought into contact with slightly-smoked paper spread over the surface of a cylinder. The cylinder is caused to rotate; the point of the barb draws a straight line on the smoked paper. The fork is caused to vibrate: the barb now describes, on the rotating cylinder, a sinuous line which records the oscillations of the tuning-fork. An independent mechanism can be made to mark the cylinder once every second, and thus the absolute number of oscillations made by the tuning-fork in the course of each second can be counted on the permanent record. The same principle may be applied to many forms of vibrating body, such as strips of metal, membranes, etc.

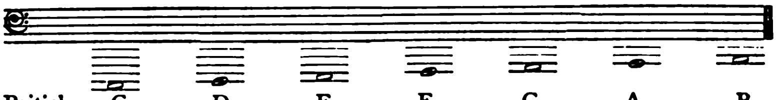
Musical Pitch.—The arbitrary scale of pitch in common use, and typified by the white keys of a pianoforte, is the following:—

Thirty-two foot Octave—Subcontra Octave.



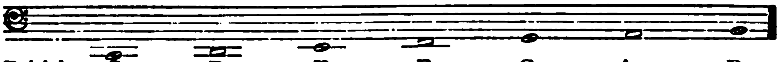
British	C_{II}	D_{II}	E_{II}	F_{II}	G_{II}	A_{II}	B_{II}
German	C_{II}	D_{II}	E_{II}	F_{II}	G_{II}	A_{II}	H_{II}
French	ut$_{-1}$	re$_{-1}$	mi$_{-1}$	fa$_{-1}$	sol$_{-1}$	la$_{-1}$	si$_{-1}$
No. of Vibrations	16·3125	18·3515625	20·390625	21·75	24·46875	27·1875	30·5859375.
Ratios	16 :	18 :	20 :	$21\cdot\dot{3}$:	24 :	$26\cdot\dot{6}$:	30.

Sixteen-foot Octave—Contra Octave.



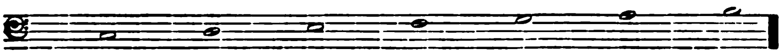
British	$C_{\vert}$	$D_{\vert}$	$E_{\vert}$	$F_{\vert}$	$G_{\vert}$	$A_{\vert}$	$B_{\vert}$
German	$C_{\vert}$	$D_{\vert}$	$E_{\vert}$	$F_{\vert}$	$G_{\vert}$	$A_{\vert}$	$H_{\vert}$
French	ut$_0$	re$_0$	mi$_0$	fa$_0$	sol$_0$	la$_0$	si$_0$
No. of Vibrations	32·625	36·703125	40·78125	43·5	48·9375	54·375	61·171875.
Ratios	32 :	36 :	40 :	$42\cdot\dot{6}$:	48 :	$53\cdot\dot{3}$:	60.

Eight-foot Octave—Great Octave.



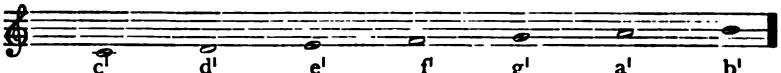
British	C	D	E	F	G	A	B
German	C	D	E	F	G	A	H
French	ut$_1$	re$_1$	mi$_1$	fa$_1$	sol$_1$	la$_1$	si$_1$
No. of Vibrations	65·25	73·40625	81·5625	87	97·875	108·75	122·34375.
Ratios	64 :	72 :	80 :	$85\cdot\dot{3}$:	96 :	$106\cdot\dot{6}$:	120.

Four-foot Octave—Little Octave.



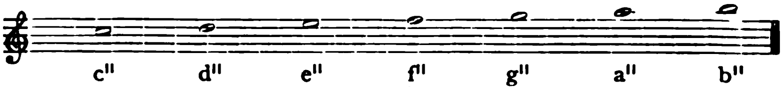
	c	d	e	f	g	a	b
	ut$_2$	re$_2$	mi$_2$	fa$_2$	sol$_2$	la$_2$	si$_2$
No. of Vibrations	130·5	146·8125	163·125	174	195·75	217·5	244·6875.
Ratios	128 :	144 :	160 :	$170\cdot\dot{6}$:	192 :	$213\cdot\dot{3}$:	240.

Two-foot Octave—One-stroked Octave.



	c'	d'	e'	f'	g'	a'	b'
	ut$_3$	re$_3$	mi$_3$	fa$_3$	sol$_3$	la$_3$	si$_3$
No. of Vibrations	261	293·625	326·25	348	391·5	435	489·375.
Ratios	256 :	288 :	320 :	$341\cdot\dot{3}$:	384 :	$426\cdot\dot{6}$:	480.

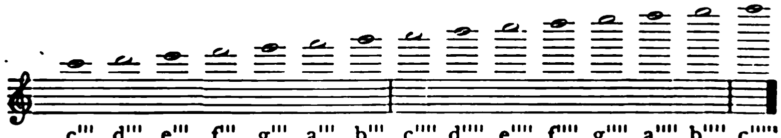
One-foot Octave—Two-stroked Octave.



	c''	d''	e''	f''	g''	a''	b''
	ut$_4$	re$_4$	mi$_4$	fa$_4$	sol$_4$	la$_4$	si$_4$
No. of Vibrations	522	587·25	652·5	696	783	870	978·75.
Ratios	512 :	576 :	640 :	$682\cdot\dot{6}$:	768 :	$853\cdot\dot{3}$:	960.

Six-inch or Three-stroked Octave.

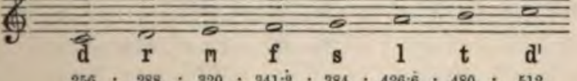
Three-inch or Four-stroked Octave.



	c'''	d'''	e'''	f'''	g'''	a'''	b'''	c''''	d''''	e''''	f''''	g''''	a''''	b''''	c'''''
	ut$_5$	re$_5$	mi$_5$	fa$_5$	sol$_5$	la$_5$	si$_5$	ut$_6$	re$_6$	mi$_6$	fa$_6$	sol$_6$	la$_6$	si$_6$	ut$_7$
No. of Vibrations	1044	1174·5	1305	1392	1566	1740	1957·5	2088	2349	2610	2784	3132	3480	3915	4176.
Ratios	1024 :	1152 :	1280 :	$1365\cdot\dot{3}$:	1536 :	$1706\cdot\dot{6}$:	1920 :	2048 :	2304 :	2560 :	$2730\cdot\dot{6}$:	3072 :	$3413\cdot\dot{3}$:	3840 :	4096.

The starting-point of this notation is the a' tuning-fork, made to vibrate 435 times per second, or the second string of the violin, made to vibrate in unison with such a fork. Under this system the c'' tuning-fork makes 522 complete oscillations per second. This is entirely a matter of convention. The number 435 was chosen by the Académie des Sciences of Paris; 440 by the German Society of Nature-researchers at Stuttgart in 1834; while a pitch $a' = 426{\cdot}6$ has been highly recommended, on the ground that under such a system the tones $C_{,,}$, $C_{,}$, C, c, c', etc., are produced by 16, 32, 64, 128, 256, etc., vibrations per second—an arrangement which has the advantage of giving very simple numbers to deal with, but which has, on the other hand, the practical disadvantage of giving a pitch which is too low to please instrumentalists, and the didactic disadvantage of tending to conceal the real arbitrariness of the convention which assigns to the a' or the c'' fork the particular number of vibrations chosen in practice. In practice there is, indeed, a great lack of agreement; instrument-makers are constantly raising the pitch for the sake of increasing the brilliancy of orchestral music, while vocalists are made to suffer. Modern concert pitch has thus risen as high as $a' = 460$ vibrations per second, about $1\frac{1}{2}$ semitone above what it was in England in the time of Handel ($a' = 424$), while the organ-pitch in England was, in the middle of the eighteenth century, as low as $a' = 388$. If the standard number of vibrations chosen for a' be any other than 435, the whole series of numbers given in the table must suffer a proportionate increase or reduction. The accuracy of such a scale depends not upon precision of absolute numbers of vibrations so much as upon correctness of the ratios of the several numbers to one another.

The successive tones of the scale of C are related to one another, with respect to their frequency, in the following manner:—



$$256 : 288 : 320 : 341{\cdot}\dot{3} : 384 : 426{\cdot}\dot{6} : 480 : 512.$$
$$1 : \tfrac{9}{8} : \tfrac{5}{4} : \tfrac{4}{3} : \tfrac{3}{2} : \tfrac{5}{3} : \tfrac{15}{8} : 2.$$

Here C ($c' = 256$) is a keynote, and upon it we have raised a diatonic major scale, **d r m f s l t d'**.

Such a scale is found by experience to be satisfying to the ears of the Western nations; and whatever tone be chosen as the keynote, there can always be sung or played on instruments of the violin or of the trombone class a scale of this kind, in which the intervals are felt to be pleasing and in tune, in which the intonation is felt to be just, and in which each tone, when it is carefully listened to while the keynote is borne in mind, is felt to have its own peculiar mental effect, this depending on its relative place in the scale, and not on its absolute vibrational frequency. Singers who have sung much together, string players who have practised together without pianoforte accompaniment, naturally use the tones of such a scale, without knowing or even caring what the numerical ratio of the frequencies of the various tones of the scale may be.

Intervals.—We may now identify the various intervals occurring within the diatonic scale—

Minor second, "*semitone*"	**m : f** or **t : d'** .	15 : 16
Grave major-second .	**r : m** or **s : l** .	9 : 10
Major second . .	**d : r**, **f : s**, **l : t** .	8 : 9
Grave (or Pythagorean) minor-third. .	**r : f**	27 : 32
Minor third . .	**m : s** or **l : d'** .	5 : 6
Major third . .	**d : m**, **f : l**, **s : t** .	4 : 5
Fourth . . **d : f**,	**r : s**, **m : l**, **s : d'**, **t : m'**	3 : 4
Acute fourth . .	**l : r'**	20 : 27
Augmented fourth .	**f : t**	32 : 45
Grave fifth . .	**r : l**	27 : 40
Fifth . . .	**d : s**, **m : t**, **f : d'** .	2 : 3
Minor sixth . .	**t_{1} : s**, **m : d'**, **l : f'** .	5 : 8
Major sixth . .	**d : l**, **r : t**, **s : m'** .	3 : 5
Acute major-sixth .	**f : r'**	16 : 27
Grave minor-seventh .	**r : d'**, **s : f'**, **t : l'** .	9 : 16
Minor seventh . .	**m : r'**, **l : s'** . . .	5 : 9
Seventh . . .	**d : t**, **f : m'** . . .	8 : 15
Octave . . .	**d : d'**, **r : r'**, etc. . .	1 : 2

Musical intervals are equal to one another when the constituent notes in each have the same relative frequency. Thus **d : s** : : $1 : \frac{3}{2}$, and **m : t** : : $\frac{5}{4} : \frac{15}{8}$; the ratio of 1 to $\frac{3}{2}$ is equal to that of $\frac{5}{4}$ to $\frac{15}{8}$—that is, it is $2 : 3$; whence the musical interval between **d** and **s** is equal to that between **m** and **t**.

Transition.—Any tone may be chosen as a keynote. Let us choose $g' = 384$ as our keynote, and then compare the tones of the scale of the key of G with those of the scale of C. Retaining the same ratios, the scale of G is

d : **r** : **m** : **f** : **s** : **l** : **t** : **d'**.
$1 : \frac{9}{8} : \frac{5}{4} : \frac{4}{3} : \frac{3}{2} : \frac{5}{3} : \frac{15}{8} : 2.$
384 : 432 : 480 : 512 : 576 : 640 : 720 : 768.

Comparing the two scales we find :—

Scale of C (" Key C").
c' d' e' f' g' a' b' c'' d'' e'' f'' g'', etc.
. . . . . $384 : 426\cdot\dot{6} : 480 : 512 : 576 : 640 : 682\cdot\dot{6} : 768.$

Scale of G (" Key G").
384 : 432 : 480 : 512 : 576 : 640 : 720 : 768.

The tones agree with the exception of the *a*'s and the *f*'s. The a' of the scale of C and the a' of the scale of G differ from one another in the ratio of $426\cdot\dot{6} : 432$, or $80 : 81$. The two tones are perfectly distinct, and an ear that has become accustomed to the pure scale of C is pained, especially in harmony, by the substitution, for the proper a' in that scale, of the slightly sharper a' which belongs to Key G. The difference between the two tones is called a Comma; and they may be respectively written a' and $'a'$. The f'' of Key C and the corresponding tone in the scale of G differ more widely from one another; their frequency-ratio is $682\cdot\dot{6} : 720$, and the interval between them, $\frac{135}{128}$, is sometimes called a semitone.

In order to play in correct tune music written in Key G as well as music written in Key C, we would require not only the tones of the Key of C, but also two additional tones in each octave. Every transition from one Key to another " more remote from" the Key of C multiplies the demand for new tones; and that to an extent twice as great as the current notation, which neglects differences of a comma, would seem to indicate.

In the table, pages 390 and 391, are given the tones of the scale of C, together with a number of tones derived from selected keys. The relative, not the absolute, number of vibrations has been shown in each case.

If a singer were called upon to produce a note of 324 vibrations per second, the feat would be impossible. This number is, however, $1\cdot265625 \times 256$; and hence if c' have 256 vibrations per second, the note required is the *re* of Key D. A tuning-fork $c' = 256$ is set in vibration; call the note of the fork *do;* sing *do, re;* fix the attention on *re* (d'); call it

NATURAL SCALE OF C WITH TONES FROM SOME RELATED KEYS.								EQUALLY-TEMPERED SCALE OF C.	
Intervals with Keynote.				Frequency-Ratios relative to the Keynote.		Logarithm of the Frequency-Ratio.	Logarithmic Increment.		
Keynote	d	C	... ...	1 : 1	**1·000000**	0·0000000		C	1·000000.
		'C	r_1 of Key B♭ . . .	81 : 80	1·012500	0·0053950	·0177288		
		—	... ...	...	...	...			
		C♯	l_1 of Key E . . .	25 : 24	$1{\cdot}04166\dot{6}$	0·0177288	·0053950		
		'C♯	t_1 of Key D, r of Key B .	135 : 128	1·0546875	0·0231238	·0049049	"C♯ or D♭"	$1{\cdot}059463 = \sqrt[12]{2}.$
Minor second . .		'D♭	d when C is t_1 . . .	16 : 15	$1{\cdot}06666\dot{6}$	0·0280287			
		D♭	r when A♭ is l . . .	27 : 25	1·080000	0·0334237	·0177288		
		—	... ...	...	...	...			
Grave major-second .		'D	l_1 of Key F . . .	10 : 9	$1{\cdot}11111\dot{1}$	0·0457575	·0053950		
Major second . . .	r	D	... ...	9 : 8	**1·12500**	0·0511525		"D or C♯♯"	$1{\cdot}122462 = \sqrt[12]{2^2}.$
		—	... ...	...	...	...	·0123337		
		'D♯	l_1 of Key F♯ . . .	125 : 108	$1{\cdot}15\dot{7}40\dot{0}$	0·0634862	·0053951		
		D♯	t_1 of Key E . . .	75 : 64	1·1671875	0·0688813	·0049049		
Pythagorean minor-third		'E♭	f_1 of Key 'B♭ . . .	32 : 27	$1{\cdot}185\dot{1}8\dot{5}$	0·0737862	·0053950	"D♯ or E♭"	$1{\cdot}189207 = \sqrt[12]{2^3}.$
Minor third . . .		E♭	d when C is l_1 . . .	6 : 5	1·200000	0·0791812			
		—	... ...	...	...	...	·0177288		
		—	... ...	...	...	...			
Major third . . .	m	E	... ...	5 : 4	**1·25000**	0·0969100	·0053950		
Pythagorean major-third		'E	r of Key D . . .	81 : 64	1·265625	0·1023050		"E"	$1{\cdot}259921 = \sqrt[12]{2^4}.$
		F♭	f when A♭ is l . . .	32 : 25	1·280000	0·1072100	·0123338		
		E♯	t_1 of Key F♯ . . .	125 : 96	$1{\cdot}3020\dot{8}\dot{3}$	0·1146388			
		—	... ...	...	...	...	·0122999		
Fourth	f	F	... ...	4 : 3	$1{\cdot}3333\dot{3}$	0·1249387	·0053950	"F or E♯"	$1{\cdot}334839 = \sqrt[12]{2^5}.$
Acute fourth . . .		'F	s when G is l . . .	27 : 20	1·350000	0·1303337			
		—		...	...	...	·0123338		

Augmented fourth .		‘F♯	t$_1$ of Key G, r of Key E .	45 : 32	1·406250	0·1480625	·0049050	" F♯ or G♭"	1·414213 = $\sqrt[12]{2^6}$.
		‘G♭	s when ‘A♭ is l . .	64 : 45	1·422222	0·1529675	·0053950		
		G♭	s when A♭ is l . .	36 : 25	1·440000	0·1583625			
		—	... ...	...	...	...	·0123338		
Grave fifth . . .		‘G	l of Key ‘B♭ . . .	40 : 27	1·48$\dot{1}$4$\dot{8}$	0·1706963	·0053950		
Fifth	s	G	... ...	3 : 2	1·50000	0·1760913		"G"	1·498307 = $\sqrt[12]{2^7}$.
		’G	Fifth above ’C . .	243 : 160	1·51875	0·1814863	·0177288		
		‘G♯	Fourth above ‘D♯ . .	125 : 81	...	0·1884251			
		G♯	m of Key E, t$_1$ of Key A .	25 : 16	1·56250	0·1938201	·0049049		
		‘A♭	f of Key ‘E♭ . . .	128 : 81	1·58024	0·1987250	·0053950	"G♯ or A♭"	1·587402 = $\sqrt[12]{2^8}$.
Minor sixth . . .		A♭	d when F is l$_1$ . .	8 : 5	1·60000	0·2041200			
		—	... ...	...	...	...	·0177288		
		—	... ...	...	...	...			
Major sixth . . .	l	A	... ...	5 : 3	1·66666	0·2218488	·0053950		
Pythagorean major-sixth		’A	r of Key G, s of Key D .	27 : 16	1·68750	0·2272438		"A"	1·681793 = $\sqrt[12]{2^9}$.
		—	... ...	...	...	...	·0123337		
		A♯	m of Key F♯ . . .	125 : 72	1·7361$\dot{1}$	0·2395775	·0053950		
		’A♯	t$_1$ of Key B . . .	225 : 128	1·7578125	0·2449725	·0049050		
Grave or Pythagorean minor-seventh		‘B♭	f of Key F . . .	16 : 9	1·7777$\dot{7}$	0·2498775	·0053950	"A♯ or B♭"	1·781797 = $\sqrt[12]{2^{10}}$.
Minor-seventh . .		B♭	d when G is l$_1$ . .	9 : 5	1·80000	0·2552725			
		—	... ...	...	...	...	·0123338		
		‘B	f of Key F♯ . . .	50 : 27	1·8$\dot{5}$18$\dot{5}$	0·2676063	·0053950		
Seventh . . .	t	B	... ...	15 : 8	1·87500	0·2730013	·0053950		
Pythagorean seventh .		’B	... ...	243 : 128	1·8984375	0·2783963		"B or C♭"	1·887748 = $\sqrt[12]{2^{11}}$.
		—	... ...	...	...	...	·0123338		
		B♯	m of Key G♯ . . .	125 : 64	1·953125	0·2907301	·0049049		
		‘c	l of Key ‘E♭ .	160 : 81	1·975301	0·2956350	·0053950		
Octave	d^1	c	... ...	2 : 1	2·0000	0·3010300		"C or B♯"	2·000000.

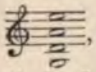
do without changing its pitch; dwell on it a moment; then sing some such phrase as *do*, *mi*, *sol*, *do*, *mi*, re, *do*; and the desired note, '*e*', a note differing by a comma from *e*', the *mi* of Key C, has been produced, the sense of tonality and key-relationship having carried the singer into the correct sound.

The column headed "logarithmic increments" contains figures which measure the intervals between the successive tones; for it gives the logarithms of the frequency-ratios between each tone and its predecessor; and the most convenient method of comparing ratios is to compare their logarithms. When the logs. are equal the ratios are equal; when the ratios are equal the intervals are equal. Thus the intervals between C and C♯, D♭ and ‘D, ‘D and ‘D♯, D and D♯, E♭ and E, E and E♯, F and F♯, 'F and 'F♯, ‘G♭ and ‘G, G♭ and G, G and G♯, A♭ and A, A and A♯, 'A and 'A♯, ‘B♭ and ‘B, B♭ and B, B and B♯, are all equal, being measured in that column by the logarithm ·0177288, which is the log. of the ratio $\frac{25}{24}$. Again we find a number of lesser intervals whose log. is ·005395, and whose ratio is $\frac{81}{80}$: these are C and 'C, C♯ and 'C♯, ‘D♭ and D♭, ‘D and D, ‘D♯ and D♯, ‘E♭ and E♭, E and 'E, F and 'F, F♯ and 'F♯, ‘G♭ and G♭, ‘G and G, G and 'G, ‘G♯ and G♯, ‘A♭ and A♭, A and 'A, A♯ and 'A♯, ‘B♭ and B♭, ‘B and B, B and 'B, ‘*c* and *c*. Between these the interval is a Comma.

The scale may be seen to be roughly divisible into 53 steps or divisions; but these are not equal to one another; if they were equal the logarithms would at each step acquire an equal increment, for the ratio between each tone and its predecessor would be equal throughout the scale. Roughly and for diagrammatic purposes it is, however, convenient to represent the interval between C and D by 9 steps, while that between D and E is represented by 8: and the table is so arranged. A thoroughly accurate table of this kind would be one engraved on metal, the intervals between any two tones in column 3 being made directly proportional to the log. increment between them.

The intervals marked Pythagorean in the table are thus derived:—Start from *c* and go upwards by successive fifths, *c*, *g*, *d*', '*a*', '*e*'', '*b*''; going downwards we arrive at F, ‘B♭$_{,}$, ‘E♭$_{,}$, ‘A♭$_{,,}$

The following exercises will perhaps aid the reader:—

(*a*) If the violin be tuned , to correct fifths, starting with *a*'; show that these notes are respectively *e*'', *a*', ‘*d*', ‘*g*.

(*b*) The scale of "B♭ major" is obtained by transition from the key of C to that of F, and from that of F to that of B♭. The scale of "G minor" has $g = la$, and thus $do = b\flat$. Show that the respective descending scales are—

. . .	d'	t	l	s	f	m	r	d			"B♭ major."
. . .	'b♭,	a,	'g,	f,	'e♭,	'd,	c,	'B♭			
. . . .			g,	'f,	e♭,	d,	'c,	B♭,	'A,	G . .	"G minor."
. . . .			l	s	f	m	r	d	$\text{t}_{\text{ı}}$	$\text{l}_{\text{ı}}$ . .	

The columns in the table, page 390, headed "Equally-tempered Scale," show the nature of the system of Equal Temperament, which is as nearly as practicable applied to the pianoforte and organ. The intervals are equal; the ratio between a tone and its predecessor and successor is in every case the same; between each pair of tones the logarithmic increment is equal: it is $\frac{\log 2}{12} = \frac{\cdot 301300}{12} = \cdot 025058\dot{3}$. The result differs widely from pure intonation; but we are accustomed to it. On a pianoforte equally tempered the fifths are not appreciably out of tune, though they are a little flat: but the thirds, three of which are forced to make an octave instead of extending only from C to B♯, are too sharp; and though this be not offensive on the pianoforte, to which indeed their sharpness lends somewhat of brilliancy, yet in slow sustained harmony these sharp thirds are really discordant, as may be well heard on a loud harmonium tuned in the usual manner, and on which thirds alone are played.

Loudness.—The physical Intensity of a sound depends initially on the square of the amplitude of the vibration of the sounding body; but the corresponding sensation of loudness depends not only upon peculiarities of sensitiveness of the ear, but also on the amount of physical disturbance of its drum, and if the sound be conducted to the ear by the air, it depends on the intensity of vibration of the air near the ear; and this varies not only (1) as the square of the amplitude of the original vibration, but also, in the open air, (2) inversely as the square of the distance of the sounding object.

To compare the relative loudnesses of two sounds of nearly the same pitch, place the sounding bodies at such distances that they become just inaudible, and no more: say that the one becomes inaudible at 10, the other at 50, yards: then the loudness of the one at 50 yards' distance is at the ear equal to that of the other at 10 yards: their initial intensities must be as $10^2 : 50^2$, or 1 : 25.

If the sound be not propagated in free air, but be confined in a tube, the loudness of sound may diminish at a much less rate, for ultimately the waves become plane-fronted, and move down the tube without any loss of intensity other than what is due to such loss of energy as is brought about by friction against the sides of the tube or by the viscosity of the air itself.

Hence sounds can be carried along sewers, speaking-tubes, etc., to great distances without great diminution of loudness.

Similarly, if sound be propagated by parallel or convergent waves in the air, as when it issues from a wide aperture, or after reflexion from a curved surface, it may lose little of its intensity, or may even concentrate its intensity on some particular place.

The loudness of a sound also depends, if it be conveyed by a gaseous medium, on the density of that medium at the place where the vibration is imparted to it. The denser the medium the greater its inertia, and the more readily it is compressed against itself: the greater the compression, the greater the amount of energy imparted to the medium, and the louder the sound produced. A body vibrating *in vacuo* produces no sound: in rarefied air or hydrogen, or any other rare or rarefied gas, it produces a comparatively feeble sound; in carbonic acid it produces a louder sound than in air. A cannon fired on a mountain-top produces little sound; one fired beneath is heard distinctly and loudly from a balloon, even at a great height.

Concentration of sound-waves renders sounds louder, as in ear-trumpets and in those stethoscopes the auditory extremity of which fits into the ear.

Quality of Sound.—If a body vibrate so as to produce a sound of the fundamental pitch C = 64, and if all the harmonics be present, the series is the following:—

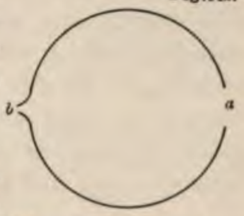
1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15, etc.
64	128	192	256	320	384	448	512	576	640	704	768	832	896	960.
C	*c*	*g*	*c′*	*e′*	*g′*	*b*♭′ −	*c″*	*d″*	*e″*	*f″* +	*g″*	*a″* +	*b*♭″ −	*b″*, etc.
						•				•		•	•	

These are all tones of the scale of C, with the exception of the 7th and its octave the 14th, the 11th and the 13th. The 7th and 14th correspond to a very flat B of 112 vibrations, lying between A♯ and ′A♯; the 11th to a sharp F of 88 vibrations, lying between ′F and F♯; the 13th to a flat A, lying between A♭ and A.

Analysis of a sound into its components may be effected by several methods, of which we shall first consider one due to Prof. Mayer. As our example we take the sound produced by a vibrating organ-reed-pipe, a sound which we recognise as peculiar and characteristic. We are provided with a set of tuning-forks, one of which vibrates in exact unison with the fundamental tone of the organ-pipe, and the rest of which respectively vibrate 2, 3, 4, 5, etc. times as rapidly. As to the organ-pipe, a part of its wall has been replaced by a piece of inelastic thin morocco leather, or some similar substance, which vibrates exactly as the air within the pipe does. To one point of this is attached a bundle of silkworm-cocoon-threads, 40 inches or so in length: each of these is attached to one of the tuning-forks and tightened somewhat. The organ-pipe is caused to sound; the leather vibrates; the silk fibres are all set in motion, and each alternately tugs and releases its own tuning-fork. If the vibration appropriate to any one of the tuning-forks be present in the original compound vibration, the corresponding fork is set in motion: if it be not present, that fork remains silent: if the vibration be ample, the fork sounds out loudly: if it be not, the sound is feeble. This arrangement analyses the sound into its components, for it can be seen which of the tuning-forks are set in vibration; and if the organ-pipe cease sounding, the forks go on sounding for some time, and by their joint action produce a compound sound closely resembling the sound of the reed-pipe which had been the means of setting them in vibration. This action is very exact: the slightest difference between the natural rate of any tuning-fork and that of the corresponding organ-pipe vibration causes the fork to sound with comparative feebleness, or not to sound at all.

Resonators are extensively used as a means of analysis of sound. A resonator consists in its most usual form of a bulb, generally of glass or of brass, with a large aperture, *a*, at one side and a small one, *b*, at the other. The air within such a bulb has a natural period of vibration which depends upon the cubic contents of the resonator and upon the size of the orifices. This period can be found by the pitch of the sound produced on tapping the resonator with a soft substance, or by blowing brief blasts of air across its mouth. If the air convey a system of waves which agree in period, either absolutely or approximately, with the natural free vibration of the air in the resonator, the air in the resonator will absorb the energy of those waves, will be set in motion, and will act as a sounding body. If we be provided with a set of such resonators, the air in one of which freely

Fig. 142.



vibrates in unison with an a' tuning-fork, and in the others respectively 2, 3, 4, 5, 6, 7, etc. times as rapidly,—then, on listening to an a' organ-reed-pipe, one ear being closed and the other adapted to each resonator in succession (this being done by fitting the nipple *b* of the resonator (Fig. 142) into the ear), we shall, if the proper sound of any of the resonators be contained in the complex sound to which we listen, hear that resonator loudly sing out its proper tone; while, if it be not present, we shall simply hear the ordinary sound of the pipe through the resonator, without any reinforcement. And further, if we fill our ears with the sound of the tone thus sung out by the resonator, and remember its pitch, we shall, when the note is again sounded out by the organ-pipe, have no difficulty, even without a resonator, in hearing the harmonic tone: and by dint of practice we may hear at will, or even independently of will, many if not all of those component harmonic tones which, by accompanying that fundamental tone to which alone in ordinary circumstances we are accustomed to listen, help to make up the note of the organ-pipe.

A very convenient form of resonator may be made of a common tall lamp chimney or a similar piece of tubing. If it be held vertical, as its lower end is immersed in water to various depths its natural pitch varies: and a tube thus gradually lowered into water is capable of resounding in succession to the different harmonics of a fundamental note, so that the ear, placed near the tube, can recognise their several presences. In Wintrich's resonator an aperture at the side may be closed by the finger. By aid of the same resonator an observer is thus enabled to listen alternately to the grave-pitched muscle-sound of the heart and to the sharper valve-stretching sound. See Gscheidlen, *Physiol. Methodik.*

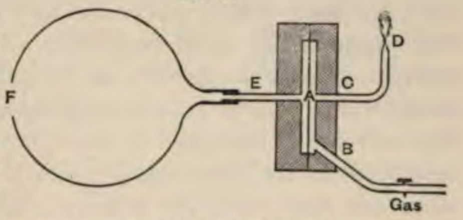
Resonators may be otherwise employed. If the small aperture *b* be stopped with wax, and if the resonator be brought near a sounding body, it will absorb the energy of any vibration corresponding to its own proper tone, and may then be removed and listened to: thus each one of a set of resonators may be made to select one tone out of the group of tones present in an ordinary musical sound, and to bring it to the ear to be listened to.

A resonator and a sounding body to which it is in response seem to be mutually repelled, in consequence of the stresses set up in the intervening air (*Dvořak*).

Resonators may further be used to transfer the energy which they thus take up to relatively-massive bodies such as tuning-forks. A resonator may be made in the form of a thin wooden box with open ends: a tuning-fork precisely in tune with it is fitted on its upper surface; a sound causes the air in the box to vibrate, the air acts upon the box, and the box upon the tuning-fork; if all be in exact unison, the energy accumulates in the tuning-fork, which comes to vibrate energetically and to produce a loud sound.

Again, the oscillation of the air in a resonator may be rendered visible by the following device:—A cavity in a block of wood (Fig. 143) is divided into two parts by a membrane, such as thin goldbeater's skin. The one moiety of the cavity is connected with the cavity of a resonator: the other is connected with a supply of coal-gas which enters at B and passes out at C on its way to be burned at the jet D. This contrivance is called Koenig's manometric capsule. When a sound is produced outside F, containing as one of its component tones the proper tone of the resonator, the air in the resonator oscillates in sympathy with that component, the diaphragm vibrates with it, and the flame at D is rendered alternately higher and lower by the action of the vibrating diaphragm on the stream of gas. The flame obviously alters its character; and the change undergone by it can be studied by looking at it while the head is turned rapidly from side to side, the eyes being kept fixed relatively to the head, or by looking at the flame through an opera-glass, which is rapidly moved across the field of view, or, best of all, by looking not directly at the flame but at its image in a rapidly-rotating mirror: in which cases the flame or its image appears to spread not into a uniform band of light, but into a band with serrated edges, or even into a chain of bead-like separate images.

Fig.143.



A sufficiently-extensive set of resonators would thus enable us to effect the analysis of sounds of any degree of complexity: but resonators do not furnish us with as delicate a means of investigation as the means first described, unless indeed they be each allied with a tuning-fork; they respond in general with excessive readiness to any tone in the proximity of their natural tone.

Synthesis of Sound.—Helmholtz has shown that any quality of sound may be built up by the superposed effect upon the ear of simultaneously sounding tuning-forks of the proper number, pitch, and relative loudness.

Complex Sound-Waves.—The pitch, the loudness, and the quality of a sound may be studied together by causing sound-waves to impinge directly upon some sensitive body without any intermediate process of selection or filtration. Thus, if instead of a resonator, as in Fig. 143, a cone be adapted to a manometric capsule, and if sound be produced at the mouth of the cone, sound-waves will impinge directly upon the membrane in A. The membrane will go through a complex motion somewhat resembling the original compound-vibration of the sounding body, and the flame will demonstrate this by its variations of height. The image of this oscillating flame will appear in a mirror, if the mirror be made to revolve, as a band of light, serrated by large teeth, whose outline is broken by subsidiary serrations; the number and size of the greater serrations indicate the frequency and amplitude of the fundamental vibration: those of the subsidiary serrations vary with the number and variety of the subsidiary vibrations. This experiment may be roughly carried out, if there be no revolving mirror at hand, by whirling the gas-flame itself (a rat's-tail jet at the end of a flexible tube) before the eye.

It is interesting to carry out this experiment by singing into the open end of the cone; even among notes of the same pitch sung to the same vowel, the association of different forms of the flame-image with different qualities of tone and different subjective sensations is very striking; and it is possible for a singer to attain to the production of very pure tone—such pure tone having, however, a somewhat hollow quality—by finding out for himself how to control the larynx so as to keep the serrations visibly open and simple.

If the membrane A have a small mirror attached to it, the mirror will share in the vibrations of the membrane, and, if it be jointed on a hinge, will reflect a beam of light in such fashion as to produce a curve upon photographic paper uniformly rolled past the vibrating membrane; this curve will indicate the frequency, the amplitude, the complexity, of the vibrations of the membrane.

Sound-waves, however complex, may again be caused permanently to record the succession and variation of their own impulses. Léon Scott's Phonautograph is a conical vessel, closed at its narrower end by a membrane; to the membrane is attached a writing-point; the extremity of the writing-point is brought into contact with a smoked revolving-cylinder. As long as there is no sound the writing-point describes a uniform line on the rotating cylinder: when sound-waves enter the cone the membrane is set in vibration, and the writing-point now describes an undulating line which varies in its form according to the frequency, the amplitude, and the complexity of the original vibration.

It must be observed that membranes thus made use of do not exactly reproduce the original motion at any point of their surface: those components are exaggerated which approximately or exactly coincide in frequency with some normal mode of free vibration of the membrane itself.

Edison's Phonograph is a phonautograph whose writing-point is somewhat blunt; and it records the vibrations of its membrane by being driven through variable distances into a sheet of soft tin fixed on a rotating cylinder: it leaves a permanently-deforming mark, a groove of varying depth. If the membrane after having made such a mark be raised from the rotating cylinder, and the cylinder turned back to its initial position; if the membrane be now readjusted in its former position, or, better, a little nearer the cylinder; and if the cylinder be again rotated in its former direction, with the same velocity as at first,—the depressions in the tin, being of variable depth, cause the blunt writing-point under which they pass to move alternately towards and away from the cylinder; this compels the membrane to execute vibrations, and in so doing to set up vibrations and sound-waves in the air, which, being received by the ear, produce a sound similar to the original. Not exactly, however: the process is not perfectly reversible. Some consonants are not well reproduced, especially the explosives (b, p, t, d, k, g) and the sibilants (s, z, th); and further, it is generally found that there has been some exaggeration of some of the higher components in the course of transmission through the membrane, the effect of which is to render the sound reproduced one whose quality is somewhat metallic, nasal, or even squeaky and Punchinello-like.

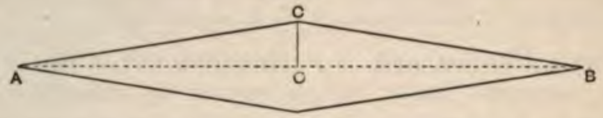
LAWS OF VIBRATION OF SOUNDING BODIES.

These laws form properly a part of kinetics; but the means of research into the phenomena of vibration which lies most readily at our disposal is the observation of the pitch of the sound produced by vibrating bodies; for which reason some part of the consideration of these laws has been deferred to this place.

In general any vibration of a vibrating or sounding body is a periodic motion, a Fourier-motion; though in particular cases we may find that the vibration is not a single Fourier-motion either simple or complex, but may be resolved into a number of such motions, simultaneous and superposed.

Transverse Vibrations of Strings.—If a string be stretched and drawn aside from its mean position, it tends to return to that position. In Fig. 144 let the string AB, subjected to the tension

Fig 144.



of a weight of T grammes (that is, Tg dynes), be drawn into the position ACB, the particle C being supposed for simplicity's sake to have been initially at O, the centre of AB: the tension tending to bring back the particle C to O is the component of the total tension resolved in the direction CO: this varies directly as CO—that is, the restitution-force varies directly as the displacement—the criterion of harmonic motion: and it can be shown as a consequence of the fact that the string is fixed at A and B, that the string will oscillate in some such manner that its aggregate motion can be analysed into a number of simple oscillations whose periods are commensurable: in other words, that the motion of the string is a Fourier-motion. According to the mode of disturbance—striking, plucking, bowing—of the string, or the duration of these operations, there may be an infinite variety in the relative amplitudes of these component simple oscillations. Some of these components may even be altogether absent: where, for example, a string is plucked at its centre, it is not possible that any of those components which have a point of rest at the centre of the string should be present, and the vibration of a string so plucked is one in which all the even components are absent. In general, a vibrating string does not present any component oscillation whose node is at the point of disturbance.

Frequency of Oscillation.—In general the velocity of propagation of a wave is $\sqrt{\frac{K}{\rho}}$; here we must substitute for K (the restitution-coefficient) the force here acting—that is, the tension per unit of sectional area of the string; this, measured in dynes, is $\frac{T\,.\,g}{\pi r^2}$ when r is the radius of the string. Hence $v = \sqrt{\frac{K}{\rho}} = \sqrt{\frac{T\,.\,g}{\pi . r^2} . \frac{1}{\rho}} . = \frac{1}{r}\sqrt{\frac{T\,.\,g}{\pi\,.\,\rho}}$. But the length of each wave is fixed by the condition of the string: it is bound at each end, and if we confine our attention to the slowest component, the fundamental tone, we see that λ, the wave-length, is equal to 2AB or $2l$, twice the length of the string.

Thus, since n, the number of complete oscillations per second, is equal to $\frac{v}{\lambda}$, we have

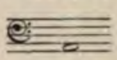
$$n = \frac{v}{\lambda} = \frac{v}{2l} = \frac{1}{2l} \cdot \frac{1}{r} \sqrt{\frac{\mathrm{T} \cdot g}{\pi \cdot \rho}}.$$

This determines the frequency of the fundamental vibration: the harmonics will have frequencies 2, 3, 4, etc. times as great.

Problem.

A wire of steel ($\rho = 7{\cdot}8$) 1 met. long and 1·2 mm. thick is stretched by the weight of 40 kilogrs. and set in transverse vibration: what will be the frequency of its fundamental vibration? What its pitch?

$$n = \frac{1}{2l} \cdot \frac{1}{r} \sqrt{\frac{\mathrm{T} \cdot g}{\pi \cdot \rho}} = \frac{1}{200} \cdot \frac{1}{\cdot 06} \sqrt{\frac{4000 \times 981}{3{\cdot}1416 \times 7{\cdot}8}}$$

$= 106\frac{2}{3}$ vibrations per second;  (when $c' = 256$).

So for a perfectly-flexible string: the effect of rigidity of wire or string is to diminish the number of vibrations, and to cause the motion to assume the character of a number of superposed harmonic motions of incommensurable period.

The vibrations of a violin string differ much from those of a pianoforte string. In the violin the oscillating string sometimes travels in the same direction as the bow, sometimes away from it. When the bow and the string travel in the same direction, the bow drags the string with it, distorts it, pulls it out to an extent greater than that which it would have travelled if allowed freely to vibrate. When the string returns the bow fails to retain it, loses it, and as it is returning bites and catches it again by means of some rough resinous particle at another part of the bow.

The friction and, consequently, the adhesion between the string and the bow are relatively greater when both move in the same direction, for at low relative-speeds friction increases.

The string is thus distorted and assumes successively a number of forms, of which no one is curved: and the form of the vibrating string at any instant presents an angle between two straight lines, a form differing considerably from the curve of sines. But it is periodic, and it is true Fourier-motion.

The mathematical problem is—What superposition of commensurate S.H.M.'s (compare Fig. 48) will produce a vibration-curve such that for a certain distance the flexures so balance one another as to produce a straight line, and then so aid one another so as to produce an abrupt angle followed by a straight line? This can be solved, and the result is that the vibration of a bowed string must be composed of a fundamental vibration, of weak components 2d to 6th, and of ample higher components. This agrees with the result of resonator-analysis of the sound of a violin.

In the sound of a violin the upper harmonics are loud and piercing; the nearer harmonics are feeble, and the fundamental tone stands apparently alone, but rendered penetrating in quality by the high mass of harmonics. Purity of violin tone depends upon perfect periodicity of the peculiar motion of the string; this is difficult to attain, for a good elastic violin, uniform strings, a uniform bow, uniformly resined and evenly handled, are necessary: and any stumbling of the bow over the string or any irregular movement of the string under the bow is revealed by scratchiness of tone.

The sharper the angle made at the point of disturbance, the richer the tone in high harmonics. A string plucked with a quill, as in the old harpsichord, has thus a metallic tinkling quality, and its fundamental tone is relatively very feeble.

A string struck suddenly at one point has a form differing greatly from that of the curve of sines. Part of the string remains unaffected, while the part struck is distorted. This distortion travels along the string, and results in a periodic motion abounding in high components; the tone produced is tinkling. If the same cord or wire be struck gently by a soft elastic hammer, the blow being deliberate, and, as it were, gradually insisting upon the displacement of the string at the point struck, the disturbance is more evenly spread over the whole string, and the fundamental component-vibration is more prominent, the higher components are relatively more feeble, and the tone is purer.

In a pianoforte string struck by an elastic soft-hammer the harmonics up to the sixth are present; the seventh is obliterated, or nearly so, by the hammer being made to strike the string at a spot one-seventh of its length from the end of the string—that is, at a spot which would have been a node of the seventh component if that component had existed in the compound vibration: and the components beyond the seventh are feebly represented.

The **Monochord** is a box of thin light wood containing air which communicates with the exterior air by lateral apertures. Upon this box rest two bridges ("banjo-bridges"), one near each end. Over the bridges is stretched a wire; of which one end is firmly fixed to one end of the box, while the other is either passed over a pulley and made to support a weight, or else connected with a tuning-peg, which may be turned by a tuning-key, the tension on the stretched wire being thus varied.

Experiments with the Monochord.—For experiments it is better to use a form of monochord in which there are two wires, of which one is tightened by a peg, the other by a suspended weight; in the latter the total tension on the wire can be directly measured, in the former it must be inferred.

1. Suppose a wire 1·2 mm. thick, whose free vibrating part is 1·2 metres long, to be stretched by the weight of 48 kilogrs., and the pitch of the sound produced to be $G = 96$ vibrations. What weight ought to be added in order to raise the pitch to *d*?

The pitch is raised G : *d*—*i.e.*, a fifth: the vibrations are rendered more numerous in the ratio 2 : 3; the tension must be increased in the ratio $2^2 : 3^2$, or 4 : 9; the stretching-weight must be increased from that of 48 to that of 108 kilogrs.; the mass which would have to be added is 60 kilogrammes.

2. Two wires of equal thickness are stretched—one by the tuning-peg and the other by the weight of a heavy mass—so as to vibrate in unison. The weighted wire is removed and replaced by one of a different thickness stretched by the same weight. A thinner wire gives a higher tone, a thicker one a lower.

If in Ex. 1 a wire 1 mm. thick be employed, what will be the pitch of the sound produced? The frequency varies inversely as the radius: it therefore exceeds that of a wire 1·2 mm. thick in the ratio of 6 : 5; the note produced will be B♭.

3. A brass wire and a steel wire of equal gauge are equally stretched: they are free to vibrate in equal lengths. Brass has a density $\rho = 8{\cdot}38$, steel $= 7{\cdot}8$. The brass wire gives a sound lower in pitch than that given by the steel wire: the respective frequencies are in the ratio of $\sqrt{7{\cdot}8}$ to $\sqrt{8{\cdot}38}$ or 1 : 1·03663.

A catgut string, whose density is small, gives a higher note than a steel wire.

4. In order to vary the free vibrating-length of wire, a movable bridge is arranged under the string. If this be so placed that 60 cm. of the wire are free to vibrate instead of 120 as before, the sound produced will be the Octave; if 40 $(=\frac{120}{3})$, the Twelfth; if 30 $(=\frac{120}{4})$, the Fifteenth; if 24 $(=\frac{120}{5})$, the Seventeenth—and so forth—above the fundamental note emitted by the freely-vibrating string of 120 cm. length.

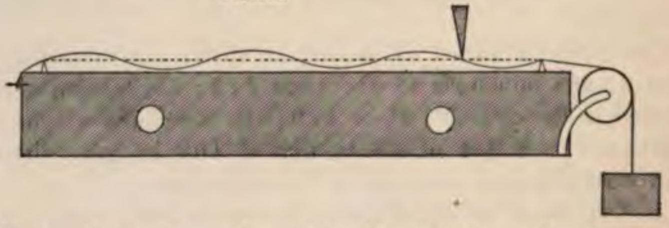
The number of vibrations of a string varies inversely as its length. Hence if we wish with a string which sounds C to produce the note D, whose frequency is that of $C \times \frac{9}{8}$, we must allow the string to vibrate not as a whole, but only in $\frac{8}{9}$ of its length. To produce the scale on one string, the parts of the string which are allowed to vibrate are as follows:—

d	r	m	f	s	l	t	d'	
1	$\frac{8}{9}$	$\frac{4}{5}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{3}{5}$	$\frac{8}{15}$	$\frac{1}{2}$	etc.

The application of this principle is familiar in violin playing.

5. Nodes and Loops can also be shown on the monochord. If the wire, 120 cm. in its vibrating length, be lightly touched at 20 cm. from the end, and if the twenty-centimetre-part of the wire be set in vibration by a bow, the whole wire is found to be in vibration from end to end; but not as a whole. It divides itself into segments or vibrating loops, separated by nodes or points of rest. Each segment is 20 cm. long: and the sound given out is that which might be emitted by half-a-dozen separate wires each 20 cm. long—that is, it bears to the note emitted by the whole string the same proportion as *g'* does to C —two octaves and a fifth. Similarly for other fractional divisions of the wire or string. Lightly stopping the string has the effect of destroying or of checking the formation of all those modes of vibration which have not a node at the point touched hence the 6th, 12th, 18th, etc., components of the vibration o the whole string are unchecked, while the other components ar rendered impossible.

Fig. 145.

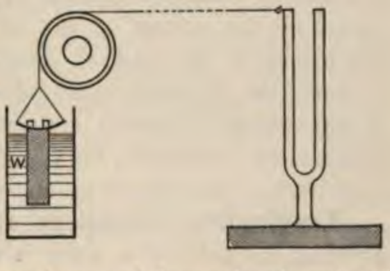


The nodes and loops of a string vibrating in this way a rendered manifest by paper riders placed—some at the nodes, so on the loops; when the string enters into vibration those rid which had been placed on the nodes will retain their place, wh those which had been placed on the loops will be jerked off.

Nodes and loops on vibrating strings may be illustrated o large scale as follows:—Take an indiarubber tube 10 feet l filled with sand, or a long spiral of iron or brass wire, and fix end of this to a wall; hold the other end in the hand. moving the hand gently the natural period of oscillation o cord can be easily found. Give with the hand a series of t verse impulses, so timed as to aid the natural oscillations: the tube or spiral will enter as a whole into wide oscillations. Now give such impulses twice as often: the cord will divide into two segments, pivoting in a striking manner round the central point. Do so three times as often as at first: the cord will divide into three segments or loops, pivoting on two nodes. By increasing the frequency of the movements of the hand, the cord can be made to oscillate in 4, 5, 6, 7, or even 8 or 9 segments, according to the dexterity of the experimenter. The experiment is a very striking one; and it may be varied by causing the hand to move in a circle or an ellipse instead of a straight line.

Melde's Experiments.—A tuning-fork is provided with a little hook on one of its prongs; to this hook is attached a fine white-silk thread. This thread is passed over a wheel and attached to a suspended mass partly immersed in water; the quantity of this mass can be coarsely adjusted by the addition or removal of sand; its effective weight can be finely adjusted by varying the quantity of water in W (Fig. 146). The fork is set in vibration; waves appear to travel up and down the thread; if the string be illuminated by a beam of light in a dark room, the effect is singularly beautiful. As the tension is increased the segments vary: at length the thread vibrates as a whole, and seems to form an opalescent spindle. Its frequency of vibration is half that of the fork; the thread, when at its limit, is pulled back by the retreating fork into its mean position, but is relaxed and allowed to swing over upon the return of the fork; whence two oscillations of the fork correspond to one of the thread. If the tension be reduced to $\frac{1}{4}$, the vibrating part of the string must be shortened to $\frac{1}{2}$ in order to keep time with the fork, or else, if the string be not shortened, the string will divide into two equal segments or loops separated by a node: if the tension be reduced to $\frac{1}{9}$, the string divides into three loops with two nodes; and so forth.

Fig.145.



If the tuning-fork be turned round through 90°, so as not now to tighten and relax the thread, but to give it a series of transverse impulses, a similar series of phenomena will be observed; but the fundamental vibration is now simply synchronous with that of the tuning-fork.

When the thread is suspended between two tuning-forks whose frequencies bear an aliquot ratio, the tuning-forks being placed at distances from one another such as to tighten the thread to the required amount, the motion of the thread becomes periodic, and presents a complex of beautiful loops and nodes which are obtained with comparative ease.

Transverse vibrations of cords may be studied with respect to the motion of each particle by casting a beam of light along a vibrating cord, and looking at a particular bright or brightened spot on the cord. The bright spot appears, when the cord is looked at end-on, to give in quick succession a large variety of such forms as we have already seen to be produced by the composition of S.H.M.'s. A bright spot on the cord may also be looked at through a microscope whose object-glass is borne upon a vibrating tuning-fork; the apparent motion of the spot produced by the motion of the object-glass (this being parallel to the length of the cord) is compounded in the eye with its real motion; the apparent up-and-down motion of the spot, as looked at transversely, is spread out into an open curve, and thus becomes more intelligible, for the eye can more readily comprehend open curves than simple up-and-down movements.

Longitudinal vibrations of a string may be excited by drawing one point of a violin bow along the string: a very shrill tone is produced.

The velocity of propagation $= \sqrt{K/\rho}$; the wave-length is twice the length of the string, or $\lambda = 2l$; the number of fundamental vibrations per second, $n = \frac{v}{\lambda} = \frac{1}{2l} \cdot \sqrt{\frac{K}{\rho}}$.

Problem.—A steel wire—elasticity 2,520,000,000 g, and density 7·8—of one metre in length, is clamped at the two ends and set in longitudinal vibration. What will be the pitch of the sound produced?

$$n = \frac{1}{2l} \cdot \sqrt{\frac{K}{\rho}} = \frac{1}{200} \sqrt{\frac{2{,}520{,}000{,}000 \times 981}{7{\cdot}8}} = 3000 \text{ vibrations per second} = f'''' + .$$

As a rule the longitudinal vibrations of a string or wire are much more frequent than the transverse ones, and thus produce a much shriller sound, and further, they are not so much affected by tension applied to the string, for a variation of tension which would materially modify the frequency of transverse vibration would have little effect upon either the elasticity K, or the density ρ, upon which the longitudinal vibrations depend.

A violin *e''*-string gives, when rubbed longitudinally by one point of the bow, a sound in the neighbourhood of ('*f*♯)''''; while, when let down so as to sound only *e'*, it gives out a longitudinal vibration-sound not so low as (*f*♯)''''; the longitudinal vibration hardly falls a comma, while the transverse falls an octave. All the catgut strings of a violin may be observed to give out nearly the same longitudinal note, for this does not depend on their thickness.

From this we see how important it is to use the bow in such a way as to bring out transverse vibrations only, and by no means to wield it so that any component of its motion over the string can excite longitudinal vibrations, resulting, as these do, in shrill discordant tones.

By means of the monochord we may learn that a string while vibrating longitudinally divides into loops separated by nodes, just as it does while executing transverse vibrations.

Longitudinal vibrations of rods resemble those of strings or wires. A glass rod grasped by its centre and rubbed longitudinally by a resined cloth will enter into longitudinal vibration and will produce a shrill sound. A glass tube treated in the same manner may be made to vibrate so vehemently that it shivers into segments.

Transverse vibrations of rods obey the rule that if θ be the thickness of the rod, l its length, K its coefficient of elasticity, and ρ its density, then

$$n \propto \frac{\theta}{l^2}\sqrt{\frac{\mathrm{K}}{\rho}}, \text{ or } n = const \times \frac{\theta}{l^2}\sqrt{\frac{\mathrm{K}}{\rho}}.$$

The constant varies according to circumstances. If the rod be free or clamped at both ends, it is 1·78; if it be free at one end only, the constant is only 0·28.

As examples of rods free at both ends and vibrating transversely, we may take the common glass or metal harmonicon—plates of glass or metal supported by threads at the nodal lines and struck by hammers. As examples of rods clamped at one end and vibrating transversely, we may take reeds such as those of the harmonium or concertina. Their pitch is raised by filing off their substance towards their free ends; it is lowered by thinning them towards their base. Tuning-forks afford another example of vibrating rods; they are tuned in the same way.

In rods of the same thickness the frequency of vibration varies inversely as the square of the length, as the formula $n = c\frac{\theta}{l^2}\sqrt{\frac{\mathrm{K}}{\rho}}$ indicates; but if the thickness θ and the length l vary together, so that different rods have the same shape, the frequency depends on the relative length only. Thus a tuning-fork 4 inches long and one 2 inches long, of the same shape, produce notes which differ by an octave: the same rule applies to reeds.

A rod does not vibrate, as a whole, in halves, thirds, etc., but the component vibrations have frequencies 1:4:9:16, etc.; but each component has its own rate of travelling through the solid, and thus the periodic nature of the vibration is disturbed. Such a vibration of a rod is an extreme case of the vibration of a rigid or thick wire.

A rod of circular section can vibrate transversely with indifference in all directions. One whose section is oblong can oscillate more widely in a plane at right angles to the broad face than it can in a plane parallel to that face—*e.g.*, a vibrating reed, in which the latter oscillation is absolutely insignificant. If a rod of circular section be filed at one side, all component oscillations which tend to bend the rod upon the filed face are retarded; those which are at right angles to these are unaffected. Thus a rod of steel, grasped by a vice at a particular spot, may be so tuned that when it is set in vibration by a violin bow its point may execute vibrations in directions at right angles to each other, and bearing to each other any predetermined ratio.

Take a knitting-needle; fix it in a vice; mark on the needle the height at which it stands in the vice; touch the free tip of the needle with a little gum: scatter a little starch or powdered antimony over the needle tip; some will adhere. Suppose the ratio desired is 4:5; refer to Fig. 38. File the rod always towards one aspect until the movement of the tip of the rod, as revealed by the brilliant particles of starch or antimony, comes to present the curve sought. If the filing have been carried too far, a little metal may be removed from the needle at an aspect at right angles to that of the previous operation. If after a needle has been tuned in this way, so that its component vibrations have been rendered commensurate, it be grasped by the vice at a point a little above or below the original point, the intervals cease to be commensurate, and the curves seen pass through a series of changes exemplified by those of Fig. 41.

A tuning-fork or vibrating reed made to write its own vibrations on a rotating cylinder describes a sinuous curve which is almost identical with the curve of sines. This shows that the motion is pendular. Again, if two harmonium reeds have their tips silvered so as to reflect light, and if they be arranged at right angles to one another; and if a lamp and lens be so placed that a beam of light falls first upon one reed-tip, then upon the other, and finally upon a screen; then if one reed be set in vibration, the spot of light opens out into a line, while if both vibrate the line opens out into some figure of the order of those shown in Figs. 35-40. This figure retraces itself and retains its form if the reeds be accurately tuned to an aliquot ratio of frequencies, while, on the other hand, if the reeds be not so in tune, the figure undergoes rapid changes—changes painful to the eye, as the accompanying beats are to the ear.

Rotatory Vibration of Rods.—When a rod is clamped by one end in a vice, and a violin bow drawn round it, it may be caused to execute vibrations in which it successively twists and untwists itself round its own axis; and it is found to do so with a frequency 0·6 × that of the longitudinal vibrations in the same rod.

Vibration of Discs or Plates.—A disc of metal or of glass may be caused to vibrate by means of a violin bow drawn across its edges. The point of support of the disc is necessarily a nodal point; any number of points may be supported or fixed, and all these must also be nodal points. The disc or plate may, under such arbitrary conditions, adjust itself so as to vibrate, according to circumstances, with great variety of nodal lines and vibrating segments.

A disc of brass or glass may be fixed at its centre to a heavy stand. If the circumference be touched at any point while the whole is set in vibration by a violin bow, the point touched will be a nodal point; the spot where the violin bow is applied tends to become the centre of a loop; according to the relative situations of the points held fixed and of the point of application of the disturbing cause, will vary the manner and the pitch of the resultant vibration.

Different discs of the same shape and vibrating in similar ways have relative vibrational frequencies varying as $\frac{\theta}{l^2}\sqrt{\frac{K}{\rho}}$; the same law as holds good in the transverse vibration of bars.

The form of the nodal lines may be studied by strewing sand and lycopodium powder upon a vibrating disc or plate: the sand collects on the nodal lines; the lycopodium, by reason of the disturbance of the air, is blown towards the centre of each vibrating segment.

Contiguous sectors are in opposite phases of vibration. If the ear be placed immediately opposite the centre of figure of a circular vibrating-disc, there will be but little sound heard: the receding and the approaching sectors neutralise each other's effects upon the air and upon the ear. If the hand be held above a vibrating disc so as partly to cut off the effect of one of the sectors, the sound heard opposite the centre of the disc is enhanced; if two contiguous sectors be thus shaded from hearing, the sound is, as at first, very feeble; if two sectors not contiguous but vibrating in the same sense be thus covered, the sound produced is much louder. If a Koenig's manometric capsule be provided with a tube which bifurcates, and if the branch tubes each terminate in a cone, one cone may be placed over a vibrating sector, while the other may be moved about over the disc. As it passes round the circumference of the disc, it will be found that the gas-flame of the capsule is alternately much agitated and steady—agitated when both cones are over sectors vibrating in the same sense; steady, or nearly so, when they are over sectors vibrating in opposite phases.

Vibration of Membranes.—If a membrane be subject to a tension equal over its whole circumference, such as that of a drum, it vibrates as a whole; its higher component vibrations are not commensurable with the slower ones, and the higher tones faintly to be heard in the sound of a drum discord with the fundamental tone.

If the membrane be not equally stretched in all directions, it will, when set in vibration, so arrange itself as to vibrate feebly along the line of least tension, and strongly in the direction of greatest tension. Thus a square piece of thin indiarubber, clamped by the two opposite edges and stretched, will vibrate at the same rate as an indiarubber cord of the same free length and exposed to the same tension per unit of sectional area; and it may be idealised as a collection of indiarubber cords, arranged side by side, attached to each other, and vibrating in unison.

Vibration of Bells.—A bell-shaped body, set in motion by being struck, or by being rubbed with the resined or wetted finger carried round the circumference, enters into vibration simultaneously radial and tangential. The bell divides into an even number of sectors; of these one half dilate, while the other half (individually alternating with the former) contract radially. At the same time sectors of the bell, moving tangentially, twist to and fro round the axis of the bell; alternate sectors are opposed to one another in the direction of their twist; hence at some of the nodes which separate the sectors there is compression, at others dilatation of the substance of the bell. The loops of the radial motion are the nodes of the tangential motion; thus where there is least expansion or contraction there is the greatest amount of twist. In the circumference of a vibrating bell there are generally four loops corresponding to each motion.

The **effect of loading** a vibrating body is to lower its pitch; if the load be distributed uniformly, all the components are lowered; if it be suspended from points of the vibrating body, those component vibrations, if any there be, which have their nodes at those points remain unaffected.

The preceding propositions have related to the vibrations into which a body may enter when it is disturbed and then left to itself. The vibration in such cases is called *free vibration*, the period of which depends on the nature and the form of the vibrating body itself. If a body capable of vibration be acted upon by a series of impulses *ab externo*, the result depends upon the period of recurrence of these impulses. These may be so timed as to aid the natural free vibrations of the body, adding energy, and therefore increasing the amplitude at every oscillation; or they may be so timed as sometimes to aid, sometimes to thwart the natural oscillations, and thus to produce, on the whole, no effect so far as concerns the amplitude of these. In the former case the interval between two successive impulses *ab externo* is equal to the period of the natural vibration; while, when this interval differs materially from the period of the free vibration, the amplitude of vibration is not increased, but the energy communicated in the successive impulses is dissipated in heat.

A heavy bell has a natural period of pendular swing, and if a person gently pull the bell-rope, a very slight swing may be obtained, perhaps barely perceptible. If the pull be repeated while the rope is tending to slacken in the ringer's hand, the original small swing is increased, perhaps doubled; a succession of well-timed pulls causes ultimately a wide oscillation of the bell; and when the bell has been set fairly ringing the amplitude of the oscillation is kept up, and all loss of energy replaced by a series of well-timed impulses. If, however, the impulses be so timed that they sometimes act in favour of the oscillations of the bell, and are sometimes delivered against a tightening rope, the bell will either not ring at all or will do so very irregularly.

If a body capable of freely vibrating with *n* oscillations per second be placed in material communication with a body actually vibrating *n* times per second, the former will take up energy and enter into vibration. If a cylinder AB be fixed at the extremity of the rod CD; if another cylinder EF of like material and dimensions be fixed at the other end of CD; and if AB be set in longitudinal vibration,—then EF will be set in vibration of equal period. Here it must be observed—(1) that expansion of AB occurs at the same time with compression of EF, and *vice versâ*; and (2) that the centre of mass of the whole system remains unchanged in its position.

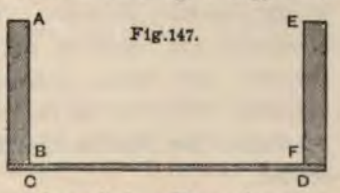


Fig. 147.

Two organ-pipes of exactly the same pitch, mounted on the same wind-chest, may fall into a similar opposition of phase, and retain it for long periods of time: as long as they do this, they can together emit but little sound, and the sound produced is rendered louder by silencing one of the pipes.

Two similar strings of equal length equally stretched over the same solid framework parallel to one another, will so adjust their vibrations as simultaneously to approach or to diverge from one another.

A tuning-fork may be regarded as made up of two equal rods connected with a common basis; the prongs simultaneously approach and diverge from one another when the fork is in vibration.

Two clocks which keep good time together beat, when placed on the same table, synchronously.

A tuning-fork can be set in vibration by another tuning-fork of exactly the same pitch vibrating within the same room. The material communication between the two forks is effected by the air; the sounding-fork causes waves in the air; these cause well-timed impulses against the second fork; the effect of these is cumulative, and the second fork takes up the vibration of the first.

In the same way a mass of air in a vessel or flask, since it has a natural period of vibration, may, if air-waves dash against the open mouth of the vessel at such intervals as correspond to its natural vibration, be caused by the accumulated effect of these waves to enter into violent oscillation. This principle we have seen made use of in resonators; and from the analogy of these instruments all phenomena of this kind may be called phenomena of **Resonance**; the law of which is, that any undulation or vibration is taken up—its energy is absorbed—by any body capable of freely vibrating synchronously with it, free to do so, and exposed to its periodic impulses. If the undulation or vibration which concusses the vibratile body be complex-harmonic, those components only which correspond to the natural period of the vibratile body are taken up by that body. On this principle resonators are used to detect the higher components of a complex sound.

Forced Vibrations.—Under certain circumstances a vibratile body may be compelled to surrender its own preference for a particular mode of vibration, and to vibrate with more or less accuracy in an arbitrary manner imposed upon it by external force.

Huyghens discovered that two clocks which did not keep time separately, kept time together when placed on the same table; the more rapid clock forced up the speed of the slower one while it was itself delayed. Two prongs of a tuning-fork, slightly unequal in size, will force one another to agree in their periods of vibration. If the one prong be powerfully pulled to and fro by an external mechanism so predominant that its period cannot be altered by any resistance offered by the fork, the fork as a whole will be forced to vibrate at a rate determined by the exterior mechanism or outside force; and it will maintain this rate as long as it is compelled to do so, but no longer; this being the case of a tuning-fork controlled by an electromagnetic interrupter, which is found to return to its normal rate of vibration as soon as the electric current ceases. The nearer the rate of the forced vibration is to the rate of the free vibration of the fork, the wider is the oscillation.

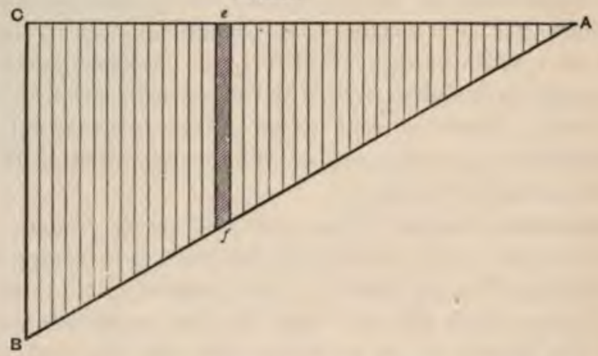
A resonating chamber of air will also on the same principle resound to some extent under the influence of a tuning-fork of pitch slightly differing from its own natural pitch. Here we observe that a steel fork can force the less massive air slightly to alter the period of its vibration; towards the end of each swing of the particles of the air, they are in very slow motion, and can with comparative ease be constrained to shorten or to prolong the period of their oscillation. Air-waves, on the other hand, cannot force a massive tuning-fork in this way, and in order that a tuning-fork may take up a vibration conveyed to it through air, the vibration of the primary sounding-body must be in exact unison with the free vibration of the tuning-fork acted upon.

There is difficulty in constraining a denser substance to take up a vibration communicated to it by a rarer one. The mechanical action of the ear involves, however, a problem of this kind; air-waves have to be communicated to the denser sense-organs; and this is effected by diminishing their amplitude and consequently increasing the force of their impulse—a principle now familiar to us.

Perhaps the best examples of bodies which can be forced into vibrations of any period are found among membranes; to some degree of amplitude any period of vibration can be forced upon a membrane, though all stretched membranes—as, *e.g.*, the orchestral drum—have natural periods of free vibration which can be modified by varying the tension. If a vibratile body which has a natural period of vibration be concussed by an external Fourier-motion, the resultant forced-motion of the vibratile body will still be Fourier-motion; but those components of the original motion which nearly coincide with the natural oscillations of the vibratile body will be represented in the resultant forced complex-vibration by components proportionately exaggerated—a principle we have already seen to affect the working of the Phonograph.

If a membrane be of unequal width and be stretched in one direction, a forced vibration imposed upon it will affect only certain parts of its area. In Fig. 148 ABC is a membrane, triangular in form and exposed to a tension parallel to CB. Consider a single very narrow strip, *ef*, of this membrane; imagine it to be isolated from the rest of the membrane. Such a strip would have a certain length, thickness, tension, density; it would therefore, if it entered alone into transversal vibrations, produce a note of a certain definite pitch. Let that note be sounded in the neighbourhood of the membrane; the membrane will vibrate strongly at *ef*, the disturbance rapidly shading off into rest on either side of *ef*; and, further, there will be some disturbance in those parts of the membrane whose length is 2*ef*, 3*ef*, and so forth. Each external sound is responded to by a different part of the membrane, which plays the part for the time being of an imperfectly-isolated string. At right angles to the direction of tension there is but little vibration.

Fig. 148.



If the external sound be complex, several such strips are set in motion.

Musical Instruments.—For the ends of musical art it is necessary that the vibrating body used as the source of sound should be capable, at the will of the performer, of producing several sounds. In the old Russian horn-bands each player had only one sound at his disposal, and by dint of practice and drill learned to produce his solitary note at the right instant; but this kind of orchestral music is quite exceptional.

We have already seen that all the notes of the scale may be produced on the monochord by varying the length of the free vibrating part of the string.

Some stringed instruments—the Lyre, the Harp, the Violin, etc., when played *pizzicato*, the Banjo, the Guitar—are played by plucking the strings. In some cases—the Lyre, the Harp—the number of sounds which can be produced is limited by the number of strings present; in others—the Banjo, the Guitar—each string is made to produce a number of sounds which depend upon the number of frets by which the finger of the executant is guided in shortening the string; in instruments of the Violin class there is no mechanical aid to the performer's fingers, and he is left to his own judgment as to the precise amount by which any given string should be shortened in order that it may emit the particular sound of which he has already formed a mental idea. All these instruments are provided with sounding boards which increase the surface by which vibrations are communicated to the air; and when their strings are plucked, the sound produced is of short duration and rich in high harmonics, poor in lower ones.

The strings of the Harpsichord were plucked by quills which were actuated by hammers. The sound was poor in quality, being feeble in the fundamental tone and disproportionately strong in the higher harmonics; and it was feeble in intensity as compared with the pizzicato notes of a violin, because the sounding-board was wanting in flexibility and had little effect on the air.

The Pianoforte contains very strong wire tightly stressed; the total stress on a Broadwood grand pianoforte exceeds the weight of 35,000 lbs.; that on a Steinway is 72,000 lbs.; whence the modern pianoforte is, as regards its framework, necessarily a very much more massive instrument than its predecessors. The longer the string corresponding to a given note, and the greater the tension upon it, the more precisely will the harmonic tones be in tune with the fundamental, and the fuller and richer will be the sound produced. The sound of a pianoforte string struck in the usual way is rich in harmonics up to the sixth; the seventh is purposely prevented by the choice of the spot at which the hammer strikes; the eighth and those beyond are feebly represented. The sounds of the higher strings approximate in character to pure tones. The compass of the modern pianoforte is from $A_{,,}$ ($=27{\cdot}1875$) in the thirty-two-feet octave to a'''' ($=3480$), or even to c''''' ($=4176$), the sound of an organ-pipe an inch-and-a-half long.

In a vibrating pianoforte-string those components disappear whose periods are $\frac{2}{3}$, $\frac{2}{5}$, $\frac{2}{7}$, etc. of the period of contact between the hammer and the string. Hence the varying quality of tone obtained from the same string by hammers of different degrees of repair or varying in the hardness or elastic softness of the leather, or by striking the pianoforte keys in different ways. The slower the stroke, the longer the contact, the greater the disappearance of higher harmonics.

The Violin, whose strings are tuned to ˋ*g*, ˋ*d′*, *a′*, *e″*, has a compass ranging from ˋ*g* to about *e‴′*; the Viola is tuned to ˋ*c*, ˋ*g*, ˋ*d′*, *a′*; the Violoncello is tuned to ˋC, ˋG, ˋ*d*, *a*; the Double-bass is tuned to $[\mathrm{E}_{\prime\prime}]$, $\mathrm{A}_{\prime\prime}$, ˋ$\mathrm{D}_{\prime}$, ˋ$\mathrm{G}_{\prime}$; these instruments give the strings of the orchestra an aggregate compass of from $\mathrm{E}_{\prime\prime}$ or $\mathrm{A}_{\prime\prime}$ to about *e‴′*, or about seven octaves. In the Violin three strings are of catgut; their pitch depends upon their thickness; the fourth string is weighted by a spiral of silver wire—an arrangement which to a great degree obviates rigidity. The belly of the violin acts as a sounding-board. The air in the cavity aids in the resonance and improves the tone, for it is easily forced to take up the vibrations of the solid parts of the instrument, especially those component vibrations which are already the most prominent; and this action of the internal air is important, as may be found on covering the *ʃ* holes with tissue paper, for then the tone of the instrument is materially deteriorated. The quality of the tone is found also to depend greatly upon the empirical form of the bridge.

Transverse vibrations of reeds are utilised in the Musical Box: reeds of various lengths are struck, vibrate in the free air, and produce sound of little intensity. In the Harmonium and Concertina the passage of air from a bellows into a resonating air-chamber is obstructed by reeds, which almost completely close the air passage. The pressure accumulates and bends the reed; the air escapes and the pressure is relieved; the reed returns and swings past its mean position, again to be driven outwards. The vibration of such reeds contain, as harmonics, tones bearing to the fundamental the ratios $\frac{4}{1}$, $\frac{9}{1}$, $\frac{16}{1}$, etc. Such reeds vibrating in the open air produce very little sound: the air of the resonating cavity acts as intermediary, and communicates the vibration by a broader surface to the external air.

In Mr. Baillie Hamilton's String-organ very various qualities of rich and full tone are produced by connecting in various ways a vibrating reed with a string stretched over a sounding-board.

In reed organ-pipes a reed vibrates at the bottom of a tube, the cavity of which it alternately opens to and closes against the access of air from the wind-chest: the air in the cavity of the organ-pipe resounds to the vibration of the reed, and does this most loudly when its own natural pitch coincides with the pitch of the reed. The Clarionet, the Oboe, the Bassoon, are instruments of this order; in them as in the organ-pipe the reed must be cut to a proper size: but each can produce several tones, for the vibration of a reed made of cane is more irregular than that of a metal reed, and as the size of the resonating air-cavity can be altered by manipulating the finger-holes and keys, to each altered size of the resonating cavity there corresponds a different note or component of the motion of the reed, which the tube is capable of selecting, and to which it resounds.

In the ordinary Organ-pipe a sheet of air is blown across the *embouchure* of the pipe, and vibrates like an invisible reed just outside the pipe: the air inside is set in vibration. The pitch of the sound produced by it is not so high as we might be led to infer from the dimensions of the column of air in the pipe, its elasticity, and its density alone: the reason is that the column of air in the pipe is not an isolated mass of air. While vibrating it has at its extremity to thrust aside the surrounding air at each expansion; the inertia of the surrounding air hampers the vibration, and, as it were, loads and retards the vibrating column.

Nodes in an organ-pipe may be demonstrated by Koenig's manometric capsules attached to the sides of the pipe. Near the centre of an open pipe there is a node, a maximum of variation of density, and hence the air at the node is alternately squeezed together and relaxed, the membrane of the manometric capsule placed at the node will vibrate, and the flame will either be extinguished or will present variations of height.

If an organ-pipe be strongly blown, the sound will rise slightly: if the force of the current exceed a certain amount, the sound suddenly breaks into the upper octave—a phenomenon familiar to flute-players—and near the centre of the pipe is now a loop, while there are now two nodes. Hence, in a pipe so overblown, the manometric capsule fixed at a point near the centre almost ceases to indicate variations of density, and the flame remains almost steady: never absolutely so in practice, for there is not even any one point at the centre of any loop at which all variations of pressure entirely vanish.

If a pipe be filled with hydrogen the sound produced is nearly two octaves above that produced by air at the same temperature; for the velocity of the sound-waves is $\sqrt{\frac{k\mathrm{K}}{\rho}}$ (p. 342);

K is the same in all gases, and in the case of hydrogen $\rho = \frac{1}{14{\cdot}47} \times$ the density of air; whence the frequency of vibration in hydrogen is to that in air as $1 : \sqrt{14{\cdot}47} = 1 : 3{\cdot}8021$ or $: : \mathrm{C} : {}'b +$.

If the barometric pressure vary, the pitch is unaffected: for $\sqrt{\frac{k\mathrm{K}}{\rho}} = \sqrt{\frac{kp}{\rho}}$; but the density ρ increases or diminishes *pari passu* with the pressure p: thus $\frac{p}{\rho}$ is constant, the velocity is constant, and the pitch is constant.

If the temperature θ vary, the ratio of K to ρ is disturbed. $pv \propto \theta : pv = \frac{p}{\rho}$; whence $\frac{p}{\rho} \propto \theta$ and velocity $= \sqrt{\frac{kp}{\rho}} \propto \sqrt{\theta}$. The pitch varies as the square root of the absolute temperature. A pipe which gives a sound $a' = 435$ at the temperature of 10° C. will vibrate 446·79 times per second at the temperature of 25° C., for as $\sqrt{283} : \sqrt{298} : : 435 : 446{\cdot}79$: the pitch is thus sharpened in the ratio $1 : 1{\cdot}027105$, or more than two commas; while the reeds are affected to a very much less degree, and thus an organ tuned in a cool room becomes discordantly out of tune with itself when the room becomes warm.

Problem.

Of what length should an organ-pipe be which sounds $\mathrm{C}_{,}$ at 0° C. and 760 mm. bar. pressure? $\mathrm{C}_{,} = 32{\cdot}625$ vibrations.

$$n = \frac{v}{\lambda} = \frac{1}{2l}\sqrt{\frac{k\mathrm{K}}{\rho}} = \frac{1}{2l}\sqrt{\frac{k\Pi}{\rho}}.$$

$$l = \frac{1}{2n}\sqrt{\frac{k\Pi}{\rho}} = \frac{1}{65{\cdot}25}\sqrt{\frac{1{\cdot}41 \times 1033 \times 981}{\frac{1}{770}}} = 508{\cdot}7 \text{ centimetres.}$$

A pipe of this length really gives a sound somewhat lower than $\mathrm{C}_{,}$; the wider the pipe the lower the pitch. A $\mathrm{C}_{,}$-pipe is called a 16-foot pipe, though really somewhat longer.

The common Flute, the Fife, the Piccolo, the Flageolet, are modifications of the open organ-pipe: the size of the vibrating column of air is altered in these instruments by opening or closing lateral apertures. In the Flageolet the mouthpiece is so constructed that the stream of air cannot but pass in the direction empirically found to produce the best sound: in the

Flute, Fife, and Piccolo the direction of the exciting stream of air, and to some extent the corresponding quality of tone, and even the pitch, are under the control of the player.

Brass instruments vary in shape from the Bugle (conical from mouthpiece to bell) to the Trumpet (slowly-widening tube and suddenly-widening bell), with the intermediate forms, the Cornet and the French Horn. The lips of the player are made to vibrate: the cavity of the instrument resounds. The scale of the instrument is confined to harmonics, which are obtained by varying the method and force of blowing and the tension of the lips; but since the size of the cavity of the instrument may be modified, any tone of the scale within the compass of the instrument may be brought in as a harmonic of some fundamental note. The size of the cavity may be modified by slides, as in Trombones—instruments which are capable of giving pure intonation; by pistons which lengthen the tube by predetermined amounts, as in the Cornet and Cornopean, the Euphonium, and Bombardon; by levers which shorten the tube, as in the Ophicleide. In some cases, however, as in the French Horn and Trumpet, the instrument is confined to one note and its harmonics; but its cavity may be altered in size by the addition of additional pieces of tubing or "crooks," whose dimensions are so adjusted as to cause the fundamental tone of the instrument to change by an interval predetermined for each crook; and the note produced may be to some extent modified both in pitch and in quality by the action of the lips, and of the right hand placed in the bell or the open mouth of the instrument.

The pitch of these instruments is that of an open pipe. Their tone is rich in quality and full of high harmonics; and though we are inclined to associate their peculiar quality with metal, it depends only on the form of the internal cavity and of the bell mouth, not on the material of the walls of the instrument, provided that these be rigid. A cornet sound will be produced by a cornet-shaped tube of guttapercha if the tube be so thick as to be rigid.

Stopped Organ-Pipes.—A longitudinal column of air steadied at one end should vibrate at one half the rate of a column free to expand at both ends; but a stopped organ-pipe does not give, as this rule would indicate, the octave of the open pipe of the same length, but its seventh, or even its minor seventh. When overblown it breaks into harmonics in the same way as an open pipe does.

Other Sources of Sound—Singing and Sensitive Flames. —A tube terminating in a blowpipe-nozzle is connected with the gas supply: the gas is ignited at the nozzle, and a small flame is thus produced. This flame is slowly passed up a wide glass tube, such as an Argand lamp-cylinder. At some particular position in the tube the flame alters its character and begins to sound forth a note somewhat higher than the natural note (in cool air) of the tube itself; or, should it not spontaneously burst into song, it may be induced to do so by singing to it a note whose pitch is nearly that which it will emit when in action. When the action has commenced, the flame is found, as by observation with a rotating mirror, to be alternately extinguished and rekindled; its image appears to be broken up into separate beads. A flame of hydrogen is prompter in its action than a flame of coal gas; for it excites more disturbance in the column of cool air surrounding it. When the action has commenced, the vibrations of the flame and of the column of air react on one another.

The flame must be of a certain height adapted to each size of tube. If the apparatus be arranged so that the flame stands about one-quarter up the tube, and if the height of the flame be carefully altered by slowly turning the controlling gas-tap, it will be found that at a certain definite height of flame the action will spontaneously commence, doing so with small initial intensity, while, when the gas-flame is lowered to one-half of this effective height, the sound breaks into the octave above.

When a jet of gas is allowed to flow vertically upwards under a pressure of about $\frac{3}{4}$ inch of water, through a fine vertical-jet, above which a sheet of fine wire-gauze is arranged horizontally at a distance of about 2 inches, the gas may be lit on the upper side of the wire-gauze, through which the flame will not descend. The distance of the jet from the gauze may be so adjusted that the flame is yellow at the tip, and at the tip only. The flame is now a sensitive flame, and responds by sinking down to the gauze when sound-waves strike it. If it be surrounded by a wide tube it will sing spontaneously. If, while the flame is so surrounded, the gas be turned down until the singing just ceases, the flame becomes extraordinarily sensitive to all very high sounds, such as hissing, the rattling of keys, etc., and it sings loudly as long as these stimuli are maintained. A simple flame issuing from an exceedingly narrow steatite burner, under a very great pressure of gas ("10 inches of water"), is sensitive to sounds too acute to be perceptible to the human ear; it alters its form under their influence.

Trevelyan's Rocker.—A mass of lead so shaped as to rest upon two long but very narrow linear feet. Placed upon a hot body, the points of contact of the rocker with the hot body are suddenly expanded by heat; the rocker is jerked upwards; before it has fallen back the heated points have cooled and returned to their normal dimensions. The process is repeated. The whole oscillates rapidly and makes a humming noise.

Radiophony.—When a beam of light or radiant heat falls upon a body capable of absorbing heat, that body becomes warmed and expands. A flash of light produces an instantaneous expansion, which immediately dies away. An intermittent beam produces a succession of expansions and contractions; in other words, the surface of the body vibrates. The amplitude of its movement may, with beams of light of moderate intensity, exceed the ten-millionth part of a centimetre. Lord Rayleigh has shown that this amplitude is sufficient for the production of sound; and the power of converting the energy of an intermittent beam of light or radiant heat into that of sound has been shown by Prof. Graham Bell to belong to all matter, with a few doubtful exceptions.

If an intermittent beam be focussed upon a mass of lampblack, at each flash of light it becomes warm, and the air within it is dilated; if it be contained in a test tube the open end of which is connected with the ear by an indiarubber tube, as the successive flashes produce successive dilatations and pulses in the air, these pulses are perceived by the ear as sound; if the lampblack be contained within a resonator, the frequency of whose natural vibration is equal to that of the frequency of succession of the flashes, the resonator emits a loud sound, audible at a distance.

PROPAGATION OF SOUND.

Propagation of Sound occurs in all elastic media, and is effected by waves of alternate Compression and Rarefaction.

Propagation of Sound in Solids.—Sound being a vibration is propagated along the ground; we may put our ears to the ground to listen for distant railway trains, distant vehicles, distant firing or marching. It is conducted along wood, as in the ordinary stethoscope; or in the experiment of closing the teeth upon a long piece of wood, to the other end of which an assistant holds a vibrating tuning-fork; or of causing a long strip of wood to rest by one end against the panel of a door, the other end being in contact with a vibrating tuning-fork,—the vibration in this case being communicated to the panel, which acts as a sounding-board, and itself sounds out loudly. Sound is conducted by wires; taps on a telegraph wire are audible at a great distance to an ear applied to the wire, provided that there be no intervening tunnel or bridge to form a resonating cavity and to absorb the energy of the vibration. In the Wire Telephone the central points of two stretched membranes or boards of thin wood are connected by a long wire, which, if sufficiently heavy and tense, may be suspended from posts, or even stretched upon carpet or bent round corners, and may thus serve the purposes of domestic telegraphy. When sound-waves impinge upon one membrane of the wire telephone, as when it is directly spoken at, the complex motion of the air is transferred to the membrane; by the membrane it is transferred to the wire; by the wire to the second membrane; by that membrane to the air, and by this to the ear of the distant listener. A slender apparatus of this kind, consisting of two parchment membranes stretched on rings, with an intervening silk thread, is sold as a toy under the name of the lover's telegraph.

Propagation in Liquids.—Divers while under water hear the sound of waves beating against the shore. A tumbler of water standing on a resonance box, if the handle of a vibrating tuning-fork be dipped in the water, will convey the vibration to the resonance box, the air in which will resound. An inverted bell filled with water and set in vibration will cause the water to assume beautiful wave-forms—an experiment which may be performed with an inverted propagating-glass set in vibration by a wetted finger drawn round its edge, or with a capacious wine-glass set in vibration with a violin bow. In the latter case the interest of the experiment is increased by substituting for the water some strongly-alcoholic liquid; the agitation breaks the surface of the liquid into drops which, by evaporation, lose some alcohol and dance on the surface of the vibrating liquid.

An organ-pipe may be blown by water under water; the water vibrates in the place of air.

Propagation in Air and other Gases.—In general, sound travels in air in concentric spherical waves. If it be restricted to tubes, the waves may become plane-fronted.

Though these waves are invisible their existence is beyond doubt, for when they strike any solid object they produce mechanical effects, and the phenomena of sound, so far as these depend upon propagation through the air, obey the laws of wave-motion.

The breaking of glass windows by the discharge of artillery, the destruction of the drum of the ear which has been known to be caused by the explosion of dynamite, the destruction of property by the explosion of a gunpowder magazine, are only exaggerated instances of that conveyance of energy by the air which is associated with the production of sound. In Edison's Phonomotor a membrane is stretched over a frame; at its posterior aspect it is connected with a broad hook which rests on the broad margin of a heavy wheel: the margin of this wheel is provided with small roughnesses so shaped that it is easy for the broad hook to slip over them in one direction, but in one direction only; on its return it is caught in the small teeth and tends to pull the wheel round. The membrane is spoken at; it trembles; the hook catches some of the teeth; on its return it gives an impulse to the wheel; continuous sound causes the wheel to rotate, and this with considerable speed and power; and if a little crank be fixed to the rotating wheel, the energy of the human voice may be made to perform obvious mechanical work.

Sound-waves in air are amenable to the laws of ordinary tridimensional Wave-motion already discussed.

In Figs. 43-48 we find curves whose forms depend upon the assumption that when two vibrations concur, the amplitude of the resultant is obtained by addition of the amplitudes of the components. To a first approximation this is true, but it leads to a curious result. If the amplitudes and periods of two vibrations be equal, the resultant vibration (Fig. 44), having twice the amplitude, will have four times the energy of either; the motions cannot be so superposed without a draft of energy from elsewhere. If two equal waves arrive in the same phase at the entrance of the same channel, there is found to be reflexion of a negative wave from the mouth of that channel.

Superposition of vibrations is familiar in Acoustics as a cause of **Beats**. Two tuning-forks or reeds are brought into exact unison; they emit jointly a smooth sound. Suppose their pitch to be $c'' = 522$. Load one with wax until it vibrates, say, 521 times per second. Once in the course of a second they will aid one another, and their action on the air at any given spot coincides; once in the course of every second they will thwart one another; their joint effect will be an alternate fading-away and swelling-out of the sound; but the pitch of that sound will be $521\frac{1}{2}$ vibrations per second. As the loading of the one fork increases, the beats increase in number until they become too rapid to be counted; but before this occurs the two notes have ceased to be blended into one note of average pitch, and the effect is that of a painful discord.

The easiest as well as the most accurate way of tuning a fork to a given note is to have at hand a standard fork which makes four vibrations less per second than correspond to that note; then adjust the fork to be tuned until it makes exactly four beats per second with that artificial standard.

Diffraction.—Sound-waves have in general the properties of waves whose wave-length is not small in comparison with the apertures through which they pass, the surfaces by which they are reflected, or the obstacles round which they flow. A sound-wave coming through a chink would suffer great lateral diffraction, as shown in Fig. 56; and the effects of obstacles intervening in the path of a wave of sound may not be such as even to produce a sound-shadow. If, however, the apertures, or surfaces, or obstacles in question be very large in comparison with the wave-length, there may be true sound-shadows and limited beams of acoustic disturbance, reminding us of the shadows and beams of light met with in optics. The air-waves produced by the note C (= 64) have a length of (33,200 ÷ 64 =) 518·75 cm., or between 16 and 17 feet; those produced by the note c'''''' have a length of about an inch and a half. When a mixture of long waves and ripples strikes an obstacle, the long waves may pass round it, while the obstacle may intercept the ripples, for relatively to these it may be wide enough to cast a sound-shadow. Thus the sound of a brass band suddenly changes in quality when the band comes round a corner into sight. It is plain that for acoustic purposes all the auditors at a concert, though they are not absolutely prevented from hearing by not seeing the performers, ought to be in full view of the orchestra.

It sometimes happens that a person at the same level as a source of sound, and in full optical view of it, hears nothing: the sound-waves, passing through strata of different density, curve upwards, and being very broad present no diffraction, and pass over the head of the observer, who may again come within their range by elevating his position. This occurs when the upper strata are cooler; when they are warmer, the sound descends.

Reflexion of Sound follows the ordinary laws of the reflexion of waves. Sound can be reflected by a mirror; a high-pitched bell ringing round the corner of a house can be rendered audible by a sufficiently-large mirror placed at a proper angle. If a stretched sheet of tracing-paper be placed at the same angle it will reflect a proportion of the sound and transmit some of it. A dry handkerchief will transmit a considerable amount of sound; so will even a half-inch layer of felt; but if wetted these become better reflectors, while they become almost impervious to sound. The reflexive power of flame is nearly the same as that of tracing-paper; and hot air above a gas-flame can reflect sound almost as well as the gas-flame itself. In clear weather the air is rarely uniform; there are ascending and descending currents of hotter and colder air; at each surface of each of these sound is partly reflected, partly transmitted, and so, ere long, it is wholly dissipated. Sound is often heard better in foggy or even in rainy or snowy weather than in clear, for then the air is more uniform. Sound coming through a fog (vesicles or minute drops), or a shower of rain, or a shower of snow, must be to a certain extent lost by repeated reflexion; but this effect is often balanced by the increase of intensity arising from the concentration of the waves in the narrowed channels between the drops or flakes.

In many buildings there are whispering galleries or places where a faint whisper uttered at a particular spot is heard at a distant part of the edifice. This phenomenon may arise in two ways:—(1) Reflexion from the vaulted roof, which acts like a concave mirror and causes the waves received by it to converge after reflexion upon a particular focus—a phenomenon very common in ellipsoidal roofs, a whisper uttered at one focus of the ellipsoid being reflected to the other focus, and distinctly heard there; a similar phenomenon also occurring in elliptical rooms where the sound is reflected by the walls: or (2) by the sound undergoing successive reflexions, and thus travelling round the walls.

Reflexion of sound is familiarly illustrated by the Echo. Sound striking a broad cliff or wall is reflected, the reflected waves sometime travelling with singular absence of diffraction and precision of direction. If a person can utter ten syllables per second, and if he speak loudly at that rate in presence of a cliff or high wall directly opposite him and at a distance of 1660 cm. (55 feet), just as he is commencing the second syllable the reflected sound of the first syllable begins to arrive at his ear, and at the instant when he ceases to speak the sound of the last syllable spoken begins to be heard; the sound travels to the cliff and back during the tenth part of a second. If the cliff were 3320 cm. distant, the speaker would hear two syllables repeated. When sound is re-echoed from cliff to cliff, or to-and-fro between two smooth walls directly opposite to one another, the result may be a multiple echo, which repeats a sound several times. The roll of thunder is partly due to multiple reflexion from cloud to cloud, partly to the varying distance of the points of disturbance, and the successive breaking on the ear of sound-waves produced along a long line.

When the distance of the reflecting surface from the source of sound is too small to produce a distinct and separate echo, the echo may be heard merely as a reinforcement of the sound produced; whence the practice of placing sounding boards behind and above pulpits and orchestras.

The action of the Ear-trumpet depends, in the first place, upon multiple reflexion; sound-waves on their arrival are reflected by the bell into the tube; then they travel, plane-fronted or nearly so, down the tube to the ear, narrowing in breadth and increasing in intensity as they do so.

Refraction of Sound.—If a lens be constructed of two large sheets of collodion cemented together at their edges and inflated with carbonic acid, sound-waves diverging from a watch placed at one side of this lens may, after passing through it, converge upon a focus on the other side of it; this shows that refraction occurs when the sound-waves enter the denser gaseous medium, the carbonic acid.

Interference of Sound.—If A and B in Fig. 75 represent the position of two tuning-forks kept accurately in unison, being driven by the same interrupted electric-current, the ear, placed successively at a', b', c', etc., perceives alternate sound and silence. The same occurs when A and B in that figure represent apertures in the side of a padded box within which an organ-pipe or bell is caused to produce sound.

Velocity of Sound.—There is but little information as to the properties of sound-waves travelling in a substance whose elasticity is not the same in all directions.

In Scotch fir the longitudinal propagation is more rapid than that across the fibres of the wood in the ratio of 5 : 4; hence sound-waves in that wood must be spheroidal. This is ascertained by observing the form of the nodal lines in a vibrating plate of that wood.

In practice we meet with spherical waves such as those in the air, and plane-fronted waves such as those which run along wires. In general the propagation of sound is amenable to the law $v \propto \sqrt{\frac{K}{\rho}}$. Where the compression produced by the wave as it runs has little or no effect upon the temperature of the medium, as in the case of solids and liquids, the velocity of sound is appreciably equal to $\sqrt{K/\rho}$; where heat is developed by the compression, as in gases, and the heat so developed has no time to be appreciably conducted or radiated away from the compressed part of the wave, the velocity of sound is equal to $\sqrt{kK/\rho}$, where k is the ratio between the two specific heats.

The velocity of sound in Solids may be found by determining the pitch of longitudinal vibrations set up in long thin rods. The length of the rod is half a wave-length, $\lambda/2$; the pitch gives n, the number of vibrations per second; the equation $v = n\lambda$ gives the velocity of sound in cm. per second.

The velocity of sound in Liquids may be determined by direct experiment, as by sounding a bell under water, and by listening at a distant station for the arrival of the sound, the precise instant of the production of which is signalled; or by comparing the pitch of organ-pipes blown in different liquids.

The formula $v = \sqrt{K/\rho}$ gives for water the velocity of 143,900 cm. per second, for $K = 2{\cdot}07 \times 10^{10}$ and $\rho = 1$; the observed value is 148,900 cm.

The velocity of sound in Gases—as, for example, air—has been directly determined by firing cannon at a known distance, and by observing the interval of time which elapses between seeing the flash and hearing the report. The objections to this method are that such violent concussions as those of cannon produce aerial vibrations which can be shown to travel faster than disturbances of less intensity, and that the velocity is not equal in all directions round the cannon or at all distances from it.

The length of an organ-pipe producing a note of a given pitch has been taken as a means of measurement. The column of air in the pipe, if it vibrated alone, would be half a wave-length in length; but the air is not isolated; the sound actually produced is graver than that corresponding to an isolated column of air, and the velocity so measured is not even approximately correct.

On similar principles the length of a pipe closed at one end and subjected at the other to the aerial impulses derived from a vibrating tuning-fork—this length being so adjusted by a movable piston that the air in the tube resounds its loudest—is taken as $\lambda/4$, and the velocity of sound is again obtained—only approximately, however (Fig. 149).

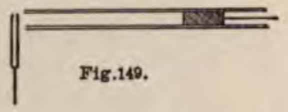
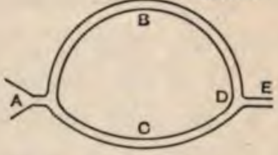


Fig.149.

Fig.150.



The velocity of sound in air may also be determined by methods based on interference. At A (Fig. 150) sound enters; the waves pass along the two channels B and C to E. B can be lengthened; it is lengthened until no sound is heard at E; B is now half a wave-length longer than C. At E may be placed the ear, a resonator, a manometric capsule, or any other indicator of sound-waves.

No energy passes down DE when no sound is heard at E: the waves are reflected at D and pass back to A, the B-waves returning along C, the C-waves along B; they arrive at A in the same phase. Even if they arrive at D in the same phase and pass on to E, there must be reflexion of a negative wave from D along B and C.

A glass tube vibrating longitudinally will emit a sound; the length of the glass is half a wave-length. If the tube contain air, the air will be set in vibration; but it must divide itself into segments, each half as long as an air-wave corresponding to the same tone. Finely-powdered silica placed in the tube will be distributed by the vibrating air in such a way as to accumulate at the nodes of the aerial vibrations. The number of air-segments thus indicated shows the comparative speed of waves in air and in glass. This is Kundt's method.

The theoretical value for the velocity in air is found from the equation $v = \sqrt{kK/\rho}$, where k can be found by thermodynamic experiments (page 342), K and ρ by direct observation.

The best average value for the velocity of sound in air seems to be about 33,200 cm. per second; the extremes being 330·6 (330·7 Regnault) and 333·7 metres.

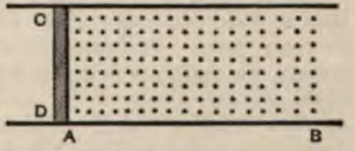
The velocity of sound in air is unaffected by variations of pressure, as we have already seen; it varies as the square root of the absolute temperature; it is affected by humidity, for damp air is lighter than dry air under the same pressure. It is also affected by wind, which not only retards the passage of sound to windward, but may also distort the waves and cause them to pass upwards. In general, the greater the intensity of sound the greater its velocity: sound therefore continuously slackens in speed of transmission as it becomes fainter.

It is still a moot point whether acute sounds are more or less rapidly conveyed through air than grave sounds: they are certainly not conveyed so far, for they are more affected by the viscosity of the air, and thus the higher components of an inharmonious sound are abolished by distance, and the whole effect is softened.

The velocity of sound being known, it becomes possible to measure certain distances by its aid. A lightning flash is seen, practically, instantaneously; if the sound take 5 sec. to travel, it must have travelled 1660 metres, or about a mile. Again, a stone is dropped down a well; the sound of the splash is heard in 6 seconds: at what depth is the surface of the water? The stone falls for t seconds through a space equal to $\frac{1}{2}gt^2$; the sound comes up for y seconds through a space equal to $33{,}200y$ cm. From the two equations $t + y = 6$, $\frac{1}{2} \times 981t^2 = 33{,}200y$, we find $t = 5{\cdot}543$, and the depth equal to 15,172 cm.

Propagation of Sound in gases according to the Kinetic Theory.[*]—Suppose AB to be a cylinder, CD an oscillating piston. The molecules which make up the gas in AB are represented by dots; for the nonce we shall suppose these molecules to be arranged in straight files, and we shall consider only one such file. The members of such a file remain in the same straight line, striking and rebounding from one another. We keep in mind the proposition that two perfectly-elastic bodies when they enter into collision rebound from one another with exchanged velocities; and that other, that a perfectly-elastic body striking a rigid obstacle returns with a velocity equal to the relative velocity with which it struck the obstacle. If the piston be at rest, it does not change the velocities of those particles which strike it, except in their direction. If it be moving towards them as they strike it, they rebound from it with a velocity greater than that with which they approached it; this increased velocity they communicate by exchange to others with which they come in collision: they then return to the approaching piston, and again rebound with increased velocity. These molecules thus act as carriers of energy; they borrow from the advancing piston a certain amount of energy, which they pass on to the molecules beyond; from molecule to molecule this energy is transmitted and a transitory crowding together of the molecules, commencing at the piston, is propagated through the whole gaseous mass. Conversely, when the piston is in retreat, those molecules which overtake it rebound with a diminished velocity; they exchange this diminished velocity with those molecules which they tardily encounter, and which they do not turn back until these have travelled farther than they normally could have done; a diminution of velocity and an accompanying rarefaction are propagated throughout the gas. If the particles all lay in straight lines, the speed of propagation of sound would be the

Fig. 151.



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[*] See Tolver Preston, *Phil. Mag.* iii. (1877).

average speed at which the particles move ; just as the rate of propagation of a message by couriers would be the average rate at which they ride. Nothing would be gained in point of speed by multiplying the number of such couriers if their horses were not susceptible to fatigue ; and so it is a matter of indifference what the number of molecules is by the intervention of which the exchange and transmission of energy are effected, provided always that the collisions of the molecules occupy time inappreciable in comparison with the intervals spent by them in traversing their free paths, and further that the size of the molecules be very small in comparison with the average length of the free path. Thus the velocity of sound does not depend, within the same gas, upon the density or on the pressure.

If the speed be changed, the case is different ; an increased average speed causes an increased velocity of propagation. This may occur within the same gas when the temperature is altered ; the absolute temperature measures the kinetic energy of the molecules ; to the square root of this the mean velocity is proportional ; the rate of sound-propagation is equal to this mean velocity ; whence the rate of sound-propagation must vary as the square root of the absolute temperature.

On comparing two gases we find that the mean velocity of the molecules varies inversely as the square root of the molecular weight, and therefore as the square root of the density ; whence the velocities of sound in two different gases are to one another inversely as the square roots of their respective densities.

We cannot, however, affirm that the particles of a gas lie in straight lines or files ; they move on the whole with perfect symmetry with reference to every point. Professor Clerk Maxwell showed that, taking this into account, we ought not to expect the rate of propagation of sound to be equal to the average velocity of the particles, but proportional to it ; and that, on the assumption that the particles were small as compared with their mean distances, and that each one was smooth and round, so as not to be set in rotation by impacts, then the rate of propagation of sound should bear to the mean velocity of the particles the ratio of $\sqrt{5} : 3$, or ·745 : 1. Kundt and Warpurg found exactly this ratio in the case of the vapour of mercury. In hydrogen the mean velocity is 184,260 cm. per sec. ; the velocity of sound in hydrogen is, according to the mean of several observations, 126,917·6 cm. per sec. ; the ratio is ·6888 : 1; less than that given above. The inference is that the molecules are to some extent set in rotatory as well as in translatory movement.

Döppler's Principle.—From a sounding body, approaching or approached, sound-waves reach the ear in greater number than when the source of sound and the listener are relatively at rest ; and conversely, if the sounding body recede or be receded from, fewer sound-waves will reach the ear. To a person standing at a railway station while an express rushes whistling through, the pitch of the whistle seems suddenly to fall as the engine passes him. Even the puffs of an approaching goods-engine seem appreciably more numerous to the ear than those of a receding one.

THE HUMAN EAR.

Aerial waves are communicated to the air in the external auditory meatus. This is short in comparison with the length of the average sound-wave. Its own proper sound is about g''''', and sounds in the neighbourhood of this tone are painfully reinforced by the resonance of the meatus.

The movements of the air in the meatus do not materially differ from those of a single point in the wave-front: the physical problem to be solved in the organ of hearing is one of the same kind as would be presented if the eye were called upon by the inspection of a single point on the surface of a multifariously-rippled sheet of water to discriminate all the component undulations of the extended surface.

The movements of the air in the meatus are, in consequence of the changes of pressure which they occasion, communicated to the drum of the ear, the *membrana tympani.*

The drum of the ear may receive some vibrations by direct transmission from the bones of the skull.

We remark here—(1.) The natural note of so small a membrane is very high; but weighted as it is by the chain of bones of the internal ear, it can take up vibrations of a much less frequency than this note.

(2.) The vibration of the membrane is a forced one, and, as regards the amplitude of very high components, does not precisely coincide in character with that of the air.

(3.) At the same time the form of the membrane is such that it vibrates more at its edges than at its centre, and the tendency of the membrane to set up vibrations of its own, or to alter those forced upon it, is mitigated.

(4.) The membrane is normally under tension: it is pulled inwards by the handle of the *malleus;* considerable pressures upon it cause very small inward movements, especially since its radial fibres have very slight extensibility.

(5.) It is easier for a rarefaction of the air in the meatus to cause an outward movement, which slackens the membrane, than for a condensation to drive the membrane inwards and thus to tighten it—a fact of importance in reference to combinational tones, of which hereafter.

(6.) When the membrane does move inwards, it pushes inwards the handle of the *malleus*, which is firmly attached to it: but only through a very small distance. This small amplitude of movement, about one-fortieth of that of the air in the meatus, implies that the handle of the malleus is wielded with considerable force—one step in the increase of the force of the aerial vibrations on their way to the internal ear.

(7.) The movements of the drum of the ear are astoundingly small. The greatest displacement seems to be about 0·1 mm. or 1-250th of an inch. A sound produced by an *f*♯ (=181) pipe under an air pressure of 40 mm. of water can be distinctly heard at a distance of 115 metres. Töpler and Boltzmann calculated that at such a distance the movements of the air must be reduced to ·000,04 mm.; but those of the more massive drum, with its appendages, cannot be more than ·000,001 mm. or the twenty-five-millionth part of an inch—an oscillation so minute as to be beyond direct microscopic observation. The drum of the ear sets in motion the handle of the Malleus. The malleus is a small bone, somewhat resembling a hammer, with a head and a handle. It is so suspended by ligaments, head upwards, that when its handle is thrust inwards, its head is made to rotate to a limited extent. The head of the malleus is connected by a smooth joint of peculiar form with a second bone, the Incus. The action of the joint is such that when the handle of the malleus is forced inwards, the head, as it rotates, locks in the incus and forces it round; while, if the handle of the malleus be driven violently outwards, the head, rotating in a reversed direction, does not pull the incus with it, but glides over it, rotating through as much as 5° before the two bones again begin to move as one piece. If air be driven through the Eustachian tube from the mouth-cavity, as it always is during swallowing, it presses against the membrane from within; if it were not for this peculiar joint there would be a decided risk of the chain of bones—malleus, incus, and stapes—being torn away from their connection with the internal ear. While the two bones are thus unlocked, as during swallowing, there is an impairment in their power of transmitting vibrations, and there arises a partial deafness, especially for loud sounds.

The incus has a process or long projection which, when the handle of the malleus moves inwards, moves inwards also. The point of this is attached to a little stirrup-shaped bone, the Stapes. Motion is thus communicated through malleus, incus, stapes; but the stapes move only two-thirds as much as the end of the handle of the malleus—another step in the increase of force and diminution of amplitude of the vibration conveyed to the ear by the air.

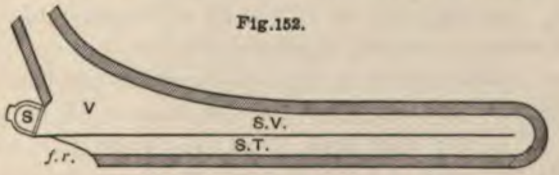
Communication of sound through the bones of the head seems to be effected, for the most part, through the malleus, incus, and stapes. Communication of sound to the bones of the head may be facilitated by means of the audiphone. This is a plate of thin vulcanite, which is bent and kept by strings under a certain degree of tension, and the edge of which is placed in contact with the teeth. Even a piece of stiff brown paper, loosely doubled and grasped by its opposite edges between the teeth, will act as an audiphone and take up waves of sound from the air, and will convey them to the bones of the head. A certain proportion of the sound travels directly to the nervous apparatus embedded in the skull, which is directly shaken; sound is thus rendered to some extent audible to those whose auditory ossicles fail in their function.

The footplate of the stirrup is blended with a membrane occupying a small aperture—the *fenestra ovalis*—in the hard mass of the temporal bone. The footplate of the stirrup has itself a form closely resembling that of a footprint of the right foot. If now the reader will place his right foot on a soft carpet and forcibly drive into the carpet the outer edge of that foot, he will see that the inner edge of his foot is tilted upwards; this describes the motion of the stapes when driven inwards against the membrane of the *fenestra ovalis*.

Beyond this membrane lies the fluid of the internal ear, contained in membranous bags, which float in channels hollowed out in the temporal bone. This system of membranous bags containing fluid consists of the vestibule, the cochlea (in front), and the semicircular canals (behind). Here we have to do with the two former. The fluid lying in the vestibule is immediately behind the membrane of the *fenestra ovalis;* the vibrations of the stapes are communicated to it. This fluid is in direct communication with the fluid lying in a part of the cochlea called the *scala vestibuli*.

The structure of the cochlea seems at first sight somewhat complex. It is a snail-shell-like object; if unwound and laid flat it might be diagrammatically represented by Fig. 152. S is the stapes, blending with the membrane of the *fenestra ovalis;* V

Fig. 152.

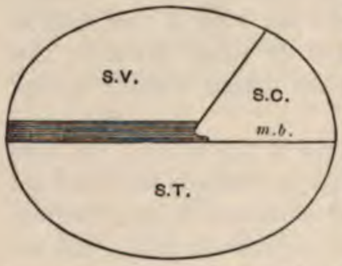


is the vestibule; S.V. is the *scala vestibuli*, a cavity extending to the tip of the cochlea; there it is continuous with S.T., the *scala tympani;* this ends at *f.r.* the *fenestra rotunda*, an aperture closed by a strong membrane. The fluid filling S.V. (the perilymph) is continuous with that in S.T., and hence strong pressure on the stapes will cause *f.r.*, the membrane of the *fenestra rotunda*, to bulge outwards. The *scala tympani* and the *fenestra rotunda* are perhaps a safety arrangement. The vibrations which are important to us are those of the fluid in the *scala vestibuli*.

Between the *scala vestibuli* and the *scala tympani* lies a partition, incomplete at the apex of the cochlea. This partition is partly of bone, partly of membrane: the purely membranous part is called the *membrana basilaris*.

Transverse section of the cochlea shows us further that we have not only to do with the two scalæ and the intervening partition, but also with a third cavity. Such a section is shown diagrammatically in Fig. 153. The *scala vestibuli* rests only upon the bony part of the partition; the third cavity, the *scala cochleæ*, S.C., lies mainly above the membranous part, the basilar membrane, *m.b.* The *scala cochleæ* contains fluid (endolymph), and is practically a closed cavity. When the liquid in the *scala vestibuli* vibrates, the endolymph in the *scala cochleæ* is at each impulse forced into similar movement before the liquid in S.V. has had time to pass into S.T.; and it in its turn acts upon the basilar membrane.

Fig. 153.



The basilar membrane is triangular in form, being widest at the *tip* of the cochlea. It takes up vibrations of definite pitch, in response to which it vibrates not as a whole but locally; just as when we sing to a piano with the dampers down, only those strings respond which are in unison with the sound produced by the voice. It responds to vibrations of considerable slowness compared with its natural vibrations, for not only does it lie between two liquids, but it is also somewhat heavily loaded; upon it are mounted certain rigid structures arranged in two rows, the rods of Corti; and upon these are arranged a number of nerve-cells, the cells of Corti, each of which is connected with a single nerve-fibre. Each of these nerve-fibres can only be stimulated to sensation by the vibratile movement of that cell of Corti from which it runs; and it is deaf to every sound but that one to which the vibration of the particular underlying part of the basilar membrane corresponds. As there are from 16,000 to 20,000 such cells of Corti, and therefore the same number of nerve-fibres which, although they merge into a common strand —the *auditory nerve*—and enter the brain side by side, are isolated from one another, we may say that we have not one, but from 16,000 to 20,000 distinct Senses of Hearing, each with its own special Organ of Hearing; the Ear, as we have described it, being merely a mechanical means for the transmission of vibration from the external world to these sense-organs, and its due distribution among them.

There is yet some difficulty in seeing how the rods of Corti, 3000 pairs in number, affect differently the half-dozen cells of Corti borne by each pair; hence some deny that more than 3000 different sounds can be perceived otherwise than by a process of comparison; even this would enable us to distinguish tones differing by less than the thirtieth part of a semitone.

When an external sound does not coincide with any of the above 16,000 or 20,000, the cells most nearly corresponding to it will be disturbed; one of these cells will vibrate more than its neighbours. The phenomenon is now one of unconscious comparison of their relative disturbances. Within the middle range of hearing, sounds differing by one vibration in three seconds can be distinguished by some persons; when the notes chosen are very high in pitch, grave errors in the discrimination of pitch may, on the other hand, be readily committed.

When a compound sound is produced, the basilar membrane is set in motion in a number of limited regions at the same time, and the effect is mingled in the brain. The nature of the compound sensation, by which we thus recognise the different qualities of sound, is a question which passes the bounds of physics.

The ear has a certain power of persisting in vibrations once set up in it; but only in small degree, and to an extent the greater the lower the pitch of the note. If the sound *c′* be broken up by alternate flashes of sound and silences of equal duration, when these each number 130 per second, the sound seems continuous. Each pitch has its own duration of persistence; and Mayer has pointed out (see *Phil. Mag.* 1876, vol. ii.) that in a mixture of sounds, alternately admitted to the ear and shut off from it by apertures in a rotating disc, some may be rendered evidently intermittent by the action of the disc, while others may appear continuous, and that thus we have a new means of analysing compound sounds.

As to the limits of hearing, sounds may be perceived by the human ear which range from 16 (Preyer), or 34 (Helmholtz), to about 32,000 (Despretz), or even 40,000 (Appunn and Preyer); there being between different individuals curious differences in the power of perception, especially of high sounds.

A small number of abrupt clicks cannot, with ease, blend into a continuous sound; if they seem to do so, it is generally some high harmonic that is really heard. A very small number of true pendular vibrations seems to produce sound when sufficient surface is acted upon, or the ear otherwise firmly enough set in vibration; whence the very deep hum of a river-steamer slackening speed, or that sound of contracting muscle—19 vibrations per second—which is heard when the forefingers are pressed into the ears and the elbows pressed against the table.

The sensible loudness of sounds does not coincide very closely with their physical intensity. This arises partly from modifications in the form of the vibration induced by so complicated a transmission through the auditory apparatus, partly from causes purely physiological.

It is curious that, as Mayer has shown, high notes are heard with difficulty in the presence of lower ones. Hence sixteen violins in an orchestra produce by no means so great an effect as sixteen violins alone. A lower note tends to drown a higher one, especially if the higher note be thoroughly in tune, or form a correct acoustic interval with the lower. In a single musical sound the fundamental tone drowns the harmonics, even though the latter may, as in a tinkling piano, very greatly exceed it in physical intensity.

The sensitiveness of different ears to sound may be compared by measuring the relative distances at which a given sound becomes inaudible; as the squares of these distances, so are the sensibilities of the listening ears. If one ear can hear a certain sound at 3 feet, the other only at 3 inches, then the duller ear is $(36^2 \div 3^2) = 144$ times less sensitive than its fellow.

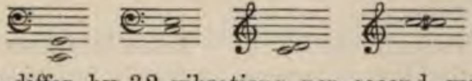
Even beyond the ear and within the brain there is some mechanism of which we are still ignorant. Professor Silvanus Thompson has shown that two sounds which beat with one another will, when conveyed separately one to each ear, produce beats which appear to jar the vertex of the brain; and further, that two sounds of the same pitch and phase, arriving separately in the ears, appear to be heard in the ears, while, if they arrive in opposite phases, the effect is as if the sound were heard not in the ears at all, but within the vertex of the cranium. The former phenomenon may be roughly shown by a tuning-fork held to each ear; the latter by a pair of telephones, one to each ear, one being provided with a commutator, by which the current in that telephone can be reversed at will.

Direction of sound can hardly be determined if the head be held fixed; we turn the head slightly while listening, and interpret unconsciously the consequent variations of intensity in the two ears.

HARMONY AND DISSONANCE.

Just as it is disagreeable to the eye to be exposed to flickering light, so it is painful to the ear to be exposed to audible flickering such as that produced by two sounds which beat when sounded together. The climax of unpleasantness or Discord is reached when the beats amount to about 32 per second. When the beats are still more numerous than this, the two notes which are sounded together become more distinctly separable by the ear, and the beats are less prominent to the sense of hearing.

Were this the only cause in operation, the intervals

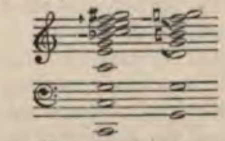


(16 : 17),

which all differ by 32 vibrations per second, would be equally painful to the ear. To some extent it is the case than an interval concordant between high notes is painful in the lower parts of the scale, as may be found on playing a major third in different octaves on a harmonium: but another cause affects the degree of painfulness of a discordant interval. When two contiguous sounds affect the basilar membrane simultaneously, the vibration of the basilar membrane is not limited to those fibres or narrow strips which exactly correspond to the two contiguous sounds heard; the result is a confused vibration of the membrane in a wider strip, and a sound is heard compounded of all the notes within the region of that strip; this is interpreted by the ear either—where the two component notes are very close—as a single note of average pitch, or, where they are farther apart, as two separate notes coupled with a painful sensation. Discord is thus due partly to beats, partly to difficulty in identifying pitch.

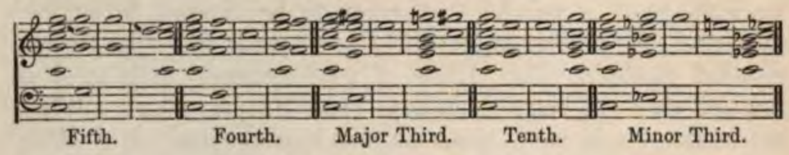
The beats produced by mis-tuned consonant-intervals correspond to curious vibrational curves, for which see Bosanquet, *Phil. Mag.* 1881.

When two notes are produced by separate sources of sound, the upper harmonics may possibly clash and beat with one another. This may be specially observed in the roughness of full chords of a brass band within an enclosed space. If, for instance, C and G be sounded together, the aggregate harmonics are the following:—



of which only *g*, *g′*, and *g″* are coincident.

If we eliminate all the harmonics beyond the sixth of the lower notes, the respective coincident and non-coincident harmonics for various intervals become the following:—



The most concordant interval is that in which the harmonics soonest coincide, and in which those non-coincident harmonics which discord with one another are as remote as possible from the fundamental tones.

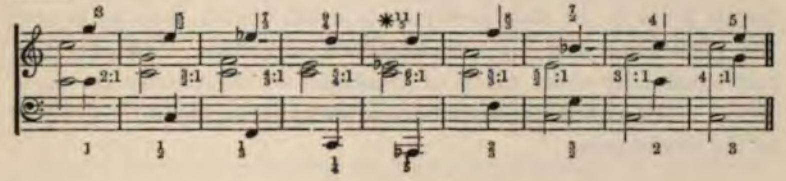
This is not the only element to be taken into account in explaining the relative harmoniousness of certain intervals. When two notes are sounded together, there are produced other tones called Differential and Summational Tones, or, generally, **combinational tones,** which are heard along with the former.

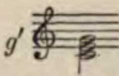
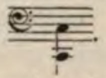
Differential Tones.—When c' $(=256)$ and g' $(=384)$ are sounded together loudly and firmly, the listening ear can distinguish the sound c $(=128)$ humming at the same time. With c' $(=256)$ and e' $(=320)$ the differential tone is C $(=64)$. The vibrational number of the differential tone is equal to the number of beats which tend to be formed by the two prime tones. For this reason it was long thought that the combinational tones were produced by the blending, into a continuous sound, of Beats too numerous to reckon. These tones are, on the contrary, distinct from the beats. If two very high notes, differing by 64 vibrations per second, be sounded together, the discordant shiver due to the beats can be distinctly perceived, though the beats themselves cannot be reckoned, while at the same time the corresponding differential tone can be heard humming. If the sounds be separately conveyed, one to each ear, the beats may be distinctly felt within the head, but no differential tone is heard. And further, if the two sources of sound be markedly distant from one another, the beats may be apparent while the differential tone is feeble.

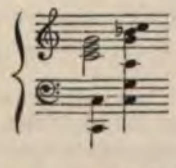
There are two causes for the formation of these tones. When a considerable disturbance has been produced in the air by one source of sound, the disturbance of the air produced by another source near the former is not simply added to the disturbance already existing, for Hooke's law—as the force applied, so is the distortion produced—is not true except for very small disturbances: the amplitude of the compound oscillation falls short of the sum of the amplitudes of the components, and this is equivalent to the introduction of a new vibration whose fundamental period is that of the differential tone. Differential tones so produced—as in the harmonium—can be reinforced and investigated by means of resonators. The more effective cause of the production of combinational tones is, however, their origination in the drum of the ear: the drum of the ear moves more freely outwards than inwards; and Helmholtz has shown that if a transmitter possessing this peculiarity be acted upon by a disturbance which is the sum of two disturbances of frequencies p and q, the energy imparted to it is in part expended in producing new vibrations of frequencies $p-q$ and $p+q$; these corresponding respectively to differential tones and to summational tones. Sounds so produced cannot be reinforced by resonators.

Summational Tones.—These tones were discovered by Helmholtz. The notes c' and g' (256 and 384) will, when sounded together, produce a faintly audible tone e'' ($=640=256+384$). Tones of this kind are only to be heard with difficulty—a fortunate circumstance, since they mostly discord with their prime tones.

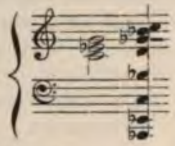
The summational and differential tones for some consonant intervals are represented in the following table, in which minims (𝅗𝅥) denote the prime tones and crotchets (♩) the combinational tones.



Harmony.—The chord c', e', g'  has a differential tone between c' and e', the tone C; between e' and g' the same; between c' and g' the tone c; together . Again, c' has a first harmonic c''; with e' and g' this makes the differential tones g and c. The note e' has a first harmonic e''; with c' and g' the difference-tones are respectively g' and c'; and similarly the first harmonic of g' produces, with c' and e', the tones c'' and b'♭—. Altogether, these form the series



all the tones of which occur as partials in the note C, and therefore blend smoothly together. The minor chord c', e'♭, g' in a similar way has the primary differential tones A♭, E♭, *c*, and the secondary differential tones *c*, *b*♭, *d′*, *g′*♭ —, *b′*♭, *c″*; and the primary chord is thus embedded in a mass of combination-tones comprising, among others, the discordant series—



The consonance of *c′*, *e′*♭, *g′* is therefore necessarily much harsher than that of *c′*, *e′*, and *g′*.

For further developments of this subject the reader must be referred to Helmholtz's *Sensations of Tone*, translated by Mr. Ellis.

When just intonation is possible, as among glee-singers or quartette-players, each listens to his fellow-performers as well as to his own voice or instrument, and gives out that note which he feels to belong to the key in which the party is performing, and learns to do so in such a way as to avoid beats: thus, as is said, the performers rub off one another's asperities. In this way mathematically-exact ratios of considerable complexity are accurately attained without necessary knowledge of them on the part of the performers.

VOICE—VOWELS.

The voice is produced by vibrations of the larynx, especially of the vocal chords, in whole or in part. Above these is placed the mouth-cavity, which may assume various forms under the action of the various muscles which regulate the position of the tongue, the soft palate, the floor, and the sides of the mouth. This mouth-cavity acts as a resonator and reflector. According to the number of upper harmonics which are reinforced, and the extent to which they are severally reinforced, will vary the quality of the sound emitted. Upon this quality, and upon nothing else, depends that character which we recognise as some particular Vowel; for every Vowel is a particular Quality of Sound.

An elementary example of this is furnished by a common pocket tuning-fork; when set in vibration and the broad face of one of the prongs presented to the ear, the fork seems to emit the vowel *u* or *oo;* when its shank is pressed against a table the fork seems to say *ō;* now the octave becomes prominent. The reason is that the fork swings in circular arcs, and not in transverse straight lines; at the end of each half-oscillation it consequently presses against the table and causes it to emit the octave as well as the fundamental tone. A tone almost pure gives the hollow sound of the vowel *ū*; one accompanied by its octave gives the brighter sound of the vowel *ō*. Each vowel gives a particular form of indentation of the tinfoil in a phonograph.

Vowel sounds can be analysed by means of resonators; and when a particular vowel is sung in presence of an open piano (loud pedal down) that vowel is repeated by the strings: each component of the complex vibration is taken up by that string which is in unison with it. On the other hand, Helmholtz showed that by causing a number of resonators of a series whose frequencies were as $1:2:3:4:5:6:7$, etc., to vibrate with independent intensities, he could at will produce by synthesis not only a great number of qualities of tone widely differing from one another, such as clarionet-tone, etc., but could also build up the different vowels themselves.

To each vowel corresponds a different form of the resonating mouth-cavity; to each such form corresponds a different natural pitch of vibration. When the larynx emits a complex sound containing as one of its components a tone of this natural pitch, this tone is strongly reinforced, and the quality of tone somewhat affected.

TRANSFORMATIONS OF THE ENERGY OF SOUND.

Sound being in its physical aspect a kind of motion, in the course of which work is done against elasticity and inertia, it is superfluous to speak of the conversion of the energy of sound into that of mechanical work. The transmission of sound is a transmission of energy, and the sound produced by a sounding body is mechanically equivalent to a definite amount of work. When a heavy tuning-fork is attached to the piston of a little pump, as in Edison's harmonic engine, it can be made to do work; but then it produces somewhat less sound than when vibrating freely. The mechanical equivalent of sound may be estimated, as it has been by Mayer, by comparing the sound produced by a free tuning-fork with the sound produced by the same fork on equal excitation when its prongs are connected by a thin strip of indiarubber, and finding the amount of heat developed in the rubber in the latter case.

Work may be, on the other hand, converted into sound. In general there are two methods of accomplishing this transformation,—firstly, by storing potential energy in an elastic body, which is then liberated; secondly, by transforming uniform into intermittent motion through the agency of friction. We have already studied the mode of excitation of a violin string. A pointed slate-pencil pushed across a slate at a certain angle produces a well-known shrill scream, and the mark produced by it will be found on close examination to consist of a train of separate dots; the action is not unlike that of the violin string. The scream of unoiled bearings in a machine may be accounted for in the same way, and in such a case much of the energy of rotation of the machine is wasted in the form of sound.

Heat may be, in some cases, transformed into the energy of Sound. Trevelyan's rocker and singing-flames we have already studied; the singing of a kettle is due to the rhythmical agitation produced by the formation and collapse of bubbles; the roar of steam issuing from a boiler is produced by the disturbance of the surrounding air by steam which, after thrusting aside the surrounding air, collapses into water-drops; the roar of a chimney is due to the oscillation to-and-fro within the chimney of heated columns of air or smoke which set the air within the chimney in vibration, of which the deep roar heard by us is generally a high harmonic: in all such cases the energy of the sound produced is obtained at the expense of the heat supplied.

But, like other forms of energy, that of sound is ultimately dissipated. When sound is produced in a room, every particle of the walls and contents of the room is set in vibration; there is, indeed, no way of protecting bodies surrounding a source of sound from this influence except perhaps by placing them upon several alternate layers of caoutchouc and soft putty within a vacuum. At last the sound degenerates, after repeated reflexion within each object, into irregular molecular motion, and its energy is converted into Heat. So when a tuning-fork is set in motion and sounded in the open air, part of the energy which was initially communicated to the tuning-fork when it was first set in vibration is lost in consequence of the viscosity of the fork, which becomes slightly warmer; while part of that energy is expended upon the external air, which, by reason of its own viscosity, gradually extinguishes the sound, beginning with the highest components, and the whole at length dies away, the energy of sound-motion becoming converted into the degenerated form of Heat, which ultimately becomes diffused throughout the entire Universe.

CHAPTER XV.

OF ETHER-WAVES.

In this chapter a variety of phenomena fall to be considered which can be explained as phenomena of undulation in the all-pervading ether, and may thus be said to be due to Ether-Waves.

It is necessary, however, to make a reservation of opinion, and to point out that all we are really entitled to affirm is that the phenomena in question are transferences of energy through the ether, accompanied by variable disturbances of that medium—disturbances whose variations follow the same laws as those of wave-motion, but which may in themselves be due to changes not necessarily of position within the ether, but possibly of its stress, of its electric condition, or of some other property of the interstellar medium as yet unknown to us. Their theory has been chiefly developed by those who considered the phenomena of Light, Radiant Heat, etc., as phenomena of Wave-Motion in the Ether; and, with this preliminary explanation, we shall in the sequel speak unreservedly of these phenomena as due to Ether-Waves.

Nature of Radiation.

The all-pervading Ether can be set in vibration by the vibration of the molecules of ordinary matter. This local disturbance sets up waves; and by these waves energy may be transferred from one place to another. This process of transference of energy is the process of **Radiation.**

The radiation of energy by the sun amounts to about 7000 horse-power per square foot of the sun's surface, of which 1/2,250,000000 part reaches the earth; this, striking the earth, amounts to about 83 foot-pounds per square foot of the earth's surface per second; hence, to use Sir William Thomson's phrase, the "mechanical value of a cubic mile of sunlight," the energy of the waves comprised within a cubic mile of ether near the earth, is about 12,050 foot-pounds.

Heat-waves and light-waves in ether are not waves of compression and rarefaction, like those of sound in air. The propagation of an ether-wave is effected after a different fashion, somewhat difficult to realise. The analogy of a **transverse** vibration running along a cord, or of a wave of up-and-down oscillation running over the surface of water or over a thin membrane, must be extended to the ether, with its three dimensions in space. At any point where the movement of the ether is examined, it is found to be an oscillation at right angles to the direction in which the wave is being propagated, and therefore parallel to the wave-front.

The vibration of the ether is initially of the nature of a forced vibration; it is probably excited by the oscillation of a part of the ether, which is in some way entangled within, or which envelops, the vibrating molecule.

The molecular vibration which excites the ether-waves is a true vibration of the molecule, not a translational oscillation from place to place.

The molecules of ordinary matter must be supposed, in virtue of their small size, to vibrate very rapidly. We have already stated that the average diameter of molecules is about the $\frac{8}{10000000}$ part of a millimetre, and that they may perhaps consist of ether rolling within ether in vibratile vortices. A steel tuning-fork 2 inches (50 mm.) long may, if it be of the proper form, vibrate 480 times a second; if it were $\frac{8}{10000000}$ mm. long, and of the same shape, it would vibrate 30,000,000000 times per second; if made not of steel, but of ether, its frequency would be greater in the ratio of the velocity of propagation in steel to that in ether, and would therefore amount to about 1,740,240000,000000 oscillations per second. The vibration of a molecule is more like that of a disc than that of a tuning-fork; but the rough analogy just mentioned may serve to show that it is, even *a priori*, probable that some such number may denote the average frequency of molecular oscillations,—an average modified in the direction of retardation by the formation of heavier molecules through the coalescence of smaller molecules, or perhaps by the reaction of the ether which is set in forced vibration, or modified, on the other hand, in the direction of acceleration by the formation of higher-pitched vibrations, which may be, to use the musical analogy, dissonant with one another when the structural arrangement of the molecules is unsymmetrical. The molecule of sodium-vapour acts somewhat like a disc which is slightly unsymmetrical: such a disc would give out two tones very near one another in pitch: and a vibrating sodium-molecule gives rise to two sets of ether-waves which differ only slightly in frequency.

Limits of Frequency.—We are not acquainted with any ether-waves except those whose frequencies lie between the limits of about 107,000000,000000 (Langley), and about

40,000,000000,000000 oscillations per second—a range, to use a musical analogy, of about eight-and-a-half octaves; but of these our eyes are sensitive to scarcely one octave—to those, namely, which range between about 392,000000,000000 per second (extreme red of the spectrum), and about 757,000000,000000 per second (extreme violet).

There may perhaps be ether-waves more or less rapid than the extreme limits mentioned, but we have no sense by which their existence is made known to us, and, at present, no experimental means of investigating them.

Velocity and Wave-length.—These waves all travel through the ether of space at the same rate, namely, about 30057,400000 cm. (186,680 miles) per second. Ether-waves while traversing the ether present no essential differences, except in respect of their frequencies, and hence also of their wave-lengths; the latter vary in a vacuum from about $\frac{1}{10000}$ cm. to about $\frac{1}{1300000}$ cm., and those waves to which our unassisted eyes are sensitive, the waves of light, have wave-lengths ranging between $\frac{1}{13042}$ cm. and $\frac{1}{25185}$ cm. These wave-lengths are usually specified in terms of "tenth-metres;" a tenth-metre being (1 metre$/10^{10}$), or 0·000000,01 cm. Extreme red and extreme violet have thus in a vacuum the respective wave-lengths of 7667 and 3970 tenth-metres.

Ether-waves do not traverse all substances with equal speed: hence their wave-lengths in different substances vary; if any particular kind of radiation have to be spoken of, it may be defined by specifying its wave-length in some specified medium, but it is better to state its numerical frequency. To do the latter implies, however, that we assume—and we are apparently justified in assuming—that all kinds of light pass through a vacuum—that is, through the ether of space—with equal speed.

Kinds of Radiation.—When a succession of waves impinges on a mass of ordinary matter, the effect varies according to the nature and the condition of the body which receives their shock; if it be an ordinary opaque mass, that mass may be warmed, the energy of wave-motion being transformed into heat, and the waves which have impinged upon the opaque mass are *ex post facto* called a beam of **Radiant Heat**; if they fall upon the eye, they may produce a sensation of light, and the wave-system is then called a beam of **Light**: falling upon a sensitised photographic plate, or a living green leaf, it may operate chemical decomposition, and it is then called a beam of **Actinic rays**. The word "rays" in the last phrase may be understood to mean not imaginary lines at right angles to the wave-front, but kinds of radiation; and hence we speak of Heat rays, of Light rays, of Chemical or Actinic rays; these names being given to one and the same train of waves according to the effects which it is found competent to produce. But while ether-waves are in course of traversing the ether, there is neither heat, light, nor chemical decomposition; merely wave-motion, and transference of energy by wave-motion. Hence none of these names can in strictness be applied to a train of waves while these are actually travelling through the ether.

Ether-waves which differ in their frequency differ to some extent in their degree of power of producing the motion of heat, the sensation of light, or of doing the work of chemical decomposition. All ether-waves can produce heat, for their energy is converted into heat when they fall upon and are absorbed by such a substance as a thick layer of lampblack, which for the most part arrests and extinguishes them.

The slowest waves known—those whose frequency is less than 392,000000,000000 per second—are too slow either to affect the eye with the sensation of light, or, in the ordinary case, to impart to molecules an agitation brisk enough to shake them to pieces, and thus to operate chemical decomposition. Such slow waves, whose presence can only be recognised after their impact by the conversion of their energy into Heat, are called **Dark-Heat-Waves.** If they fall upon an ordinary photographic plate they do not operate chemical decomposition; but if the molecules upon which they impinge be specially heavy and complex, even these slow heat-waves may be found to toss and shake them with briskness sufficient to break them up.

The waves may, on the other hand, be so rapid—above 757,000000,000000 per second—as to produce no visual effect on the eye; the eye is normally, physiologically, blind to them, and is unable to feel their impact; but they may effect chemical decomposition; their successive impulses may aid the natural free vibrations of the molecule, which thus become increasingly ample: and just as a resonant tumbler into which its own note is steadily sung vibrates, shivers, and breaks into fragments, so a molecule, quivering under the steady, regular, and continuously well-timed blows of the rapid ether-waves, may yield and break up into its constituent atoms, or into groups of atoms, which constitute simpler molecules. Such rapid waves are called Invisible or Ultra-violet **Chemical Rays**.

The power of operating chemical decomposition possessed by the more rapid waves depends upon their frequency more than upon their intensity.

The slowest waves may thus produce heat, or perhaps chemical decomposition of heavy complex-molecules; waves of medium rapidity may produce heat, the sensation of light, or chemical effect; the more rapid ones may produce heat or chemical effect according to the substance upon which they fall.

The invisible chemical rays, though they can operate chemical decomposition, are yet of very feeble physical intensity; their aggregate kinetic energy is, in the radiations from the sun, as we receive them filtered through our atmosphere, millions of times less than that of the slower red or dark heat rays: even those rays which are visible are effective not so much in virtue of their intensity, which is but small, as in virtue of the extraordinary sensitiveness of the eye to light—that is, to the impact of ether-waves of a certain range of frequency.

Colour.—Within the limits of visibility—392 billions to 757 billions—there is an indefinite variety of integral and fractional numbers, each of which represents the frequency of a particular kind of radiation, a particular kind of light. Physically there are as many kinds of light as there are possible frequencies between the limits mentioned. These kinds of light, each physically characterised by the number of waves which strike the eye during a second, are recognised by the eye as being distinct, not as the result of any conscious process of counting the number of impulses suffered by the eye during a second, which would be absolutely impossible, but in consequence of the distinct and peculiar Sensation attending the reception in the eye of wave-motion of each particular frequency—a sensation known in each case as that of a particular **Colour.** Thus, when we look at a Bunsen burner, the flame of which is caused to emit a dingy-yellow light by contact with common salt, we recognise the sensation as one of yellow light. Colour is a sensation: it is not a material existence; but the physical basis and cause of the special sensation of yellow light is in this case the joint simultaneous impact on the eye of two kinds of ether-waves, which have the respective frequencies of 508,905810,000000 and 510,604000,000000 per second, or the respective wave-lengths in air of 5895 and 5889·04 tenth-metres.

Either of these trains of waves impinging singly on the eye would produce a sensation of yellow, the slower one giving a yellow very slightly more orange in its tint than the other does. The term "yellow light," which means primarily a certain sensation, means, secondarily, the physical cause of this sensation—that is, a train of ether-waves of a particular frequency. Any particular colour is best specified by a statement of the frequency of the single wave-motion, which can produce that colour when it enters the eye; the analogy between light of any given Colour and a sound of any given Pitch being obvious.

When there fall successively upon the eye trains of light-waves which differ only slightly in their frequency, the respective colour-sensations produced by them may resemble one another generically, though not precisely. When, in gradual succession, luminous waves of all possible frequencies are caused to strike the eye, we obtain in successive gradation the sensations of all the colours of the spectrum. The slowest waves which can affect the eye produce a sensation of red, those somewhat more rapid a sensation of scarlet; then in succession we find, as the frequencies increase, that the sensations produced are those of orange-red, reddish-orange, orange, yellow-orange, orange-yellow, yellow, greenish-yellow, yellowish-green, green, bluish-green, greenish-blue, blue, blue-violet, violet. Waves of still greater rapidity than those which produce the sensation of violet are practically invisible; but it must be admitted that they are not perfectly so.

Even beyond the ordinary range of visibility some eyes are affected by ultra-violet ether-waves; a sensation of lavender-gray colour results: a spectrum is often seen, especially if the dispersion be small, to contain three bright bands of lavender-gray in the ultra-violet region. This light is in intensity about 1-1200th part of that which shines in the same region of the spectrum when it is rendered visible by fluorescence.

The following table, modified from Ogden Rood's *Modern Chromatics* and Sir William Thomson's Royal Instit. Lecture, Feb. 2, 1883, gives the frequencies and the wave-lengths in air of the several undulations which correspond to the several leading colours of the spectrum, and to some of the so-called Fraunhofer Lines:—

	Frequencies.	Wave-lengths in centimetres.
Line A	395,000000,000000	·00007604
Centre of red	... ...	·00007000
Line B	437,300000,000000	·00006867
Line C	457,700000,000000	·00006562
Centre of orange-red . . .	... ...	·00006208
Centre of orange . . .	... ...	·00005972
Line D^1	508,905810,000000	·00005895
Line D^2	510,604000,000000	·00005889
Centre of orange-yellow . .	... ...	·00005879
Centre of yellow	... ...	·00005808
Centre of green	... ...	·00005271
Line E	570,000000,000000	·00005269
Line *b*	... ...	·00005183
Centre of blue-green . . .	... ...	·00005082
Centre of cyan-blue . . .	... ...	·00004960
Line F	617,900000,000000	·00004861
Centre of blue	... ...	·00004732
Centre of violet-blue . . .	... ...	·00004383
Line G	697,300000,000000	·00004307
Centre of puce-violet . . .	... ...	·00004059
Line H_1	756,900000,000000	·00003968
Line H_2	763,600000,000000	·00003933

When a source of light is receding from the eye, fewer waves per second strike the eye; the light approximates towards red. Conversely, the light of an approaching luminous object is, as it were, sharpened in pitch. The characteristic lines in the spectrum are thus somewhat displaced; and by this application of Döppler's principle, the speed of relative approach or recession of the earth and many fixed stars has been estimated.

That which we call **white light** is, in the state in which we receive it from such a body as a white-hot bar of iron or, perhaps in its purest form, from the crater of the positive pole of the electric light, a mixture of long and short waves; waves of all periods are either continuously present, or, if absent for a time, are absent in such feeble proportions or for such short intervals that they are not appreciably missed by the eye. White light of this kind is comparable to an utterly-discordant chaos of sound of every audible pitch; such a noise would produce no distinct impression of pitch of any kind; and so white light is uncoloured. If a parallel beam of light of one kind, one wave-length, one colour,—homogeneous or **monochromatic** light,—be caused to pass through a slit in an opaque screen, it may be received upon a white screen, and it will cast upon that screen a coloured image of the slit. If the light, on issuing from the slit, instead of being received directly upon a screen, be made to pass through a glass prism, the narrow edge of which is held parallel to the slit, it will be refracted by that prism, and the image of the slit will now be found in a new position on the screen. If a beam of white light be so dealt with, a number of coloured images of the slit will be formed, each in its proper place on the screen, each image overlapping its neighbour if the slit be of appreciable width; there will thus be formed a many-coloured band of light, in which the colours are marshalled in the order of the frequency of their waves,—the slowest waves, the red, being least refracted by the glass prism; the quickest waves, the violet, being most refracted. This is the **spectrum**: every component of the original white-light is displayed in the spectrum, each in its distinct place; and thus the prism furnishes us with a means of analysing light—that is, of finding what its components are.

But the spectrum extends beyond the visible part of it; the more rapid invisible rays, being more refrangible than the violet, form an invisible part—an **ultra-violet region**—which we detect by the phenomena of fluorescence (p. 467), or by casting the whole spectrum upon a sensitive photographic-plate, upon which we afterwards find a record of a region of the spectrum invisible to the eye; and the slower dark-heat rays form an invisible part of the spectrum beyond the red, the **heat spectrum** or ultra-red region, not visible, but demonstrable by means of any apparatus such as a thermometer or a thermopile (Fig. 212), which is sensitive to heat. If the prism used be made of quartz, or if the spectrum be produced by reflexion from a diffraction-grating (p. 508), it will be found that the ultra-violet region is, if the light analysed be that of the electric arc, from six to eight times as long as the whole of the visible part of the spectrum; while if the prism used be of glass, it absorbs to a remarkable degree these rapid ultra-violet waves. If the light analysed be that of the sun, the ultra-violet part of the spectrum is comparatively very short, on account of absorption by the atmosphere.

This effect of the atmosphere is of extreme importance. Sunlight is originally bright blue, and is extremely rich in the more refrangible rays, but filtration through two absorbent atmospheres—that of the sun and that of the earth—renders it a yellowish-white (Langley). The ultra-violet part of the spectrum is enormously brighter at high altitudes.

Compound Coloured-Light.—Let us now cast a beam of sunshine or of electric light, shining through a slit in an opaque screen, upon a piece of greenish-blue glass, and receive upon a white screen the light which passes through this coloured glass: by the aid of a lens we may obtain a greenish-blue image of the slit upon the screen. So far as we have yet learned, such coloured light, whatever be the mechanism of its production, is a single kind of light—perhaps due to waves of only a single frequency: whether this be so in the particular case may be tested by interposing a prism in the path of the coloured beam of light: if the greenish-blue light be homogeneous, we shall again have on the screen an image of the slit, altered in position, but not in colour. This is not what we find: a short and imperfect spectrum is produced; the transmitted greenish-blue light is analysed by the prism into green light, blue light, yellow light, with perhaps some other colours, more or less faintly represented.

This phenomenon is very singular. It shows that two widely-differing physical causes are capable of producing exactly the same colour-sensation: the one being, as we have already seen, the impact of ether-waves of a single definite frequency, the other being the joint impact on the retina of a number of wave-systems, each of which is capable, if it were to act independently, of producing a distinct sensation; and the colour-sensation which is produced by the joint action of these wave-systems may differ from that which characterises any one of them. It is as if a listener to concerted music were to hear the strains of an orchestra compounded into some sort of loud melody of average pitch, he being wholly unable, by his unaided ear, to recognise the really compound nature of the sound heard by him. Then, whether the instruments all played in unison or diverged into pre-calculated harmony, the effect on his ear might remain the same.

Further, many such mixtures may produce the same apparently simple sensation; and, accordingly, such a phrase as "green light" or "orange light" is perfectly vague, unless it be accompanied by a specification of its physical cause.

Complementary Colours.—The greenish-blue glass in the instance just alluded to has in whole or in part prevented the transmission of violet light, of red, of orange, and of other kinds of light which are present in white sunlight; the complex of undulations thus denied transmission would, if collectively allowed to impinge on the eye, have produced a single sensation of red light. If this compound red-light had not been obstructed by the coloured glass, the transmitted beam would have been white; this compound red-light thus obstructed by the greenish glass, and the compound greenish-light transmitted by it, will pass together through a piece of clear glass, and will together produce the sensation of white light. To the eye it is a matter of indifference whether the red or the greenish light be monochromatic or compound; monochromatic red-light and monochromatic greenish-blue light, allowed to fall upon the same spot in the eye, will mingle, and, if they be of the proper tint, will produce the compound sensation of white light. These colours, red and greenish-blue, each of the proper tint, are thus **Complementary** to one another; together they make up white light.

The following pairs of colours are, among others, thus complementary to one another:—Red and a very greenish blue, orange and cyan-blue (a rather greenish blue), yellow and ultramarine blue, greenish-yellow and violet, green and "purple," the latter being a colour not in the spectrum, but formed by the superposition of blue and red.

The expression "white light" standing alone is thus also wholly vague; physiologically it means light which produces the sensation of white; physically it may mean (1) a mixture of all possible light-waves, long and short, in certain proportions; or (2) a mixture of two complementary simple colours; or (3) of a simple colour blended with a complementary compound one of any degree of complexity.

The white light of sunlight at sea-level is made up (Vierordt and Rood) by a mixture (= 1000) of the following coloured lights:—Red, 54; Orange-red, 140; Orange, 80; Orange-yellow, 114; Yellow, 54; Greenish-yellow, 206; Yellowish-green, 121; Green and blue-green, 134; Cyan-blue, 32; Blue, 40; Ultramarine and blue-violet, 20; Violet, 5.

RADIATIONS OF A HOT BODY.

The hotter a body the greater the intensity of the aggregate disturbance which it sets up in the ether; and further, the greater the frequency of the most rapid components of that disturbance. A white-hot iron ball is visible in a dark room; it emits dark heat-rays, light-rays, and also the rapid ultra-violet rays: it can be seen and photographed, and its warmth can be felt at a distance. If it be intensely hot it may emit so great a proportion of violet and blue light that it appears bluish; it is "blue-hot."

As it cools down, the more rapid vibrations die away; the ultra-violet waves cease to be formed; the mass becomes somewhat less easy to photograph by its own light. Gradually the violet rays cease to be emitted: the light radiated is now apparently tinged with yellow: the apparent colour becomes orange, then red; a body at a red-heat is difficult to photograph, though it continues perfectly visible in the dark. When its temperature sinks to a point below 525° C., it ceases to radiate light and becomes invisible in the dark; it continues, however, to radiate heat, as may be felt for some time by the cooler hand placed near it.

The luminous radiations of an Argand oil-lamp are $2\frac{1}{2}\%$ of the whole: of a gas-flame, 5; a glow-lamp, 5-6; a small electric arc, 10; a 5000-candle arc, at 3000° C., 25%. Of the solar radiation, 25% is luminous (Sir C. W. Siemens).

It never ceases to radiate heat; it could not cease to do so unless it were cooled down to absolute zero. Since the molecules of all bodies are in repeated collision with their fellow-molecules, as they rebound at each collision, they shiver and they vibrate. They must therefore continuously originate ether-waves—waves which, when the temperature of the body is below 525° C., are too slow to affect the eye.

Exchange of Radiations.—Two bodies placed opposite to one another with intervening ether of which we cannot get rid, and with or without intervening air, may present the two following cases:—

1. Both may be of the same temperature, in which case the one loses by imparting to the other exactly as much energy as it takes up from those ether-waves which strike it, having been originated by the other hot body; whence two bodies equally hot exchange their energies by radiation, but do this to an equal extent, and there is thus no change in their relative temperatures.

2. The one may, on the other hand, be hotter than the other. The hotter body sets up a more vehement system of ether-waves than the colder one can; in doing this it expends its energy to a greater extent than the colder one does; the hotter loses more energy than it gains; the colder gains more than it loses; in course of time their energies, and therefore their temperatures, become equal: when the temperatures have become equal, though the two bodies still go on imparting energy to each other, neither profits by the exchange, and their temperatures remain relatively equal.

The absolute amount of radiation of energy from a body does not depend on the condition or even on the presence of surrounding objects, but solely on the condition of the body itself. It is easy to see that the absolute physical brightness of the sun or of a candle is at any moment independent of the presence of illuminated objects; it is not, however, at first sight so clear that a fire not only warms a room, but the room also warms the fire;

that the sun warms the earth while the earth—to a lesser extent it is true—warms the sun; and that the warming of a colder body by a hotter one depends upon the difference of two similar but unequally-opposed actions.

When a lump of ice is placed near an object at the ordinary temperature, that object is cooled; it loses to the ice more heat than it gets from the ice: the ice apparently radiates cold.

When one body is surrounded by another, the body enclosed and the walls of the enclosure come to have the same temperature if they be relatively at rest. A thermometer whose bulb is immersed in a cavity will come to indicate the temperature of the walls of the cavity, whether it be in contact with them or not. This equalisation of temperature by radiation is quite independent of the form of the walls of the cavity; a cavity of any form acts in the same way as a spherical cavity would do. In figure 154 the irregular hollow body ABC surrounds a body E; both E and ABC assume after some time a common temperature, and remain at an equal temperature.

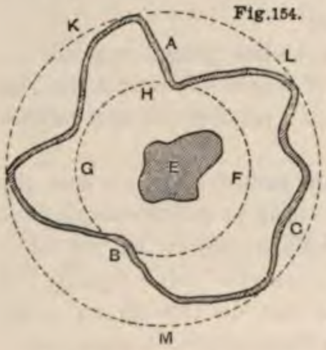


Fig.154.

The irregular body ABC might be replaced by the hollow spherical-body FGH, or by the hollow sphere KLM. From this ensue the following propositions.

1. The amount of energy received by a receiving surface per unit of its area—the amount of heat received, the brightness of light there—varies inversely as the square of the distance from the source of radiation. The advantage of extensive surface possessed by the larger sphere KLM is exactly neutralised by its disadvantage of distance; its surface is greater, the radiation received by it per unit of area is less, both in the ratio of the squares of the radii, and the total radiation received by it is the same, whatever be its radius.

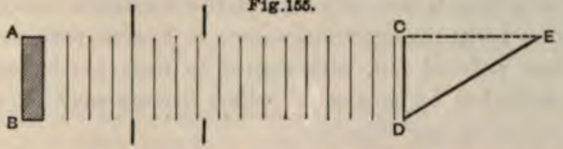
A candle at the distance of 1 foot can illuminate a printed page as brightly as a 25-candle gas-burner at a distance of 5 feet.

A bright wall is equally bright at all distances when looked at through a narrow tube. Close at hand it appears brighter, but less of it can be seen; at a distance it appears dimmer, but more of its surface can be seen; in all cases the amount of light falling on a given area of the retina is the same.

2. When a plane wave whose area is AB strikes squarely and simultaneously all parts of a surface whose area is also AB—the normals to the wave being also normals to the receiving surface—the receiving surface receives a certain number of units of energy per second. In Fig. 155 AB is a hot or bright body radiating ether-waves towards CD; CD receives e units of energy per second per unit of its area.

If the receiving surface be tilted, say into the position DE, the wave-front, striking obliquely, is now able to cover the larger surface DE; no more of the wave can now reach DE than would previously have reached CD.

Fig. 155.



The same quantity of energy is thus distributed over a larger surface: the quantity of energy communicated to it per unit of its area is diminished in the ratio $\frac{CD}{DE}$, or is equal to (e × cos CDE) units.

In accordance with this, the intensity of sunlight at noon is greater than during the earlier and later portions of the day, when the surface of the earth is presented obliquely to the sun's radiation.

3. Let AB receive energy from CD or DE; then, whether the surface be the smaller CD vertically facing it, or the larger DE arranged obliquely, is a matter of indifference; in either case there will be radiated towards AB the same amount of energy. DE therefore radiates towards AB in the direction DB, less energy per unit of its surface than CD does in that direction when equally heated, and that in the ratio of cos CDE : 1.

Were this not so, and did a hot surface radiate equally in all directions, then a body placed within a hot enclosure might become hotter than the walls of that enclosure.

This principle explains the apparent uniformity of brightness of the sun's disc. Towards the margin of the sun's apparent disc, areas which seem equal to similar areas near the centre are in reality much larger; but we see them obliquely; their larger superficial area exactly compensates the effect of their oblique aspect.

4. Radiation reflected from a mirror to a focus can never make an object placed at the focus radiate more energy per sq. cm. than the Source does; the temperature of the object cannot exceed that of the Source; but the object may, if sufficiently small, come to the same temperature as the source, after which there is between it and the source an equilibrium of radiation. Whence a thin wire in the focus of a very large mirror in sunlight ought (atmospheric absorption, etc., apart) to come up to the Sun's Temperature (3000° C., Siemens), but not to exceed it.

The law just stated—that bodies are always radiating and receiving energy—that the amount of radiation depends on the temperature of the radiating body—that at constant temperatures bodies radiate as much energy as they receive—is known as **Prevost's Law of Exchanges.** From this it follows that good radiators are good absorbents; and conversely, good absorbents are good radiators.

If a hot-water vessel be intended to retain its heat for a comparatively long period in the open air, it must be polished externally; a polished surface, being a good reflector, is a bad absorbent, and is therefore a bad radiator; while a blackened surface, being a good absorbent, is a good radiator, and heat is with comparative rapidity lost through a coating of lampblack, provided that it be not so thick as to impede conduction of heat to the surface.

Prevost's Law is not only true of the aggregate energy gained or lost by a body through radiation; it is also true, as Balfour Stewart has pointed out, with regard to each particular form or kind of radiation by means of which energy may be conveyed between neighbouring objects.

If a piece of yellow glass be placed within a hot shell of iron, the glass and the iron may both shine by their own light, and the glass may be looked at through a minute aperture in the wall of the hollow shell. Yellow glass absorbs ultramarine light, and a white-hot object looked at through it appears yellow, provided that the glass be colder than the source of the white light; but when the yellow glass is itself as hot as the source of white light, as it must be in this instance, in which we look through the white-hot glass at the white-hot wall of the iron shell, the glass seems perfectly transparent to the whole white light,—a phenomenon which may be interpreted as showing that while the glass only transmits yellow, it itself radiates blue light; the aggregate radiation, the transmitted yellow and the radiated blue, producing in the eye an aggregate effect of pure white. If the yellow glass be hotter than the source of light behind it, it seems relatively blue. The conclusion is, that as yellow glass absorbs blue light, so when itself heated it radiates blue light.

Stokes's Law.—A body which absorbs any particular kind of radiation will in general, when heated, become a source of radiation of the same kind; just as a resonator will, when it vibrates, impart to the air the same kind of sound of which it may rob the air when it is relatively at rest.

If a screen of strings tuned, say to the note of *a*, be arranged between a sounding *a* organ-pipe and a listener, the latter will hear comparatively little of the sound produced by the pipe; by resonance the strings have taken up the energy, and have converted part of it into heat. If a mixed sound were produced on the farther side of such a screen, the sound of *a* would not be transmitted to the listener; the rest of the mixed sound would be heard by him.

When mixed ether-waves strike a system of molecules of which some are tuned to particular frequencies, those molecules will take up the energy of vibrations of those frequencies: the body will appear to be opaque to the corresponding waves.

From the reciprocity of absorption and radiation it follows that if a given substance be divided into portions of which the one, A, is hot, while the other, B, is comparatively cool, radiations from A will be absorbed by B; the cooler portion, B, is opaque to radiations from the hotter portion, A. Thus, if carbonic oxide be burned, its flame contains hot carbonic-acid; the radiations from such a flame cannot pass through pure carbonic-acid, and are checked in very large proportion by air containing even a very small percentage of that gas or, curiously, of CS_2-vapour.

A hydrogen flame contains hot aqueous-vapour; the heat radiated from this—very slow dark heat-waves—cannot pass through aqueous vapour: in this way, as Prof. Tyndall has shown, while the sun's light and heat can reach the earth's surface through the humid atmosphere, their effect is to warm the earth and cause it to produce slow waves of dark heat; these resemble in frequency the waves produced by hot aqueous-vapour in a hydrogen flame, and they cannot pass away through the aqueous vapour of the atmosphere. The atmosphere thus acts as a kind of heat-trap, and the surface of the earth is preserved from extremes of cold produced by excessive radiation. But for the atmosphere the earth's temperature would be below − 45° C., even under the vertical rays of a tropical sun (Langley).

Burning sodium-vapour emits a particular yellow light; if looked at through a mass of sodium-vapour, it can hardly be seen; sodium-vapour absorbs the light given out by hotter sodium-vapour. Even though light of that particular kind do not happen in any particular instance to have been emitted by burning sodium, if the attempt be made to transmit it through sodium-vapour, the sodium-vapour will be found opaque to that kind of light. If an electric lamp produce a beam of light which contains amongst others this particular kind of light, and if a spirit lamp have salt (NaCl) placed in its wick so that it gives out this particular yellow light, this denoting that the spirit-lamp flame contains incandescent sodium-vapour; and if the electric arc be looked at through the spirit-lamp flame,—then the colour of its light would appear, if the eye were sufficiently sensitive, to be altered; it is bluer; the sodium-yellow light of the electric arc is absorbed as it passes through the comparatively cool spirit-lamp flame, which, by its own comparatively-feeble radiation, does not repair the damage done by it, and the light which has passed through the spirit-flame is comparatively (not absolutely) wanting in that particular kind of yellow. The beam may be made, after passing through the sodium-vapour, to traverse a slit and a prism, and thus to form a spectrum on a screen. It will be found, if this be done, that the spectrum is discontinuous; at the place where the particular yellow light ought to have been found, and would have been found had no spirit-lamp flame intervened, we find a **dark line**—a dark image of the slit, which, if the slit be fine and the focussing accurate, is found to be a double line; a line not absolutely lightless, but shining with the comparatively-feeble rays of the spirit-lamp, and therefore dark in comparison with its environment. If the temperature of the spirit-flame be increased, the dark line brightens up; if the temperature of the absorber be equal to that of the source, there is no dark line; if the temperature of the absorber be higher than that of the source, more of the particular light is emitted than is absorbed by it, and the line is relatively bright. The prism, which resolves any compound light into differently-coloured linear images of a slit,—images which stand side by side so closely as to blend into one another, but any defect or redundancy of brightness in any one or in any group of which can be at once detected,—offers a more delicate means of investigation than the eye can afford. In **Spectrum Analysis** a prism or a diffraction-grating is used to disperse into a spectrum the light which passes through a narrow slit from a luminous body; by inspection of the spectrum we can at once see what kinds of light are emitted, and what kinds are not emitted, by a luminous body. But the kinds of light emitted by incandescent substances are generally (since they depend on the vibrational frequencies of the molecules of the substances) distinctively characteristic of each chemical element, and, to a certain extent, of each physical state—of each degree of temperature—of the incandescent substance.

The spectrum of the limelight is continuous; that of the sun is not. It presents dark lines; among others, the double sodium-line: the presence of this indicates a bright central source of light, a hot region of the sun's atmosphere, containing incandescent sodium-vapour, the light from which is absorbed by the cooler sodium-vapour in the upper and cooler regions of the same atmosphere. These lines, discovered by Fraunhofer, and named after him, are distinguished by letters; and the best-marked of the numerous Fraunhofer-lines are known as A, B, and C in the red, D (a double line) on the orange side of yellow, E in the green, F in the blue, G at the beginning and H_1 and H_2 near the end of the violet. The position of any colour is often roughly specified by stating its proximity to one or other of these Fraunhofer-lines.

When the body radiating energy consists of a gas, each molecule, as it proceeds in its free path, executes free vibrations, like a vibrating tuning-fork thrown through the air; and the mass thus vibrating may impress upon the ether only one kind of vibration, or perhaps, if the structure of the vibrating molecule be complex, a large though not an indefinite number of simultaneous oscillations whose frequencies may or may not be commensurable. Thus a white-hot vapour may emit only a few distinct kinds of light, and may produce a **line-spectrum**—a spectrum consisting of a few isolated, linear, diversely-coloured images of the slit.

A very rare gas may emit very little heat or light even at such temperatures as 1500° C.: gases are bad radiators. The outer shell of a flame is non-luminous. Vapours are nearer their points of liquefaction than gases are, and are better radiators.

When the particles are so close together as to have no free path, or but a small one, they very frequently collide and rebound, and thus vibrate in an irregular manner; no rate of vibration is long enough absent for the eye to detect its absence. From the radiations of an incandescent solid or liquid, no kind of radiation appears to be absent up to the most rapid which is given out by the incandescent body; and the spectrum of such a body is **continuous,** so far as it extends. It is not, however, necessarily equally bright throughout, for didymium and erbium oxides give well-marked bright bands in the spectrum of the light which they emit while incandescent.

If a heated gas or vapour be compressed, the shocks between its molecules become proportionately more numerous: if its temperature be increased, the energy of each shock becomes greater; in either of these cases the vibrations of the molecules tend towards irregularity and complexity; and there may, in addition to the main free-vibration of the molecules—which is well-marked if there be any appreciable free path—be a number of additional vibrations of all or of many frequencies: a condition which is indicated by the broadening of the lines in a linear spectrum of a gas into the bands of a **band-spectrum.** As pressure is relieved the spectrum merges into that of an ordinary incandescent gas, or, on the other hand, as the pressure is increased, into the continuous spectrum of an incandescent liquid or solid.

Continuity between the gaseous or vaporous and the liquid states is thus indicated on an independent ground.

Light from incandescent solids or liquids travels from some distance within the surface; for it is polarised at right angles to the plane of incidence; this shows that it has been refracted on its outward passage through the surface of the incandescent body and into the rarer surrounding medium. Light from incandescent gases is not polarised; sunlight is not polarised; hence sunlight is due to incandescent gas or vapour.

Variations in the light emitted by one and the same substance under different conditions, and therefore in the spectrum of that light, serve to indicate molecular changes in the substance which radiates light. Salts have a different mode of vibration, and therefore a different spectrum, from their component elements; heat or, if heat fail, a discharge of high-tension electricity will break them up into their elements. Even the elements are reduced to comparatively-simple forms of aggregation by high temperatures; their continuous spectrum breaks up into one of bands; a still higher temperature, such as that of a high tension electric spark if other means fail, converts the spectrum into a line spectrum,—the line spectra being perhaps due to atoms, the band spectra to molecules. The spectra of the same substance at different temperatures are often remarkably dissimilar.

At temperatures beyond our reach, such as those of some of the fixed stars, or the lower levels of the sun's atmosphere—the high temperature of which may be inferred from the great amount of the highly-refrangible rays emitted by them—the elements themselves appear to be broken up and reduced to simpler forms of matter. This lends probability to the belief that the various elements are modifications of one kind of matter—a belief somewhat strengthened by numerous coincidences between the lines of the spectra of different elements.

TRANSMISSION, REFLEXION, AND ABSORPTION.

When ether-waves fall upon a **transparent** body, they pass through it: they are propagated through the ether which lies between the molecules. When a body is thus pervious to light it is specially said to be transparent; when pervious to dark heat it is said to be diathermanous,—no special term being used to denote transparency to actinic radiation. A body impervious to light is opaque; impervious to dark heat it is adiathermanous.

A perfectly-transparent body is invisible. Colourless thin glass, with a dustless, polished, clean surface, approaches this character: objects are seen beyond it, and, as we say, through it: they appear, if the glass be thin, inappreciably distorted. Light may be reflected from the polished surface of glass, and the presence of the glass may thus be rendered manifest to one standing in a particular position; the sun shining on the windows of a distant house makes the window-glass visible.

If glass be roughened at its surface, it presents numerous facets which reflect light so as to make the glass visible in all directions; and light passing through it is irregularly turned out of its path in all directions; objects beyond cannot be seen distinctly, though light can pass through the whole mass, and roughened glass, though not perfectly transparent, is translucent.

When glass is powdered, the powder presents so many facets and reflects so often the light which falls upon it that the whole is practically opaque: it is a powder which reflects in every direction the light incident upon it—in white light a white powder, in red light a red powder.

When ether-waves of any kind impinge upon a body impervious to them their progress is arrested; in part they are reflected or scattered; in part they are absorbed by the impervious body; the ether loses energy, ordinary matter gains it, and the impervious body is heated to an extent corresponding with the amount of energy absorbed—this heat being first communicated to the superficial layer of the body.

If ether-waves impinge upon a body which is transparent and diathermanous, that body is not heated, for the ether-waves pass through it and are not absorbed. Thus, clear mountain-air is not heated by the sunshine which streams through it; in the shade it may be very cold. Sunshine may stream through clear ice, or even through hoar-frost, without melting it. If there be any particles of dust in the air or in the ice, these, being opaque, will become heated, and the air is then, by conduction, rendered warm, or the ice is melted.

Some bodies are impervious to all kinds of radiation; others, having a power of Selective Absorption, are impervious to some kinds only.

Thus radiant heat can pass, while the more rapid light-waves cannot pass, through a thin piece of black vulcanite, or through a strong solution of iodine in bisulphide of carbon: while a solution of alum in water is, on the other hand, transparent to light, but is adiathermanous, impervious to heat-rays. Lampblack, again, is very transparent to the slowest heat-waves.

A soap-bubble film is remarkably adiathermanous, cutting off about half the heat of an incident beam.

Glass is transparent and diathermanous, but is somewhat opaque to the ultra-violet rapid ether-waves; a quartz prism or lens allows a great amount of ultra-violet radiation to pass through it which a glass prism or lens would extinguish.

The absorptive power of a substance may not be so extensive as to enable it to absorb and extinguish light-rays or heat-rays of all kinds; it may arrest some only. A piece of green glass can only allow a certain number of kinds of light to pass through it; by their joint impact on the retina these produce the sensation of green. Sunlight contains other waves than these; they have been absorbed; the green glass is opaque to them. These waves would together have produced a sensation of purple-coloured light. If this purple light had alone fallen upon the green glass, it would not have been transmitted; the glass would have appeared to be opaque. When sunlight is directed first through purple glass and then through green, the eye perceives blackness: the two pieces of glass are together opaque, though each of them is transparent to its own kind of light.

Very dark-red glass and green glass together produce a similar effect of blackness: pale-red glass allows some green light to traverse it, and so, when it is combined with green glass, the result is dark-green light.

Nickel nitrate absorbs red and violet, and is therefore green when in solution. Cobalt solution is red. A mixture of strong solutions of the two metals is black: diluted it becomes, however, almost colourless.

Copper, when it receives the impact of white light, emits orange light, together with superficially-reflected white light. Electrically-deposited copper, while immersed in a solution of sulphate of copper, which does not allow the transmission of orange light, looks as white as plaster-of-Paris does in the same liquid.

The colour of a coloured object, as seen by transmitted light, is produced by subtraction of the light absorbed from the light incident upon the object.

The kind of light transmitted may vary with the thickness of the absorbing medium. A solution of chloride of chromium in a thin layer absorbs much yellow, orange, and yellowish-green light; in a thicker layer it absorbs all but the red and some green and blue; in a still thicker layer the only colour transmitted is red. Thus a wedge-shaped layer of this solution appears to vary in colour, according to the thickness, from a greenish-blue, through purple, to red. Iodine vapour transmits a blue group and a red group, as also ultra-violet rays; together these produce an impression of purple: in thicker layers the blue rays alone are transmitted, and the vapour appears blue.

When a strong solution of blood is interposed in the path of a beam of light, no light but red is transmitted; dilute the solution gradually, and successively the solution appears more and more yellowish, and of increasingly paler hue.

The special absorptions of absorbent bodies are most thoroughly studied, not by means of their visible colours, but by the prismatic analysis of the light which passes through them. It is then found that some substances absorb several distinct kinds of light, belonging to different regions of the spectrum.

Transparent coloured-objects, through which light is filtered, give dark bands across the spectrum—the so-called **"Absorption-bands,"** which indicate what kind of light has been stopped and extinguished by the absorbent object—these bands varying in breadth with the degree of concentration of the absorbent solution employed, and varying in position with its nature.

When a strong solution of blood is interposed in the path of a beam of light which is on its way to form a spectrum on a screen, all the spectrum, with the exception of the red part of it, disappears. As the liquid is diluted the spectrum lengthens out: orange, yellows, greens, blues, are successively added; but there always remain two relatively-dark absorption-bands in the spectrum, in the yellow and in the green, between the Fraunhofer lines known as D and E.

If the blood be treated with sulphide of ammonium, it will be reduced; its oxyhæmoglobin will become reduced hæmoglobin; the chemical constitution changes, and with it the absorbent power; the absorption-band is now a single band placed between the two preceding.

If absorption-bands be numerous and pretty uniformly distributed throughout the spectrum, or if they be in complementary regions, the absorbent substance may present no distinctive colour, *e.g.*, benzene.

On the evidence of absorption-bands Capt. Abney has brought to light the existence of traces of benzene vapour between the earth and the sun, and Prof. Langley has shown that there are very peculiar gaps in the heat-spectrum which are probably due to absorption by the upper regions of the solar atmosphere.

The kind of light absorbed by a body may also vary with its molecular constitution.

It is supposed by Helmholtz that each absorption depends on the presence of a particular kind of molecule, differing from the simple chemical molecule. Chlorine has many absorption-bands in its spectrum, and it must either arrange itself in many kinds of molecules, or else its ordinary molecules, considered as vibrating bodies, must be extremely complex.

By changes in the absorption-bands we may learn that substances change their molecular constitution when heated. Iodine vapour gives an extensive absorption; when highly heated, the absorption-spectrum becomes reduced to a few bands; when the vapour is still more highly heated, some of the absorption-bands disappear, and one of them is replaced by a group of fine lines.

Sulphur-vapour changes its absorption-spectrum when its density changes at 1000° C.; N_2O_4, when it becomes NO_2, changes its spectrum, though it does not do so when it becomes a liquid; iodine, on the other hand, when dissolved in carbon disulphide, has the same absorption-spectrum as when it is in the state of vapour.

In a red solution of cobalt—the chloride, for example—when heat is applied to it, the salt enters into a different state of hydration; its molecular structure is changed; the solution becomes blue.

If there be no molecular difference between a substance incandescent and the same substance absorptive of light from a hotter object, a condition probably realised in the case of didymium, erbium, and terbium compounds, the incandescence- and the absorption-spectra will be mutually complementary; the one presenting bright lines where the other presents dark.

The **Colour** of a coloured object seen by reflected light is also generally due to absorption. An object seen by reflected sunlight does not appear to be coloured in any degree unless there have been absorption of some of the components of the incident white-light, and the colour of a coloured object is complementary to the colour which would have been produced by these absorbed components had they jointly impinged on the eye.

Some of the light incident on a piece of coloured glass is reflected at its surface; there is no absorption; if the incident light be white, the light reflected is also white. If a piece of green glass be laid upon black paper, and if it be looked at in such a direction that daylight is not directly reflected from it into the eye, it will be nearly invisible, and will be devoid of colour; it will appear black. If coloured glass be ground to powder, the powder is white; white light is reflected at every facet, while the light reflected from the lower surfaces of the fragments, and again issuing into the air, has nowhere traversed a layer of sufficient thickness to cause the extinction of all the absorbable components of the incident sunlight. The finer the powder, the whiter it is; the coarser it is, the more marked is its colour. If the upper surface of a sheet of green glass be ground, it will appear almost white; if the ground surface be looked at through the glass, it will appear green, for the light issuing from the glass is white light, which has undergone a certain amount of absorption.

If the green powder be immersed in water or oil, there is less reflexion at the several facets; there is deeper penetration of the light into the mass, and consequently more absorption; the colour appears to deepen. Hence the value of oil as a medium in painting.

A solution of chloride of copper placed in a deep black-walled vessel will not appear to have any colour; it will seem black; it reflects no light except from its surface. If powdered chalk be mixed with it, light is now reflected from the white particles of chalk, and passes out in every direction, through every part of the surface; so much of the reflected light is absorbed that it appears green when it reaches the eye,—the milky mass appears green. In a similar way a piece of malachite is penetrated by light to a very small depth; internal reflexion occurs; absorption of all the outpassing light takes place, with the exception of certain kinds, which jointly appear green; the malachite is green. A piece of polished gold reflects white light at its surface; it also reflects interiorly, and from within the substance of the gold at a very small depth there is reflected in all directions a quantity of light which, by absorption before leaving the surface, has become of an orange colour.

If the layer of gold be very thin, that part of the light which would be absorbed by a thicker layer may, in part, pass through and issue into transparent media before its energy is wholly converted into heat. A thin piece of gold leaf thus appears transparent and allows a greenish-blue kind of light to pass through it, which, if the leaf be rendered very thin by the action upon it of a solution of cyanide of potassium, may become violet, for both green and violet light then find their way through.

The object-glass of an astronomical telescope may be covered with a thin layer of silver, which will reflect the heat and some of the light, allowing a pleasant greenish light to pass.

When a beam of light enters the eye after undergoing repeated reflexion from gold to gold, it is of a deep-orange colour; this is the true colour of gold. As we ordinarily see gold, the orange light coming from its deeper particles is mixed with much white light irregularly reflected from its surface. The true colour of copper is scarlet, of silver a yellowish-bronze colour, of brass a rich golden-red. By reason of such repeated reflexion, a deep metal-vase, equally polished within and without, appears to be of a much richer colour internally than it is externally, and velvets appear of a richer colour than silks, for light undergoes repeated reflexions between the vertical fibres which constitute the outer aspect of the former.

When there is little opportunity for reflexion from the inner particles of a body, as where light falls exceedingly obliquely upon a gold mirror from a white object and is reflected into the eye, the image of the white object in the polished gold-mirror appears not gold-coloured, but white.

Some metals can be rendered transparent, not by being reduced to thin films, but by being reduced to the liquid state: potassium and sodium can be dissolved in anhydrous liquid-ammonia; the solution is blue, and the true colour of these metals is therefore a copper colour.

If the incident light be already coloured, it may be that the whole of it is absorbed. An object, blue or red in daylight, if illuminated by a sodium-flame, may absorb all the light that falls upon it; if it do so, it appears black; a bunch of flowers, looked at in such a light, where it is not yellow, appears black; it must either reflect some or none of the light which falls upon it. A piece of red cloth illuminated by the red regions of the spectrum glows with a bright red; when moved into other regions it becomes black, for it absorbs the incident light.

The **blue colour of opalescent bodies**, which in general present a multitude of reflecting particles embedded in a uniform matrix, and of which we may take as a type the sky-blue liquid obtained by adding to water a very small proportion of milk, is not primarily due to absorption. The principle is an established one, that where there is most refraction of light there is the greatest proportion of reflected light. A beam of mixed light falls upon a colourless transparent-body: all the rays are both refracted and reflected; the blue and violet are the more sharply refracted, and a greater proportion of them is reflected than of the less-refrangible rays. Even after one reflexion the image of an object in a mirror is bluer than the object itself. After multiple reflexion light may become distinctly blue. Multiplicity of reflexion is favoured by smallness of the individual particles.. The light which is not reflected is wholly, or in part, absorbed; the sun, looked at through a thin layer of dilute milk, appears yellow; through a thicker layer, orange or red; through a still thicker layer, it cannot be seen. Similar phenomena are presented by water into which a little very dilute alcoholic-solution of resin or mastic has been dropped with stirring, by salt water into which a few drops of a very dilute solution of nitrate of silver have been stirred, by a thin haze, by smoke; all these appear blue by reflected, yellow or red by transmitted, light. Even the Sky itself is a haze of this kind, the air being rendered visible against the dark background of black space by sunlight reflected from its fine suspended dust or water-particles; while the light transmitted is always more or less yellowish, and, in the afternoon and evening, when sunlight comes to us through a greater thickness of the more dusty layers, verges towards orange or even red. Such a dust-haze is more opaque than adiathermanous.

When the particles of a haze increase in size they jointly offer a greater resistance to the entry of light into the fog: light is reflected more promptly, and the reflected light presents a large proportion of white light. This phenomenon is familiar to the smoker; the thick clouds of smoke produced by vigorous smoking are obviously different from the thin fine blue columns which ascend from a cigar laid aside for a moment.

The **colours of metals** may be partly accounted for in a similar way. Steel and zinc have a normal refraction; the violet is most refrangible; they appear blue. Bell-metal, brass, Au, Cu, Ag, have abnormal dispersion; the red end is most refrangible and most reflected; they appear red or reddish. Speculum-metal refracts red more than green, but also violet more than green; on the whole it is reddish (Jamin).

Those rays which are absorbed in the greatest proportion by any substance are reflected by it in the least; when a beam of sunshine falls on a green leaf, the actinic rays are absorbed and spent in doing chemical work; the light reflected from such a leaf is feeble in actinic rays, and foliage is consequently not easy to photograph. Light which is absorbed is generally converted into Heat; this may presently be radiated away; shorter, quicker light-waves strike the body; longer, slower waves of dark heat leave it.

FLUORESCENCE, PHOSPHORESCENCE, AND CALORESCENCE.

Fluorescence and Phosphorescence.—The molecular disturbances of the interior particles of a body impinged upon by light may, however, give rise to other waves which are not so slow as to be invisible; the ether-waves absorbed may thus give rise to Light. In this case the body may not only reflect light, but it may also seem to emit light from within; it is **fluorescent.** The particles down to a very small depth, being set in agitation, originate a new set of ether-waves, which are propagated from each particle in every direction.

The phenomena of Fluorescence may be shown by a solution of æsculin, which may be very simply prepared by stirring some horse-chestnut twigs in water; a beam of light is caused to pass through this solution, and then for some distance within the solution the liquid seems self-luminous and shines in a dark room with an opalescent shimmer along the track of the beam of light. This effect is partly due to the impact of the light rays, but is principally due to the rapid invisible ultra-violet waves. If a piece of paper be wetted with a solution of æsculin, and if this paper be then used as a screen on which the image of a slit is thrown through a quartz prism, the ultra-violet part of the spectrum is rendered visible; a compound blue light radiates from the paper over an area six or eight times as long as the ordinary visible coloured spectrum; the light refracted by a prism may, with the same effect, fall on the walls of a glass vessel containing the fluorescent solution. Quinine chloride or disulphate, on paper or in solution, gives a blue light—that blue which is seen about the edge of the upper surface of a solution of quinine in a phial; petroleum or shale oil a green; turmeric solution in alcohol, or much better in castor oil, a green; uranium compounds, especially uranium glass, a green light; chlorophyll in solution, or lying undissolved in the cells of leaves, a red; an alcoholic solution of soot or one of *datura stramonium*, a greenish blue. Among fluorescent substances we find also such compounds as eosin (tetrabromofluorescein), fluorescein (resorcin-phthalein), anthracene, fluor-spar (especially chlorophane, which, when heated by conduction or by radiant heat, shines with an emerald-green light), many sulphides, especially those of barium and calcium, and, to a slight degree, the cornea and the crystalline lens, and the rods and cones of the retina.

Very frequently a body goes on vibrating for some time after ether-waves have ceased to strike it; this is familiar when the waves given out by it are Heat-waves. Sometimes, however, the body thus vibrating produces Light, and such a body—Balmain's luminous paint, for example—which goes on visibly shining or fluorescing for some time after ether-waves have ceased to impinge upon it, is said to be **phosphorescent.**

Among such bodies we find barium and calcium sulphides, diamonds, chlorophane, dry paper, silk, sugar, teeth, the alkalies and alkaline earths and their salts in general, and compounds of uranium.

These substances may be placed in a Geissler tube in a dark room ; an electric current passes ; the solids commence to fluoresce in the light produced by the discharge, but the observer's eyes are kept shut ; the current is stopped, and the eyes are at once opened to look at the tubes ; the solids are seen shining in the dark room.

For substances the duration of whose phosphorescence is very small Becquerel's Phosphoroscope may be employed. In rapid succession a phosphorescent body is exposed to bright light and brought against a dark background before the eye of an observer situated in darkness. Most objects are found by this means to be to some extent phosphorescent.

The compound nature of the light produced by fluorescence or by phosphorescence can be ascertained by means of a slit and a prism.

It is a very singular fact that the red rays of the spectrum and the invisible heat-rays have the effect of accelerating the exhaustion of a phosphorescing body. If a body, phosphorescing after exposure to white light, or better, to violet and ultra-violet rays, have a spectrum instantaneously thrown upon it, the body thereafter phosphoresces more brightly in the area occupied by the ultra-red part of the spectrum ; if the exposure to the spectral image be relatively prolonged, the phosphorescence becomes exhausted in those regions on which heat-rays had fallen, and now the Fraunhofer dark lines in the invisible part of the spectrum are rendered manifest by the survival of local phosphorescences in those parts of the screen which have not been affected by the impact of heat-waves (Becquerel).

A similar action of these rays has been long known : they often reverse the chemical action of the actinic rays.

As a rule a fluorescent or phosphorescent body emits for a longer or shorter time, on exposure to light, or, specially, on exposure to actinic rays, the same kind of light which, when light falls upon it, it absorbs ; and thus, in some instances, the light emitted by fluorescent and phosphorescent bodies presents bright bands where the absorption-spectrum of the same substance presents dark bands ; but the whole series of phenomena of fluorescence is one full of anomalies ; we do not fully know the laws of the molecular groupings of different substances, simple and compound, their necessary modes of vibration, or their relations to the ether.

A mixed beam of sunlight which has passed through a fluorescent solution cannot affect another solution of the same kind ; fluorescent solutions rapidly absorb those rays which are the effective cause of their luminosity.

We sometimes find transformation of slower waves into more rapid ones. When a solution of naphthaline-red has been shone upon by a beam of deep-red light, it emits by fluorescence an orange-yellow light. Chlorophyll presents an analogous phenomenon; it fluoresces with a red light, even though it be shone upon by a slower red-light. In the case of chlorophane, the impact of slow radiant-heat-waves is competent to set up an emerald-green light.

Calorescence.—When a beam of light is filtered through a solution of iodine in bisulphide of carbon, so that dark heat-rays can alone pass through, these heat-rays may be brought to a focus by a lens, and absorbed by a piece of platinum placed at the focus; this will become luminous and give rise to ether-waves of all kinds; if its light be examined by a prism it will be found to give a continuous spectrum. This phenomenon is called by Tyndall the **calorescence** of heat-rays.

SOURCES OF ETHER-WAVES.

Vibrations of Molecules.—Light, Heat, and Chemical Radiation being primarily due to the vibration of particles of ordinary Matter in the midst of Ether, the energy of ether-waves is derived from the kinetic energy of vibrating particles; and whatever increases the Kinetic Energy of these vibrating particles increases their vibratory movement, and gives rise to increased radiation. When by any action a given amount of energy is liberated in or communicated to a system of material particles, the rapidity of their resultant vibration, and therefore that of the ether-waves set up by them, depends on the rapidity with which that action occurs. When energy is slowly imparted to or liberated among them, the vibrations of the particles may remain relatively slow, and radiant heat may alone be the result; while if the particles be suddenly set in violent commotion, their vibration will be complex and irregular, the particles will become incandescent, and they will at once originate not only heat, but also light or even actinic waves.

When a flash of lightning or an electric spark rushes through the air it jars the particles of air, and renders the air incandescent and luminous; and it even originates actinic waves, for an electric spark can be photographed as well as seen. When the electric discharge through the air is continuous or rapidly intermittent, its light is, to the eye, apparently continuous, and we have the Electric Light. When a flint and steel are struck together the concussion agitates the molecules of those particles of steel which are knocked off, and a luminous spark is produced; so also when a bullet strikes a target there is a flash of light. Within a gas-flame molecules of a hydro-carbon are robbed of part of their hydrogen by a process of destructive distillation; the residues are heavy, almost purely-carbonaceous molecules, and these, in virtue of the energy supplied by the combustion of the hydrogen, become strongly agitated and incandescent, oscillating within the gas-flame, and therein acting as sources of light until the current takes them into the zone of perfect combustion in the outer region of the flame; there they become completely oxidised into gaseous carbonic acid, and thereupon lose in great part their radiative power. The brightness of a gas-flame is favoured by external pressure, or by a relatively small internal pressure and velocity of outflow, by the long continuance of carbon particles or other solid particles (which in a candle-flame cast a shadow in sunlight) within the flame in which they are incandescent, and by heating the gas before it enters the flame.

When a crystal is cleft it often emits a flash of light; work is done in splitting the crystal: the energy of part of this work appears as that of ether-waves.

When salts suddenly crystallise out of a liquid menstruum it not unfrequently happens that the formation of crystals is attended with a flash of light; the salt leaves the water and coheres with particles of its own substance; the agitation attending this process causes ether-waves to be set up.

Even the application of moderate heat, falling far short of such a temperature as might produce incandescence, may cause a body to become luminous, as in the case of the fluorescence of fluor-spar (chlorophane) and the diamond, which shine when heat is imparted to them by conduction.

We have already seen that light may result from the impact of ether-waves upon a body.

Chemical union is often attended with both heat and light: as when we drop copper filings or powdered antimony into chlorine gas, or in the ordinary phenomena of combustion. Even slow combustion, such as that of eremacausis or decay may cause light, as in the luminosity of decaying wood; or the green luminosity visible on the surface of some fish when in a state of incipient decay; or the slow oxidation of a piece of phosphorus in the air at ordinary temperatures, or of sulphur or the metal arsenic at higher temperatures. Even during the life of organisms they may become luminous either abnormally, as when the skin of the human body evolves phosphuretted hydrogen; or normally, as in the glow-worm, in the *noctiluca*, in medusoids, and in many other invertebrate animals: light being in these cases produced at the expense of the animal heat which might otherwise have been evolved.

Vibrations communicated to the Ether.—In all these cases the origin of the light plainly is in the agitation of ordinary matter, but there is a certain deficiency of knowledge in respect of the next step in the transference of energy. How is any ether-wave set up in the Ether by the motion of any particle of ordinary matter within it? A full answer to this question would involve a full knowledge of the constitution of the Ether, and of the relation of the Ether to the particles of ordinary matter which are embedded in it—a question still under discussion.

Some hold that the Ether is entirely independent of ordinary matter, being unaffected in density by its presence; others hold that it is of various densities in various substances, these densities being in different transparent substances inversely proportional to the squares of the velocities of light within them. Some hold that it is so independent of ordinary matter that a moving solid body moves freely through ether like an ideal net through ideally-frictionless water; in which case it would be difficult to understand how a vibrating molecule could set up vibrations in it. If this were so, the most rapidly-moving solid transparent object would allow the transmission of light through the ether which permeates it, as if it were itself at rest. The contrary view seems probable; a ray of light is said to be retarded a little by being made to pass up a running stream of water; the effect, quite perceptible in the case of water circulating at the comparatively-slow rate of two metres per second, is, however, imperceptible in a current of air.

A beam of light was found by Fizeau to be retarded when made to pass through a rotating cylinder of glass in such a direction that the rotation of the glass tended to carry back the light while in the act of passing through it.

The consequence of such an adhesion between the ether and the matter embedded in it is, that the earth must to some extent drag the ether with it as it rolls through space; yet Aberration (p. 473) tells somewhat against this.

The whole subject is as yet one of the most recondite in Physics.

PROPAGATION OF WAVES THROUGH THE ETHER.

At present it is usual in discussing the propagation of ether-waves to assume the wave to have been effectually set up; the wave-motion is studied as it diverges from a small wave-front formed in the immediate neighbourhood of the vibrating molecule; and in discussing the transmission of ether-waves of different wave-lengths through different transparent bodies, we shall have to take for granted that the interaction of the Ether and the ordinary Matter—an action which cannot be very great, for, if it were, Transparence would be impossible—is such as, in different media, unequally to retard ether-waves of different wave-lengths. This retarding effect depends somehow upon the nature of the transparent body; and this holds good not only with regard to light in general—as where a diamond is found to transmit light much more slowly than water does—but also with reference to each particular kind of light. Each transparent substance has its own rate of transmission for ether-waves of each particular frequency; and this is found for each case only by experiment. A denser substance may sometimes transmit ether-waves more rapidly than a rarer one does: light passes more rapidly through water, for example, than through alcohol or oil of turpentine. A substance through which light travels more slowly is said, however, to be optically denser.

On the assumption that the density of the ether is different in different substances, it would follow that all wave-lengths must be diminished or increased in equal proportions, that all kinds of waves must be equally retarded or accelerated, and all kinds of light, heat, or chemical rays therefore equally refracted, on passing from one medium into another—a conclusion contradicted by the simplest experiment with a prism. Cauchy, on the arbitrary assumption that the ether consisted of separate particles of an average size extremely minute as compared with the average distance between them, found that the amount of retardation was affected by the frequency of undulation, and that thus prismatic dispersion became explicable; an assumption which more modern writers—unwilling to admit that ether, which is not found to be capable of having waves of compression and rarefaction set up in it, and whose parts yet preserve or tend to preserve their mean positions, can be a fluid composed of separate molecules—have converted by interpretation into the following, namely, that there is some kind of discontinuity in the relations between the ether and the ordinary matter which it permeates; a discontinuity which is held to show that while ether may be considered to be a homogeneous jelly-like solid, which can yield to powerful stresses after the manner of a fluid, the matter, apparently homogeneous, which is embedded in it, is not truly homogeneous throughout.

The index of refraction, n, varies with the wave-length, λ, being connected with it by the law $n = A + (B/\lambda^2)$, where A and B are constants to be determined by experiment.

The Ether resembles a very weak solution of gelatine: to relatively-great momenta it acts as a fluid, and it closes up behind moving particles; to small stresses it acts as a solid, and it suffers tangential strain under the influence of a tangential stress.

Ether-vibration transverse.—When any part of the ether is displaced by a vibrating molecule, the displaced portion always tends to return to its normal position: in doing so it sets up waves. These are waves of transverse vibration like those of an elastic string or rod plucked laterally.

According to some the ether is absolutely incompressible, and it is impossible to form waves of compression in it; according to others waves of compression are at first formed, but very rapidly die out. The latter view assimilates the motion of the ether to that of an ordinary elastic-solid in which both longitudinal and tangential displacements occur and waves both transverse and compressional are produced.

According to Clerk Maxwell's view the ether is a homogeneous body, a non-conductor of electricity: periodic electric stresses applied to this produce waves which travel at the rate of about 300,000000 metres per second; these waves are waves of transverse vibration, and there is no vibration longitudinal or normal to the wave-front. These waves, due to electric displacement, are quite competent to explain the ordinary phenomena of light, and this theory explains on mathematical grounds that absence of the normal or compressional vibration which is a source of great perplexity in all the mechanical theories of light. According to this view, each particle of a body through which light is shining is in rapid succession exposed to alternately-opposite electric stresses: at each half-vibration it becomes oppositely electrified; but the ordinary effects of electricity are not generally observed when light shines through or on a body, for the electrification produced by any one half-vibration simply reverses the effect of that produced by the previous half-vibration.

The **Velocity of propagation** of ether-waves through the Ether of space is found by two astronomical methods.

1. Jupiter's Satellites.—These pass out of sight behind the mass of Jupiter and again reappear: when the earth is nearest to Jupiter the eclipses and reappearances appear to take place 8¼ minutes earlier, when the earth has wheeled round to the opposite side of its orbit and is at its farthest from Jupiter 8¼ minutes later, than they would have appeared if the earth had been at the centre of its orbit. The suddenly commencing or ceasing light takes 16½ minutes to cross the earth's orbit, a distance of 299,270,000000 metres: it therefore travels 302,300,000 metres per second. According to the latest determinations, the velocity is 299,336,000 cm. per sec.

2. Aberration.—No star is seen in its true place: every star seems to describe a little ellipse in the heavens, and seems to travel round the ellipse once a year. The reason is, that as the earth wheels onward in its orbit, bearing the observing telescope with it, rays of light coming from distant stars on their way down the telescope tend, short though the telescope tube be, to verge towards the hinder side of that tube: for which reason, in order to see the star in the centre of the field, the eyepiece must be tilted appreciably backwards in a direction opposed to that of the earth's orbital motion: the telescope, when the star is seen in the centre of its field, is therefore directed not towards the true position of the star, but towards a point in advance of it. In the course of a year, therefore, as the earth bowls round its elliptical orbit, the successive points to which it is necessary to direct the telescope are found to have been situated on the circumference of an ellipse. The size of this ellipse indicates the amount of tilting of the telescope: from

this can be inferred the proportion between the length of the telescope and the distance traversed by the ocular during the time spent by the ether-waves in passing down the telescope tube ; the speed of the waves of light can be calculated from these data, and is found to be 299,300,000 metres per second.

Do waves of different frequencies travel at the same or at different rates ? If their rates were different, then a suddenly-appearing satellite of Jupiter, or a suddenly-brightening variable star, would be first rendered visible by that light which first arrives at and enters the eye, and it might consequently appear violet or blue ; and when it disappears it would continue for the longest time visible by that component of light which is slowest in travelling, and therefore might appear red before vanishing ; or again, aberration of light would necessarily have the effect of giving us an image of each star drawn out into a spectrum. Nothing of the kind is observed ; all kinds of ether-waves must therefore travel through the ether of space at the same rate.

Terrestrial experiments for ascertaining the velocity of light are based upon one of two principles.

1. Fizeau's principle.—A ray of light is rendered intermittent by flashing between the teeth of a rotating cogwheel. It travels to a distant mirror ; each flash is there reflected along its former path. Before a flash can again reach the cogwheel, the cogwheel may have rotated so far that one of its cogs now obstructs the returning ray ; if a sufficiently-increased speed be imparted to the cogwheel, the light is allowed again to pass between the teeth of the wheel through a neighbouring notch, which has now come to occupy the position at first occupied by that notch through which the light had flashed on its onward journey. Given, then, that the light has travelled to a certain distance and back, and that in the meantime the cogwheel has been rotated through a certain angle, it is, in principle, easy to find the speed of propagation of the light. Fizeau found this to be 314 million metres : Cornu, by similar experiments, obtained the value 300,400,000 metres.

2. Foucault's principle.—A beam of light starts from a source S ; it strikes a mirror M, and is reflected to a distant mirror R, on which it is focussed by a lens : it is there reflected and retraces its journey : it is again reflected from M and returns to S. If, however, the mirror M have, in the meantime, been rotated perceptibly before the beam of light has had time to return from the distant R, the light can no longer be reflected from M towards the original point S ; it illuminates some other point T. The distance between S and T can be measured ; the amount of rotation of the mirror M in the time taken by the light to go from M to R and back can be inferred from this ; the amount of rotation of the mirror M can be read off on a speed-indicator attached to the rotating apparatus : the distance traversed by light in one second can be ascertained by calculation from these data. There is no need to use instantaneous flashes of light from S ; the steady beam from S reflected from the rotating mirror M only encounters the small fixed mirror R for an instant once in the course of each revolution, and is thus rendered practically instantaneous.

By this means, with a mirror rotating 1000 times in a second, Foucault demonstrated that light takes a measurable time to pass through a distance of 7 or 8 yards. Lord Rayleigh has shown that these different methods cannot be expected to give the same results, for it is not precisely the same thing which is observed in all these cases. In some (aberration method) the speed of single waves is observed; in others (Fizeau, Jupiter's satellites) the speed of propagation of a certain peculiarity, intermittence; in others (Foucault) these are blended.

As a mean result it may be stated that the velocity of ether waves in a vacuum—that is, in the ether of space—is 300,574,000 metres, or 30,057,400000 centimetres per second.

From this it follows that K, the rigidity of the ether, and ρ, its density *in vacuo*, are definite in amount, and bear to one another the relation $K = \rho \times (30057,400,000)^2$; for $v = \sqrt{K/\rho}$ centimetres. The mean velocity in air is less than that *in vacuo* in the ratio of 1 to 1·000294.

It is generally believed that light of all colours travels with equal velocities through air, though some doubt has been cast on this result by the recent experiments of Forbes and Young, who find that blue light travels *more* rapidly in air than red light does, in the ratio of 1018 to 1000.

By a modification of Foucault's method, above described, the relative speeds of light in two different transparent media, or in the same medium at different temperatures or under different pressures, may be compared. The light between M and R has to traverse a space in which a certain thickness of the medium, whose retarding power is to be examined, may be laid in the path of the beam: the beam may be exposed, by having to pass through this medium, to a retardation, which is rendered manifest and measurable by an alteration of the position of the image T.

The **Physical Intensity** of light at a place is measured by the energy transmitted through that place in a second of time; for light of constant colour this intensity is also proportional to its brightness as perceived by the eye.

Hence there are two methods of measuring the intensity of a beam of light:—1. Calorimetrical: allow the beam to fall upon a thermopile, and estimate the intensity of the light by the amount of heat into which it is converted upon absorption; the beam in this case having undergone a preliminary sifting through alum-water, which absorbs the heat-rays. 2. Photometrical: two sources of light are placed at such distances from an illuminated body that they appear to produce the same effect, such as equal shadows, or equal illumination of the two sides of a disc; but this method is only accurate when the two lights to be compared are of exactly the same colour. The intensity of actinic radiation may be estimated by observing the depth of tint produced in a piece of photographic paper exposed for a given time. The total radiation may be measured calorimetrically.

As a unit of photometric intensity the Paris Electrical Standards Committee has recommended (May 1884) the light emitted by 1 sq. cm. of melted platinum at its solidification-temperature.

MODE OF PROPAGATION—POLARISATION.

Waves of light—under which term we shall for the moment include all forms of ether-waves—have the peculiarities of propagation characterising waves whose wave-length is generally small in comparison with the breadth of their wave-front. They do not usually diverge laterally from the directions mapped out by the normals to their wave-fronts; or, as it is commonly expressed, Light travels in straight lines; they can only so diverge when they are made to pass through apertures or round obstacles not very much greater in breadth than their own wave-length.

The light from a single luminous point is propagated in spherical waves; that from such an extended object as a candle-flame in waves which, at some distance from the source, are approximately spherical. If light from a wide source be made to pass through a narrow tube, or successively to traverse equal apertures in two opaque screens, at such a distance from the source that the wave passing through the second screen has a plane front (see Fig. 57), then on the farther side of the second screen there may be an unwidening or parallel beam of light. Such a parallel beam of light, as it traverses space, may be compared to a bundle of vibrating strings of ether, isolated in the ether, vibrating independently, and practically unaffected by the ether situated laterally with respect to them. Each such imaginary individual cord may enter into transverse vibrations of different kinds, analogous to the vibrations of strings.

1. It may transversely vibrate simply up-and-down, or from side to side, or in any other single direction,—its vibrations are restricted to one plane; the whole beam is then called a beam of **Plane-Polarised** Light.

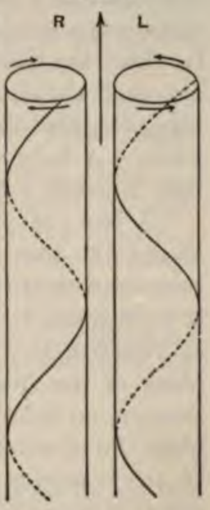
By a convention as to which there is some dispute, a plane cutting the beam in its whole length, but at right angles to the plane in which the ether vibrates, is called the **Plane** of **Polarisation.** The vibration of the ether is thus effected at right angles to the plane of polarisation.

2. Its vibration may be resoluble into simultaneous transverse-vibrations in two planes at right angles to one another.

(*a*.) These may be of equal period, and the vibration in one plane may be $\frac{1}{4}$ period behind or in front of that in the other; looked at endwise, *any part* of the ether in such a beam would necessarily be seen—if it could be rendered visible like a bright point on a vibrating string—to execute small circular vibrations. Such a beam of light is said to be **Circularly Polarised**. Looked at from one side the vibration would apparently progress like a screw.

A common corkscrew is a right-handed spiral. A simple experiment with a string, one end of which is fixed to a wall, while the other is held in the hand, will show that, in order to impress upon the string the right-handed spiral form, we must rotate the free end in a direction opposed to that of the hands of a watch. Such is the movement in a so-called **right-handed circularly-polarised beam** of light. When the rotation is in the opposite sense the circularly-polarised ray is **left-handed**. Fig. 156 shows the direction of propagation and of rotation, and the forms assumed by the vibrating ether in a right (R) and in a left-handed (L) ray respectively.

Fig. 156.



(*b.*) The circle may, by a difference of phase other than $\frac{1}{4}$ period, be converted into an ellipse. A beam, the ether in which rotates in ellipses, is called a beam of **Elliptically-Polarised** light; this again may be right or left-handed.

(*c.*) The periods of vibration in the two planes may not be equal, but may be commensurable. A beam of light of this kind would present movements which, looked at end-on, would present, for each portion of the vibrating ether, figures like those of 35-40, etc.

(*d.*) The periods of vibration in the two planes may not be commensurable or even constant, and further, the vibration in each plane may be variable in its amplitude. In such cases the vibrations would rapidly run through a great variety of figures, circles, ellipses, figures of eight, and non-reëntrant complex harmonic curves of every kind. This is the condition of a beam of **Common Light**. No single plane has any advantage. If for a moment the amplitude in any particular plane preponderate, this is but momentary: and since the most irregular transverse-vibration can be resolved into a vibration up-and-down, and one side-to-side, or may be resolved in any other two planes arbitrarily chosen at right angles to each other, a beam of common light may be held to be the result of the superposition of two simultaneous irregular transverse-vibrations, each plane-polarised, each possessed of half the energy of the whole vibration, and both propagated with the same velocity through the ether.

The doctrines of composition and resolution of harmonic motion are applicable to each small portion of the ether within such a transversely-vibrating beam, just as they are to transversely-vibrating strings.

A beam of common light encountering an object which is selectively transparent to vibrations in one plane, but opaque to vibrations in a plane at right angles to this, will have the latter vibrations extinguished; it will lose half its energy; the beam to which the object is transparent—the transmitted beam—will have half the energy of the original beam; and all its vibrations being executed in one plane, it will be a beam of Plane-polarised Light. A body which acts in this way on a beam of common light is called a **Polariser.**

A beam of plane-polarised light falling on a polariser will, should the plane in which its vibrations are executed happen to coincide with the plane of those vibrations to which the polariser is transparent, be found to pass through it freely: if the plane of vibration be, on the other hand, a plane at right angles to the plane of the freely-transmitted vibrations, no vibration can get through, no light is transmitted, and to light polarised in such a plane the selectively-transparent polariser proves perfectly opaque. If the condition be intermediate—that is, if the plane of the actual vibrations and the plane of free transmission through the polariser be neither coincident nor at right angles to one another—then the actual vibrations of the plane-polarised light must be resolved into two sets of component plane-polarised vibrations at right angles to one another; these, looked at end-on, carry out the principle of Fig. 42; and of these components—the one in the plane of free transmission, the other at right angles to that plane—the former is transmitted, while the latter is extinguished by absorption, its energy becoming converted into heat.

When ordinary light has had a certain proportion of its vibrations in a given plane quenched, while in the plane at right angles to this they are not quenched at all, or not quenched in equal proportion,—it is in a state of partial polarisation, and is called **Partially-Polarised** Light.

A plane-polarised beam of light may not only be resolved into two at right angles to one another and coincident in phase, but also (see Figs. 46 and 47) into two circularly-polarised beams, the one left-handed, the other right-handed. If it be supposed that a transparent body or a region of space is so peculiarly constituted or stressed that a left-handed circularly-polarised beam travels more rapidly through it than a right-handed one can, then, on passing a plane-polarised beam of light through such a region, the left-handed circular component emerges with its phase less advanced than the right-handed one; but the plane-polarised light equivalent to the synthesis of two such circularly-polarised beams can no longer be due to a vibration in the original plane; the plane has been turned round a longitudinal axis in the centre of the beam; and the farther a plane-polarised beam travels through such a body or region of space, the greater, in a direct ratio, will be the rotation of the plane of polarisation of that beam—a result observed in many cases, and to be described under the head of the so-called **Rotatory Polarisation**.

When the left-handed component is relatively accelerated in its transmission, or retarded in its phase of emergence, the plane is rotated to the left—*i.e.*, to an observer stationed at the source of light the plane of polarisation of the receding beam is seen to rotate in a direction opposed to that of the hands of a watch: the more-rapidly travelling left-handed component is at any point less advanced in phase than the more-slowly travelling right-handed component at the same point; the contrary-to-clock rotation of the right-handed ray prevails over the less advanced clockwise rotation of the left-handed ray.

Reflexion and Refraction.

When a ray of light travelling in a rarer medium strikes the surface of an optically denser transparent medium, some light is reflected, some refracted; and if there had been neither absorption nor scattering, the energy of the reflected ray, together with that of the refracted ray, would have been equal to that of the incident ray.

The incident ray and the reflected ray are in one plane: this is called the **plane of incidence.** The plane of incidence is at right angles to the reflecting surface at the point of incidence and reflexion.

The vibration of the incident light may be either at right angles to the plane of incidence,—*i.e.*, parallel to the reflecting surface,—or it may be in that plane, in which case the vibrating ether will not brush, but will strike the reflecting surface: or it may be in any intermediate direction or sequence of directions.

Fresnel, in investigating this subject, made the following assumptions, viz.—(1) that of the conservation of *vires vivæ*,[*] or, as we would now say, the Conservation of Energy; (2) that the movement of the incident ray merges continuously into that of the refracted ray; (3) that while it does so continuously, it does so very rapidly at the surface of separation; and (4)—a very arbitrary assumption—that differences of velocity of ether-waves in different substances are due to differences of density of the ether, whose elasticity remains unaffected.

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[*] *Vis viva* $=\Sigma mv^2$; energy $=\Sigma(\frac{1}{2}mv^2)$.

From these postulates he showed by mathematical reasoning—

(1.) If the incident beam be a plane-polarised beam, vibrating parallel to the reflecting surface, the refracted and the reflected are also plane-polarised beams whose vibrations are parallel to the original direction.

(2.) The frequency of vibration is unaffected by reflexion and refraction : the colours of the incident, reflected, and refracted rays are the same.

(3.) The angle of incidence is equal to the angle of reflexion.

(4.) The sine of i, the angle of incidence, bears to the sine of r, the angle of refraction, a constant ratio, n, the "Index of Refraction;" the numerical value of n depends upon the nature of the two media. $\sin i = n \sin r$.

(5.) The amplitudes of the three rays are related to one another in the following way:—

The angle of incidence is i; that of refraction is r; the intensity of the incident ray $= f$. Then v, the amplitude of the reflected ray, is equal to $\left(\frac{-\sin (i-r)}{\sin (i+r)}\right)$, while u, the amplitude of the refracted ray, is equal to $\left(\frac{2 \sin r \cos i}{\sin (i+r)}\right)$, times the original amplitude. From the former formula we learn—

- (*a*) That the greater the angle of incidence i, the greater is v, the amplitude of the reflected ray.
- (*b*) That when the incident ray is so nearly parallel to the surface of the glass as simply to graze it, the reflected ray is equal to the incident one and the refracted ray is *nil*.
- (*c*) That when i is greater than r, v is negative; while when i is less than r, v is positive ; or in words, when a ray strikes the surface of a denser medium, the reflected ray is a direct continuation of the incident ray, changed in direction ; while when light travels in a denser medium, half a wave-length is lost on reflexion at the surface of a rarer medium. This conclusion is independent of any such hypothesis as to particles as that by which we have already illustrated the same propositions on pages 122 and 123.

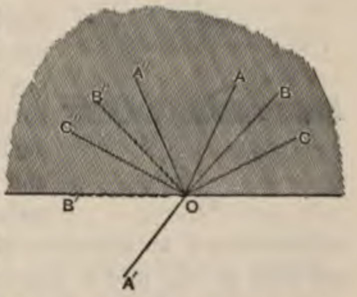
(6.) The respective intensities of a reflected, a refracted, and the incident ray are in the ratios of

$$\frac{\sin^2 (i-r)}{\sin^2 (i+r)} : \frac{\sin 2i \sin 2r}{\sin^2 (i+r)} : 1.$$

(7.) When light travelling in a denser medium strikes the surface of separation between the denser and a rarer medium at such an angle i that $\sin i$ is greater than n, both reflexion and refraction are possible; but if the angle of incidence be such that $\sin i = n$, then $\sin r = 1$, and the ray refracted into the rarer medium grazes the reflecting surface, for $r = 90°$; and any light falling still more obliquely upon the surface will be **totally reflected,** the reflected ray possessing the whole energy of the original incident-ray. In the last case $\sin r = \sin i./n$, which is greater than 1; and r, the angle of refraction, is an impossible angle; there is therefore no refraction.

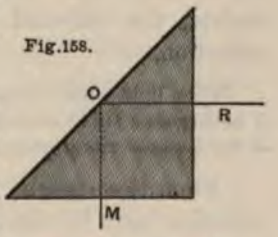
In Fig. 157 the ray AO is partly reflected to A″ and partly refracted to A′; the ray BO is partly reflected to B″, and partly refracted to B′, the refracted portion grazing the refracting surface; the ray CO is wholly reflected to C″, and is not refracted at all into the rarer medium.

Fig. 157.



As examples of Total Reflexion we may take a tumbler of water held above the head; it will give a clear mirror-image of the objects on the table below it; a bubble of air in water, or a test tube containing air immersed in water, will, when looked at at a certain angle, appear to have as bright a mirror-surface as that of mercury. In Fig. 158 light entering a total-reflexion prism at M is totally reflected at O, and travels towards R, which it reaches without refraction.

Fig. 158.



(8.) The energy of a ray totally reflected is equal to that of the incident ray; there is, however, a slight retardation of its phase.

These laws all apply to vibrations **executed at right angles** to the **plane of incidence,** and were deduced by Fresnel from the fundamental hypotheses already mentioned.

Let us now turn to the reflexion and the refraction of plane-polarised light whose vibrations are at right angles to these, and are thus executed **in the plane of incidence.**

In the refracted and reflected rays the vibrations will still be in the plane of incidence, but they cannot, after encountering the refracting surface, remain parallel to their original direction. The consequence deduced by Fresnel from this is, that while the ordinary laws of refraction and reflexion are obeyed by such plane-polarised light so far as directions are concerned, the relative intensities of the incident, the reflected and the refracted light, are not the same as they were in the preceding case, but are now respectively in the ratio

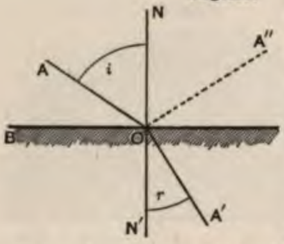
$$1 \quad : \frac{\tan^2 (i-r)}{\tan^2 (i+r)} \; : \quad \frac{\sin 2\, i \sin 2\, r}{\sin^2 (i+r) \cos^2 (i-r)}$$

incident : reflected : refracted.

The intensity of the reflected light is, in a particular case, *nil;* that is, when $\{\tan^2 (i-r) \div \tan^2 (i+r)\} = 0$, or when $(i+r) = 90°$.

In Fig. 159 the ray AO, whose vibration is *in* the plane of incidence, falls at such an angle i that it is refracted along OA′; if it had been reflected at all it would have been reflected along OA″; at one particular angle of incidence, AON, the refracted ray OA′ and the reflected ray OA″ (if there had been such a ray) would have been at right angles to one another. When the angle of incidence and the angle of refraction together make up a right angle, there is no reflected ray; no vibration effected *in* the plane of incidence is reflected at all, and a plane-polarised ray of this kind falling at the appropriate angle of incidence, however bright it may be, will fail to be reflected from a mirror. This is a case of **Total Refraction;** the whole of the energy of the incident ray is in the refracted ray. When the incident light grazes the refracting surface, the reflected beam also grazes it, and there is no refraction.

Fig. 159.



When light whose vibration is in the plane of incidence is totally reflected, it undergoes a slight retardation of phase, less than that observed in the case of light whose vibration is at right angles to the plane of incidence.

From these results it is easy to pass to the case of light whose vibration may be considered to be the result of the composition of vibrations parallel to the plane of incidence with others at right angles to that plane. Such light may be plane-polarised, vibrating in some plane neither that of incidence nor one at right angles to it; it may be circularly- or elliptically-polarised light, or it may be common light. In all these cases each component is reflected and refracted according to its own laws. In this way the reflected and refracted rays may come to differ in character from one another, and from the original ray. As an extreme case, let a beam of common light fall upon a piece of glass at such an angle that

$i+r=90°$ (Fig. 159); of that part of the incident beam which is due to vibrations executed at right angles to the Plane of Incidence there is reflected a certain proportion; of that part of the incident ray which is due to vibrations parallel to this plane there is reflected none. The light reflected from glass at such an angle has its vibrations thus restricted to a plane at right angles to the plane of incidence; it is plane-polarised light. A piece of glass held in the course of a beam of common light at the proper angle may thus be used as a simple means of obtaining a beam of light Plane-Polarised by Reflexion.

The precise angle of incidence i, for which $i+r=90°$, is called the **Angle of Complete Polarisation.** It depends upon n, the refractive index of the refractive substance, and the angle i is such that $\tan i=n$: for $\sin i=n \sin r=n \cos i$. With metal reflectors there is no angle of complete polarisation.

Such are the consequences deduced by Fresnel from the hypothesis that the Ether is condensed around the particles of ordinary matter while its elasticity remains unaffected.

Neumann and MacCullagh, from the contrary hypothesis—that the density of the Ether is the same in all substances, while its elasticity is different in different substances—deduced a set of conclusions precisely similar to those above given, with this remarkable difference, that the properties attributed by Fresnel to plane-polarised ether-waves whose oscillations are effected at right angles to the plane of incidence are by the latter writers found to be associated with plane-polarised light whose vibrations are parallel to that plane, and *vice versâ*. The fundamental postulates of the two theories are closely associated with these consequences.

We must now turn to a point of terminology. When a beam falls upon a mirror at the angle of complete polarisation, the reflected ray, if there be any, is plane-polarised; and it is said to be polarised **in the plane of incidence.** According to Fresnel's view, the vibrations in this beam are supposed to be executed in a direction at right angles to the plane in which the beam is said to be polarised.

The question between the followers of Fresnel and Cauchy on the one hand, and those of Neumann and MacCullagh on the other, may thus be stated: Are the vibrations of plane-polarised light executed at right angles or parallel to the plane in which the light is said to be polarised—a plane which by convention is called the **plane of polarisation**? This question is still *sub judice*. Though it seemed until recently that Fresnel's view had definitely prevailed, and that the vibration of polarised light is at right angles to its plane of polarisation, yet in recent times the contrary view has met with increasing favour, especially since the electromagnetic theory of the propagation of light has been promulgated; this theory, due to Clerk Maxwell, demands that the vibrations of a plane-polarised ray be *in* the plane of polarisation, not at right angles to it. A further consideration, which lends probability to this view, is derived from certain mathematical difficulties which arise from the admission of Fresnel's second hypothesis, that the movement of the ether in the second medium is continuous with that in the first; for this hypothesis is found to lead directly to the conclusion that the density of the ether in the two media must be the same, and is therefore one which is incompatible with his fourth hypothesis.

Professor Stokes, on the other hand, reasons in favour of the movement perpendicular to the plane of polarisation, and the following is a sketch of his argument. The vibrations in a beam of light are admittedly transverse to the direction of propagation. Consider a polarised reflected beam; the vibrations are admittedly symmetrical with regard to the plane of reflexion; they must be either parallel to it or at right angles to it. Now, suppose a horizontal beam to strike a haze and then to be reflected vertically upwards into the observer's eye. The reflected light is undoubtedly polarised in a plane passing through the source of light, the point of the haze looked at and the observer's eye; that is, it is polarised in the plane of reflexion. If the vibrations be parallel to this plane in the ascending beam, they must either have been originally parallel to the direction of propagation of the incident beam, which is impossible; or else they must have been changed in their vibratory direction by impact against obstacles smaller than their own wave-length, which is improbable. If, on the other hand, the vibrations be at right angles to the plane of reflexion, the general direction of vibrations is the same after reflexion as before it. The latter view is preferable: and according to it, the vibrations are at right angles to the plane of polarisation.

When ordinary monochromatic-light is reflected at any angle other than that of complete polarisation, the reflected and the refracted beam must both be partially polarised, and each will be polarised to an equal extent, though in contrary senses. In the reflected beam, light polarised in the plane of incidence preponderates until the incidence is a grazing one: in the refracted ray, light polarised at right angles to that plane preponderates to an exactly equal extent, so far as the energy of the vibration is concerned.

When mixed light, such as white light, falls upon a refracting surface, then since n, the index of refraction, is different for each kind of light, the proportions of each coloured light present in the reflected and the refracted rays respectively are different; white light, when reflected from a normally refracting surface, always becomes bluer, the refracted light redder; and we have seen this to account for the blue colour of haze.

The intensity of a reflected ray is represented by $\{\sin^2(i-r) \div \sin^2(i+r)\}$. If we pass to a ray of greater refrangibility we alter r to $\bar{r}$, and our intensity becomes $\{\sin^2(i-\bar{r}) \div \sin^2(i+\bar{r})\}$, which is always greater than the former. Where the actual refraction is greater, the corresponding angle of refraction is less; wherefore in this case $\bar{r}$ is a smaller angle than r.

Near the incidence of total reflexion some colours may be totally reflected, others in part refracted; near the incidence of total reflexion or complete polarisation an analogous result is obtained. The slower waves of heat have a lower refractive index, and must therefore strike a refracting surface at an angle somewhat more vertical or nearer the normal than those of light, in order to become completely polarised.

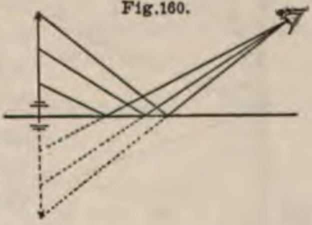
We thus learn that refraction and reflexion may materially modify the character of light which strikes on a refracting surface. If, however, we attend only to the Direction of the respective rays, and not to their states of polarisation, or to their colour, etc., we may study the effect of mirrors or lenses in modifying the direction of an incident beam of light, whether this be plane-fronted, convergent, or divergent. We shall first consider the case of monochromatic light.

Mirrors.—Plane mirrors reflect light in such a way that the reflected waves are, as regards their direction, precisely such as might have come from an object or source of light situated behind the reflecting surface, and at a distance behind it equal to the distance between the object and the mirror. This is illustrated in Fig. 61, and it is a matter of familiar knowledge in the use of looking-glasses, and in the appearance presented by the inverted image of objects on the shore when these are seen reflected on a surface of smooth water, the images then seen being apparently at the same horizontal distance from the eye as the objects themselves; while the image of a slightly-clouded sky, as reflected in very smooth turbid water, appears extremely deep.

The apparent inversion of an image in a mirror is a natural result of the fact that the image of each point is apparently situated behind the mirror. Fig. 160 explains this result.

The reader may construct a diagram to show how it is that a mirror about half a man's height, and placed opposite the upper half of his body, will give him a full-length image of himself.

Fig. 160.



The use of mirrors fixed at an angle of 45°, in order to cast the light of the sky into a room, or in order, when fixed outside a window, to enable a person within a room to see the passengers in the street outside, is sufficiently intelligible.

In medicine the same principle is utilised in the Laryngoscope. Light falling from a lamp upon a concave mirror is cast upon a small plane-mirror, held by means of a long handle at an angle of 45° within the pharynx of the patient; the light is reflected and passes down towards the larynx, which is illuminated and becomes a source of light; rays returning from this, passing upwards, strike the small pharyngeal-mirror, and are diverted by it so that they traverse, horizontally, the cavity of the mouth, and pass through a small aperture in the centre of the concave mirror into the eye of the observer, who is thus enabled to see the larynx and windpipe.

The brightness and distinctness of an image depend upon the polish of the mirror, and on its not scattering the light which falls upon it. A smooth clean mirror is, itself, almost invisible.

A piece of polished platinum reflects light as well when it is white-hot as when it is cold.

The image of an object in a mirror can never be brighter than the object itself, however smooth the mirror be. Hence, if a candle be held between two parallel mirrors, the long series of images produced by multiple reflexion grows fainter as the images seem to grow more distant.

When we have multiple reflexions of light between two polished plates, if the plates be parallel and the incidence oblique, the reflexions are more numerous the nearer the plates are to one another. If the two plates be inclined to one another at an angle of 60°, the images of a point lying between them, the image of which is multiplied by repeated reflexion, are so situated that the first, second, ... sixth, form together a symmetrical Kaleidoscope-image of the point, a group of images ranged round a central axis, while the seventh and further images coincide with their predecessors. Similarly for such angles as 90°, 45°, 30°, 24°, and other aliquot parts of 360°.

When a mirror is rotated a beam of light reflected from it is deflected through an angle equal to twice that of the rotation of the mirror itself. In Fig. 161 S is a source of light; AB a mirror; SM an incident ray; MR a reflected ray. If the mirror be turned into the position A′B′, the reflected beam is now MR′: the reflected ray has swept through the angle RMR′, which is equal to twice the angle AMA′.

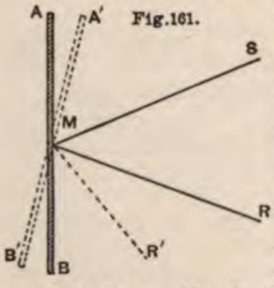


Fig. 161.

We have already (p. 120) considered some cases of the reflexion of waves at parabolic and elliptical mirrors, and on segments of spheres. Parabolic mirrors are used when it is desired to bring the plane-fronted light of a distant star accurately to a focus, or to produce a parallel beam of light: in the latter case the source of light is placed at the focus of the paraboloid. Spherical mirrors may be considered, if we restrict our attention to those rays which fall very near the centre of the mirror, to have an approximate focus at the tip of their "caustic by reflexion" (Fig. 64).

When the beam is broad the rays do not converge accurately to a focus, and the image of a point is a circle, brightest towards its centre. This is the **Spherical Aberration** of a mirror, which renders its definition, especially of somewhat broad objects, very bad. In consequence of this it is often necessary to cut off the lateral rays by a diaphragm, which increases the clearness of definition, though it diminishes the brightness of the image.

When light coming from a source at a positive distance, f, is reflected by a concave spherical-mirror, it is reflected back to an approximate focus at a positive distance f'. The distance of the source f, the distance of the approximate focus f', and r, the radius of the mirror, are connected by the law $1/f + 1/f' = 2/r$.

Here r, the distance of the centre of curvature of the mirror is to the right, positive.

When the source is infinitely distant, $1/f = 0$, and $f' = \frac{1}{2}r$; the case of Fig. 64, page 121. The focus to which the light converges in this case is called the Principal Focus of the mirror. If the course of the light be reversed, and $f = \frac{1}{2}r$, $f' = \infty$, and the light is reflected to a focus at an infinite distance; it is approximately parallel-rayed or plane-fronted.

When the source is at a definite distance, beyond the centre of the sphere, the focus is between the principal focus and the geometrical centre of the sphere; conversely, when a source of light is between the principal focus and the centre of the sphere, the reflected light converges upon a point at a definite point beyond the centre.

Light radiating from the centre of a spherical mirror, after reflexion, again converges upon the same point. When the source is between the principal focus and the mirror, *i.e.*, when f is less than $\frac{1}{2}r$, f' is negative, and the reflected rays seem to diverge from a point on the other side of the mirror, and the **Image** of the point is imaginary or **Virtual.**

Pairs of points at the respective distances f and f', as defined by this formula, are called **Conjugate Foci.**

Conjugate foci are found in one of the four following relations :—

(*a.*) Coincident, both being at the geometrical centre of the mirror.

(*b.*) One between the principal focus and the centre; the other beyond the centre.

(*c.*) One at the principal focus; the other at an infinite distance beyond the centre.

(*d.*) One between the principal focus and the mirror itself; the other, a virtual focus, apparently behind the mirror.

If the source of light be not in the axis of the mirror, the light is not brought to a focus in the axis, but at some point situated laterally. In Fig. 162 the rays proceeding from A converge upon A′, and an eye situated in the direction of B looking towards the mirror will receive rays which appear to diverge from a **Real image** at the point A′, a point which the waves really traverse. If the source of light be an extended object, AZ (Fig. 163), there will be formed an inverted real image, A′Z′, an image produced by an actual divergence of the rays from a series of points situated at Z′A′. In Fig. 163 the object AZ produces a diminished and inverted real image at Z′A′; or, conversely, an object at Z′A′ will present an inverted and enlarged real image to the eye placed beyond AZ, an image which appears nearer than the object itself.

Fig.162.

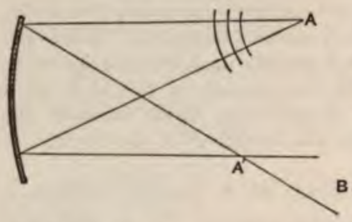
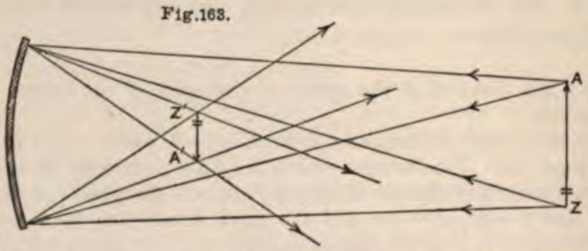


Fig.163.

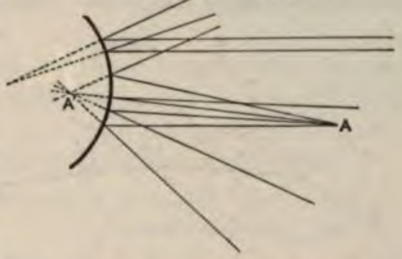


Suppose that the object AZ is the face of a person looking at a concave mirror, it is plain that the eyes in that face are eyes situated beyond Z′A′: a real image of the face is seen, diminished and inverted, and apparently situated at Z′A′, between the observer and the mirror. As the observer approaches the mirror, the image approaches him: it appears to grow larger. When his eye is at the geometrical centre of the mirror, the image is blurred: when the eye is placed between the centre and the principal focus, the rays not having either really or apparently crossed one another, the image is still blurred. When the eye passes between the principal focus and the surface of the mirror, the rays reflected seem to come from a virtual image behind the mirror, which is erect, and is larger the nearer the eye is to the mirror.

The student will have little difficulty in drawing the diagrams appropriate to each case, and in verifying the results even by means of such a simple concave-mirror as the inside of a watch-case, or of a common large spoon.

Convex Spherical-Mirrors.—In Fig. 164 we see that a beam of light, plane-fronted or parallel-rayed, and therefore coming, apparently or really, from an infinite distance, is so reflected that it appears, after reflexion, to diverge from the principal focus of the principal mirror, this point being the tip of its caustic; while a beam diverging from a point A appears, after reflexion, to diverge similarly from a point A′ situated nearer the mirror than the principal focus.

Fig. 164.



The formula is $1/f + 1/f' = -2/r$.

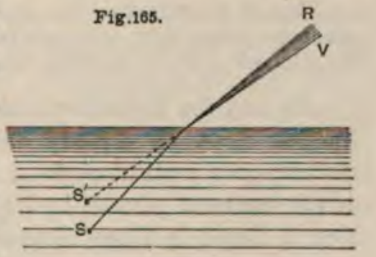
This formula is really the same as the preceding; but r is now negative.

When **mixed coloured-light** falls upon a mirror, all the reflected rays, whatever be their wave-lengths or relative intensities, are reflected in the same directions.

When a mirror is flexible and has a variable curvature, as the form of the mirror is made to vary irregularly, intermittently, or in accordance with some harmonic law of simple or complex variation, so the intensity of light at any point near the focus varies irregularly, intermittently, or harmonically.

Refraction of Light.—Light-rays are bent when they reach the surface of separation between an optically rarer and an optically denser medium. In Fig. 165 a body situated at S seems to be situated approximately at S′. In this way, if a coin be placed in a basin, and the eye placed in such a position as just not to see it, water poured into the basin will bring the coin into view. The sun is seen before he has astronomically risen, and continues to be seen after the true "sunset." A stick placed in water appears bent, for the image of each point of its surface appears raised within the water by an amount proportioned to its depth beneath the surface of the water; each point of the image appears indistinct, being brought only to an approximate focus, and the image of the whole is blue on one side, red on the other, for, as shown in Fig. 165, the different colours of white light travelling from any point S are unequally refracted into the air.

Fig. 165.



When light passes through a number of parallel-sided transparent plates of different densities, the total angular-deviation produced is the same as if the last of them had alone stood in the course of the transmitted light.

Prisms—Monochromatic Light.—If we confine our attention to a single ray of light impinging upon a prism surrounded by a single medium, we may trace the course of the corresponding wave-front, as in Fig. 166. The light travelling from S strikes at the angle of incidence i; it is refracted at the angle r; $\sin i = n \sin r$ (i.) It strikes the second face at the angle r'; it leaves the glass at the angle i'; $\sin i' = n \sin r'$ (ii.) Further, the angles $r + r'$ can be proved together equal to the angle A[*]; $r + r' = A$ (iii.) Lastly, if the angle between the incident ray SD and the deviated ray ES′ be V, then $i + i' = V + A$ (iv.)[†] From these four equations, which involve n together with the six angles, i, i', r, r', A, V, we may, if we know any three of these angles (say A, i, and V), determine, for the particular monochromatic light employed, n, the Index of Refraction of the material of the prism; this numerical quantity n expresses, as compared with unity, the relative slowness of ether-waves in the prism as compared with that in the medium surrounding the prism.

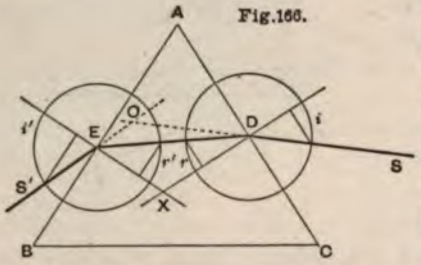


Fig. 166.

Thus, if a particular glass prism have the index 1·5 in air for a monochromatic yellow light, that light travels 1·5 times as fast in air as it does in the glass out of which the prism has been cut.

To find the Refractive Index of a Liquid, the amount of refraction must be directly observed. Two telescopes arranged radially on a vertical circle or alidade; light traverses the one, and is rendered parallel by it; it then impinges on the level surface of liquid in a central vessel, penetrates it, leaves the vessel, passing normally through the glass bottom of the vessel, and enters the second telescope, which it traverses. The relative angles made by the two telescopes with a fixed bar indicate the angles of incidence and of refraction, which, being known, the index n is known.

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[*] The normals to the faces drawn at the points of entrance and exit of the ray cross one another at X; the quadrilateral AX has its internal angles equal to four right angles; two of these (at D and E) are right angles: the remaining two (A and X) are therefore equal to two right angles. In the triangle DEX, the angles $r + r' + X$ are again equal to 180°; whence $r + r' = A$.

[†] The angles $r + r' = A$; $r + r' + V = V + A$; $V = OED + ODE$; $\therefore r + r' + OED + ODE = V + A$; but $r + ODE = XDO = i$; and $r' + OED = XEO = i'$; $\therefore i + i' = V + A$.

The Refractive Index of a Gas is found by processes which depend on the amount of retardation suffered by light in a long column of that gas, as will be seen under "Interference," and as has already been explained under "Velocity of Light" (the rotating-mirror method, page 474).

If instead of a prism filled, say, with water in air, we use a prism filled with air submerged in water, the deviation of a ray travelling in the water will be equal, but of opposite sense to that of a ray travelling in air and refracted by the water-prism.

The refractive index of organic substances is found to have a close relation to their chemical constitution. If a substance contain carbon, hydrogen, and oxygen, its formula being $C_aH_bO_c$, and if its density be ρ, it is found that the numerical quantity $\left\{\overline{(n-1}\cdot/\rho) \times \text{molecular weight}\right\}$ is constant, even though the density be varied by changes of temperature, and it is called the Molecular Refractive Power of the substance. This refractive power may be predicted from the chemical formula by finding the value of $\left\{5a + 1{\cdot}3b + 3c\right\}$. Isomeric substances have thus the same Refractive Power.

If in the last figure (166) light start from S′, it will retrace the line S′EDS; and the same result will follow if a mirror at S′ turn back the light coming from S: on its direction being reversed, light will retrace its path—a proposition applicable to waves in general.

Minimum Deviation.—It is only when the prism is turned into such a position that i, the angle of incidence, becomes equal to the angle i', that light diverging from a source S (Fig. 166) can converge upon a single focus lying towards S′; and this is the position in which the incident ray of light is, on the whole, least deviated by the prism.

If **mixed coloured-light** be passed through a prism, each colour has its own index of refraction, n, its own path through the prism; the whole light is broken up by **Dispersion** into a bundle of monochromatic rays, each travelling in a separate direction, and diverging the more widely from one another the farther they travel; and, received on a screen, these form a **Spectrum.**

Each kind of light is differently deviated by the prism; and for each kind of light the minimum deviation of the prism is different; whence an image of a slit looked at through a prism, or cast upon a screen, cannot be sharply in focus in all parts of the spectrum at once; only one colour can be accurately in focus at any one time; and to put any particular colour of the spectrum into focus, the prism must be rotated one way and another in the beam of light until that position is found for it which corresponds to the greatest approach of the particular colour towards the red end of the spectrum: this is for that colour the position of minimum deviation and of most accurate definition.

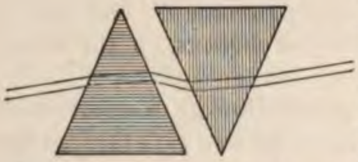
The **Rainbow** is produced by reflexion and refraction of sunlight in the drops of water which make up falling rain. Parallel sunlight falls from behind the spectator; in each drop the light is dispersively refracted, and then reflected from the farther face of the drop; it travels back through the drop and emerges in a state of chromatic dispersion. Drops which for the moment are situated at a certain angular height send violet light into the eye of the observer, but the red light from them misses his eye, for it strikes too low. Drops at a greater angular height send red light into the observer's eye, but violet light proceeding from them is, as it were, aimed too high, and does not enter his eye. Drops in intermediate positions send intermediate-coloured light into his eye. Drops above or below a certain range of angular height do not send light into his eye at all. The whole phenomenon is symmetrical round an axis containing the sun and passing through the observer's eye, and the bow is, according to the height of the sun in the heavens, a greater or lesser portion of a circle whose parts are equidistant from that axis. The rainbow as seen by the one eye is not formed by the same water-drops as the rainbow seen by the other eye.

Repeated reflections and refractions in raindrops frequently give rise to secondary, tertiary, etc., rainbows, which, under experimental conditions, have been observed to the number of eighteen.

The **halo** seen around the sun when it shines through a frozen cloud is due to refraction of sunlight through the crystals. Conceive a circle of prisms round the sun arranged in such a position as to send a maximum of sunlight into the eye: a circular spectrum would be seen, red internally; among the particles of ice in the cloud some must be in the favourable position of these prisms.

Recomposition of White Light.—If a bundle of coloured rays, emergent from a prism, be received on a second prism, similar to the first but reversed in position, these monochromatic rays are again recombined, and again form, on emergence, a beam of the original mixed coloured-light, parallel to its original direction; on the whole there is neither dispersion nor angular deviation.

Fig.167.



Deviation without Dispersion.—If white light fall upon a flint-glass prism of such an angle as to make on a screen, at a distance of, say, 10 feet, a spectrum whose length between two definite colours or Fraunhofer lines is, say, 3 inches; and if another prism of crown glass which is able to produce a spectrum of an exactly-equal length between the same definite colours or lines be so placed as to reverse the dispersive action of the former prism, according to the principles of the last paragraph, the light leaving the prism is approximately white. It is not, however, parallel to its original course; for when we pass from prisms of one substance to prisms of another, we find a phenomenon known as the **Irrationality of Dispersion**; we find that the relative amounts of mean deviation and the relative amounts of dispersion produced by two given prisms are independent of one another; and hence, to reverse dispersion is not necessarily to reverse deviation, if this be effected by a prism of a second, a different substance.

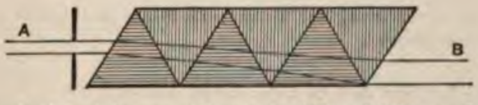
A flint-glass prism and a crown-glass prism thus combined may produce deviation without producing any dispersion, and the emergent light is approximately white; and thus, for any two kinds of light a flint-glass prism may be **achromatised** by a second prism of crown glass. The recomposition of the colour is not perfect, except for the two colours (or lines) chosen; it would have been perfect were the spectrum of crown glass precisely similar to that of flint glass in respect to the proportionate lengths of the coloured areas in it; but it is not so; in the crown-glass spectrum the orange and yellow are proportionately more refracted, and are spread over a proportionately greater area than they are in the flint-glass spectrum, and the blue and violet less so; the former are for accurate recomposition too much, the latter two little, refracted by the achromatising crown-glass; the issuing beam, white at the centre, is yellowish on one side, bluish on the other. Three prisms may be combined so as to blend three colours in the emergent ray; and so forth.

Reflexion at the surface of a mirror may be said to furnish an example of deviation without dispersion. Sometimes a reflecting prism (Fig. 158) is preferred to a mirror, especially in lantern projection-apparatus. Light enters normally at one face of the prism, is totally reflected at the second, and passes normally through the third. The image produced by such a prism is inverted, and if the incident beam be parallel there is no refraction, and therefore no chromatic dispersion, while the loss of light is very small.

Dispersion without Deviation.—If crown-glass prisms and flint-glass prisms be alternated they can be made to produce dispersion while the issuing rays are parallel to the original direction of the entering light.

This principle is applied in the Direct-vision spectroscope, which simply consists of a train of such prisms, to which

Fig.168.



the light is admitted by a slit at A (Fig. 168), and from which the light issuing at B is caused to pass through a lens which can be so adjusted as, for each colour, to give the eye a clear image of the slit; and the eye accordingly receives the impression of a continuous spectrum situated at the focus of the lens.

Abnormal Dispersion.—The amount of deviation of each kind of coloured light can be directly measured when a spectrum is formed; so can the angle of the prism and the angle of incidence; thus for each transparent substance, and for each wave-frequency of incident ether-waves, the index of refraction may be calculated and recorded.

The spectra produced by similar prisms of different substances differ not only in their absolute lengths, but also in the proportionate length of each colour, and even in their arrangements. A hollow glass prism filled with iodine vapour refracts red light most, and violet least; it gives a spectrum the order of succession in which is ultra-violet, violet, blue, red, the reverse of the ordinary; the intermediate parts of the spectrum are lost by absorption. A weak alcoholic-solution of fuchsin in a hollow glass prism refracts the blue and violet less than it refracts the yellow and red; the spectrum thus presents the following order:—Fraunhofer lines, F to H—*i.e.* green, blue, violet—then A to D, red, orange, and yellow, not quite up to the E line: the green from E, nearly as far as F (E inclusive), being absent on account of absorption. Aniline violet, aniline blue, indigo carmine, give a green-blue-orange spectrum. A concentrated solution of cyanin in a hollow glass prism gives a spectrum consisting of—first, green-blue, then a dark band, then red, and traces of orange.

In general all bodies which, like many of the aniline dyes or crystals o permanganate of potash, act upon some kinds of light as metallic reflectors and present, when in the solid state, superficial colours differing from the body-colours, are found to give, when their solution occupies a hollow glass prism, an analogous refraction-spectrum—not to be confounded with the ordinary absorption-spectrum—in which the order of the colours is not that of the spectrum as produced by a prism of solid glass. On the redward side of an absorption-band the index of refraction is found to increase so rapidly as to throw the part of the spectrum near the redward side of the absorption band over towards the violet; and conversely, the part of the spectrum situated near the violet-ward side of an absorption-band is thrown back towards the red. The effect may be simply to render the absorption-band narrower than it would have been had there been no effect of this kind, or to cause overlapping of different parts of the spectra; or, as in the case of fuchsin and cyanin, actually to throw over the redward and the violet-ward parts of the spectrum into each other's places, and to make red light more refrangible than violet. These phenomena, experimentally systematised by Kundt, have been discussed by Helmholtz on the supposition that the absorbed light being in tune with the molecules of the body, these molecules are set in motion which is modified by intramolecular friction so as to react on the transmitted light. The question is still obscure.

The heat region of the spectrum is very much shortened or compressed by the use of glass prisms.

Lenses.—Lenses are of two main kinds:—

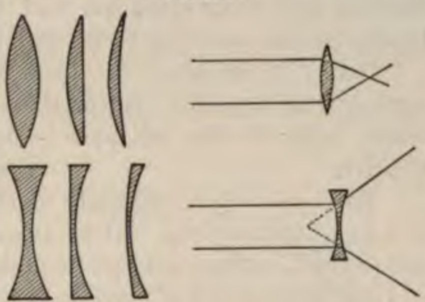
a. **Thin-edged,** thick in the centre; either convex on both sides, plano-convex, or convexo-concave, with a shallow concavity;

a parallel-rayed beam of light falling upon one of these lenses is made to **converge** after transmission upon a **Real Focus** at the opposite side of the lens.

b. **Thick-edged,** thinnest in the centre; bi-concave, plano-concave, or concavo-convex, with a deep concavity: a plane-fronted beam of light after transmission **diverges** so as to seem to come from a **Virtual Focus** on the same side of the lens as the source of light itself.

Fig. 169 shows these different forms and their action on a parallel beam of light travelling from left to right.

Fig. 169.



Every lens has a **principal focus**; this is the point to which a parallel beam of rays is caused to converge, or from which it is apparently caused to converge, as the case may be.

The distance of this principal focus from the lens is called f, the **Focal Length** of the lens.

If r and r' be the distances of the centres of curvature of the right and left faces of the lens respectively, both distances being measured from the centre of the lens positively—that is, towards the right on a horizontal line; and if a parallel beam come from the *right*, $1/f = (n-1)\ (1/r - 1/r')$, where n is the index of refraction of the lens in air, and f the positive distance of the focal point to the right. If f be found negative, then the focal point is to the left, beyond the lens, a real focus for rays coming from the right.

This general formula applies without alteration to concavo-convex lenses, the convexity of which is turned towards the left, and both of whose centres of curvature are therefore situated in a positive direction. In a bi-convex lens r is negative, and $1/f = (n-1)\ (-1/r - 1/r')$; in a bi-concave one r' is negative, and $1/f = (n-1)\ (1/r + 1/r')$; in a convexo-plane lens (convexity to the left) $r = \infty$, and $1/f = (n-1)\ (-1/r')$; in a concavo-plane lens (concavity to the right) $r' = \infty$, and $1/f = (n-1)\ (1/r)$. The expression $1/f$ measures the increase of divergence produced by a lens; it is called its Power. In thin-edged lenses, which are convergent, it is negative; in thick-edged, positive.

Reversibility of Lenses.—Let us reverse a lens. Then r becomes $-r'$; r' becomes $-r$; the value of $1/f$ remains unchanged.

To find the focal length of a thin-edged lens, find by experiment the distance at which it will make a clear image of the sun upon a screen: light coming from the sun is practically plane-fronted, and is caused to converge upon the principal focus.

Lenses, like mirrors, have Conjugate Foci at distances and $p_{,}$, + or − according to their direction with regard to th optical centre of the lens; rays coming from an object placed one focus are caused to produce an image, real or virtual, at th other focus.

Convergent or thin-edged lenses have their principal foc distance and their conjugate foci (or distances of object an image) related according to the formula, $1/p - 1/p_{,} = -1/f$.

Here f is as before the focal distance for a parallel bea coming from the *right:* but for these lenses f has itself a negati value, being to the left, and $(-1/f)$ is therefore numericall positive.

We must adhere carefully to our convention as to positi and negative directions. If we attend merely to the numbers an not to the directions, the equation takes the form $1/p + 1/p_{,} = 1/$ if p and $p_{,}$ be found of opposite signs we infer a virtual, if of th same sign a real image; and this equation is more easily applie than the former.

A beam comes from an infinite distance; $p = \infty$; then $p_{,} = f$; lig converges really upon the principal focus. Conversely, if a beam come fro a source of light at the principal focus, $p = f$; ∴ $p_{,} = \infty$; the image is re but at an infinite distance beyond the lens.

If the source of light be at a distance less than infinity, but greater th twice the focal length, the image is real, and is on the other side of the le at a point between the principal focus and a point twice the focal leng from the lens; that is, if the object be to the right at a distance $(2f + d)$, t image is to the left at a distance $-f\,(2f + d) \div (f + d)$, which is greater th f, less than $2f$. Conversely, if the object be at a distance greater than f, b less than $2f$, the image is real, at a distance greater than $2f$.

If the object be at a distance $2f$, the image is also at a distance $2f$, the other side of the lens; for when the distance of the object is $+(2f +$ that of the image is $-f\,(2f/f) = -2f$. The object and the image are th equal in size.

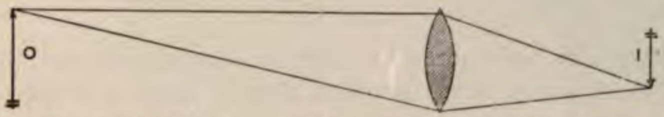
Hence another method of finding the focal length of a thin-edged le Adjust an object, a lens, and a screen, so that the image on the screen equal in size to the object: the screen and the object are now both situat at distances equal to $2f$ from the centre of the lens; half the distance either of them from the centre of the lens is the focal length.

If the object be between the lens and its principal focus, the rays a not made sufficiently convergent to cross at any place; they seem to co from a virtual image behind the principal vocal-point, farther from the le than the object, and therefore behind it; but the virtual image rapidly gai on the object as the object approaches the lens.

As to the inversion or erectness of the image produced a thin-edged lens: an object at O (Fig. 170) produces a small inverted image, **a real image,** at I, and an eye placed beyond

—that is, at a sufficient distance from the lens—will perceive the image of a distant object, inverted, smaller than the object, and

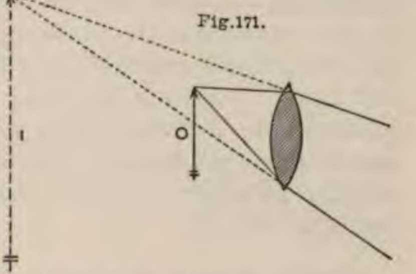
Fig. 170.



apparently situated between him and the lens; the contrary being the general impression.

If, however, the object be brought so near the lens as to lie at a distance from the lens less than the focal length, then to an eye situated at any distance on the other side of the lens a **virtual image** will be apparent, erect, magnified, and more distant than the object. Hence these lenses are commonly used as magnifying glasses (Fig. 171).

Fig. 171.



Divergent or thick-edged lenses.—The formula for these is $1/p - 1/p_{,} = 1/f$.

This formula is essentially the same as the preceding; but in these lenses f, the focal distance for a parallel beam coming from the *right*, is positive, to the right, on the same side as the source, and the image is therefore virtual. If we confine our attention to numbers only, the formula for divergent lenses is $1/p - 1/p_{,} = -1/f$.

When p is of any value > 0, $< \infty$, $p_{,}$ is less than f.

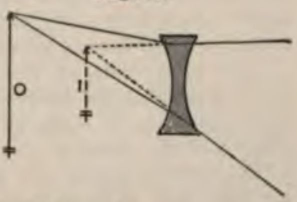
When $p = +f$, $p_{,} = +\frac{1}{2}f$; virtual image at half focal length.

If rays from the right converge towards a point to the left, at a distance p, which must be less than f, the lens will make them converge upon a point at a greater negative distance $p_{,}$, determined by making p negative in the above equation.

The focus of a thick-edged lens is most conveniently found by coupling it with a thin-edged one, found by trial among a sufficiently extensive series, so that together they shall produce no change in the apparent size of an object seen through them.

As to inversion and size of image, Fig. 172 shows that the image is erect, diminished, virtual, and nearer the lens than the object itself; and, since there is no subsequent crossing of the rays beyond the lens, there is no inversion of the image. Lenses of this kind may be used as diminishing glasses.

Fig. 172.



For some purposes flexible lenses may be used in which the curvatur may be slightly varied. Cusco's ophthalmoscopic lens consists of two piec of thin microscopic cover-glass fixed in a frame : water fills the cavity betwee them ; by forcing more or less water into the cavity the curvature may b varied.

Even when monochromatic light is transmitted through lense the focussing can never be exact if their surfaces be spherica each point of an extended object forms a slightly-blurred image this effect can be reduced somewhat by the use of diaphragm which cut off the external parts of broad beams of light, and thu diminish the Spherical Aberration of the lens (as in the pup of the eye), but it can only be brought to a minimum by modif ing the curvature of the lenses used. Some lenses are so curve as to bring all the points of an extended image into the same plan and thus to produce a flat field; others to bring points differin in distance to foci which differ very little from one another, an thus to secure penetration. The calculation of the various curve surfaces necessary for these ends often involves considerab mathematical skill.

Gauss's Method of treating Lens-problems.—It is not possibl owing to considerations of space, to follow up the subject of Geometric Optics which, after all, is not a part of Physics. Yet a few words on Gaus method may perhaps induce the reader to extend his studies into this regi of Applied Mathematics.

Gauss found that every possible system of lenses could, if well-centre be reduced to a region of space to be traversed by the incident light, and pr senting six characteristic or Cardinal Points, ranged along the axis of t system. These are (1) Incidental Focus; (2) Refractional Focus; (3) Incident Principal Point; (4) Refractional Principal Point; (5) Incidental Nod Point; (6) Refractional Nodal Point. These are reciprocal, so that (1) b comes (2) and *vice versâ* when the direction of the rays is reversed.

All rays proceeding from (1) become after refraction parallel to each oth and to the axis; all parallel incident rays parallel to the axis of the syste pass through (2). An object at (3) or in the same plane (at right angles the axis of the system) with it forms an equal and erect image at (4) or the same plane with it : there are only two such points. Any ray apparent making for (5) before refraction is after refraction parallel to its former cour but appears to be coming from (6) : there are only two such points. T distance (1, 3) is the Incidental Principal Focal Distance ; (2, 4) is the Refr tional Principal Focal Distance.

These six points, all in the same line, are closely related. The distan (1, 5) is equal to the distance (2, 4) ; and (6, 2) = (1, 3). Therefore (3, = (4, 6) = (1, 3) − (2, 4) ; and (3, 4) = (5, 6). Further, if μ be t relation of the index of refraction of the medium nearer the source to th of the medium beyond the lens (1, 3) = μ (2, 4) ; in the case of a lens air these two principal focal distances are equal, and further, (3) coincid with (5), and (4) with (6).

Planes passing through (1) and (2) at right angles to the axis of the system are called its Focal Planes.

Rays diverging from a point in one of these focal planes (of which rays one might be towards the corresponding nodal point) emerge parallel to one another; and since the ray from the divergence-point to the corresponding nodal point would have emerged parallel to its original direction, all the rays must necessarily emerge parallel to the original direction of that ray. Thus they retain parallelism with that ray, the direction of which is, on its emergence, determinate.

Rays parallel in the first medium converge on a point in the second focal plane. That ray which travels towards (5) emerges as if it had come from (6) parallel to its former course. Hence a line drawn from (6) parallel to the originally parallel rays will cut the second focal plane in a certain point; towards that point in the second focal plane all the rays, originally parallel, must converge.

The artifice of Gauss's method (for which see his *Collected Works*, or, for an elementary exposition, Helmholtz's *Physiolog. Optik*, and Clerk Maxwell, *Qu. J. Mathem.*, 1858, p. 233; or Pendlebury, *Lenses and Lens-Systems*) is, so to speak, the identification of an incident set of rays *as they cross the first Focal Plane;* the rays are then traced until they arrive at the second plane; from the data thus obtained their subsequent course can be ascertained. Mathematical difficulties are thus minimised, for the problem becomes mainly one of finding these cardinal points for a lens-system of any given form and of any degree of complexity.

The accompanying diagram (Fig. 172a) may serve to illustrate the method as applied to a biconvex lens: in this case the focal distances are equal and the principal points, which coincide with the nodal, are within the lens.

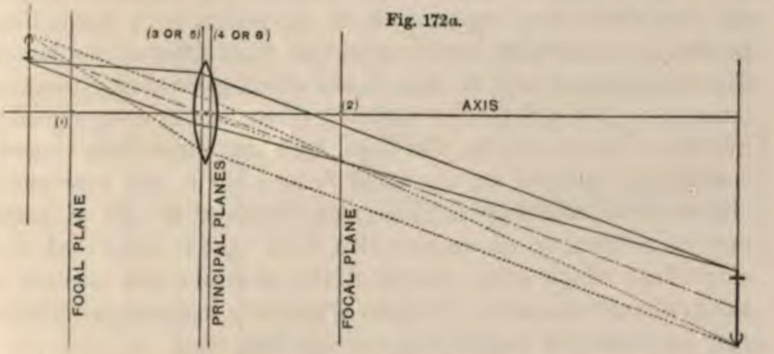


Fig. 172a.

If we represent the distance between the object and the incident principal plane by p, between the refractional principal plane and the image by $p_{,}$, and the principal focal distance by F; if r be, as before, the radius of the right-hand, and r' the radius of the left-hand surface of the lens, and d its thickness at its centre; and if n be the index of refraction of the medium surrounding the lens, and n' that of the lens itself; then we have, giving each quantity its proper sign, $1/p + 1/p_{,} = 1/\mathrm{F}$, where $\mathrm{F} = -nn'r'r \div \{(n'-n) [n'(r-r') + (n'-n)\,d]\}$. The distance between each principal point and the corresponding surface of the lens is for the incidental surface and point, $ndr'/\overline{n'(r-r') + (n'-n)d}$, for the emergent $-ndr/\overline{n'(r-r') + (n'-n)d}$.

Chromatic Aberration. — When mixed coloured-light is passed through a thin-edged lens, violet light is most refracted, and comes to a focus sooner than the red rays do; beyond the red focus is the heat-focus; between the violet focus and the lens is the region of the photographic focus.

Makers of photographic lenses have shown much skill in making the photographic and the visual focus coincide; for special photographic work, such as Rutherford's lunar photography, lenses have had to be constructed whose curvature is calculated with reference to the focus of the highly-refrangible actinic rays alone; and, while nothing can be distinctly seen through such lenses, photographs of extraordinary clearness have been taken by their aid.

If a beam of white light be passed through a single convergent-lens, a screen placed at the violet focus will give an image with a red border—the red rays not having yet converged; if it be placed a little farther off, at the red focus, the image is now surrounded by a violet border, for the violet rays are already divergent. Consequently no clear definition can be obtained by the use of such simple lenses, and it is necessary to render them Achromatic. A biconvex lens of flint glass, more convergent than is necessary, is coupled with a biconcave lens of crown glass of proper curvature; the latter destroys the dispersion, by bringing two colours to the same focus, without wholly doing away with the deviation; the couplet acts on the whole as a single lens, producing a somewhat smaller refraction than either of the lenses. This arrangement may be seen in the object-glass of any common telescope. For still further accuracy three, four, or even a greater number of lenses may be combined, by which three, four, or more colours are brought to the same focus; as in the achromatic objectives of microscopes. The most complex system of lenses may be reduced, so far as its action upon light is concerned, to a single lens, which gives images of the same size and position as the system in question. Thus an achromatic microscope-objective may be treated in diagrams as a single lens.

The Eye may be ideally reduced for many purposes to a single lens composed of aqueous or vitreous humour, having its back coincident with the retina, and its anterior aspect a spherical surface of 5·1248 mm. radius, situated at its most anterior point 2·3448 mm. behind the actual anterior surface of the cornea. Such a lens would refract incident light, and bring images of distant points to a focus upon the retina in the same way as the actual eye does.

Radiant Heat may be shown to be reflected and refracted like Light by concentrating rays of dark heat upon a thermopile by means of a lens or a mirror, or by refracting them by means of a prism into a new path, in the course of which the thermopile must somewhere be placed before it will indicate the impact of Heat-waves: by photography of the infra-red region of the spectrum; by Langley's Bolometer (p. 663); by Becquerel's Phosphorescence-effect, p. 468.

INTERFERENCE.

Ether-waves are capable of Interference. Two systems of equal waves arriving at the same point in opposite phases will produce at that point no effect either of light or of heat or of photographic action: at that point the ether will be at rest; and thus light added to light may produce darkness. In Fig. 75 the two points A and B are centres of wave-motion, and at the points b', d', f', on the screen MN, there is no disturbance, while at intervening points, a', c', e', the amplitude of disturbance is doubled.

Interference of waves thus affects the distribution of energy in a system of undulation, and such a screen produces a system of negative reflected waves from a', c', e', etc.

Let us now consider a monochromatic beam of plane-polarised light. Such a beam may be divided in two parts by reflexion from a silvered or platinum mirror bent in the middle at an angle very nearly equal to 180°, or else by refraction through a biprism, whose angle is very nearly 180°. The last case is shown in Fig. 173. S is a source of light; the light from it is transmitted through a polariser N: it is now a polarised beam. The rays are received by a convergent lens, which makes them converge upon S''. In its course it is passed through a piece of glass R, coloured red with suboxide of copper: it is now to a rough approximation monochromatic. It is then passed through the biprism P, which refracts it in such a way that it seems to come from two equal and equally-distant foci at S''' and S''''. The light may then be received, either on a screen, or directly in the observer's eye placed in the onward path of the beam. A series of dark and bright fringes will be seen corresponding to the

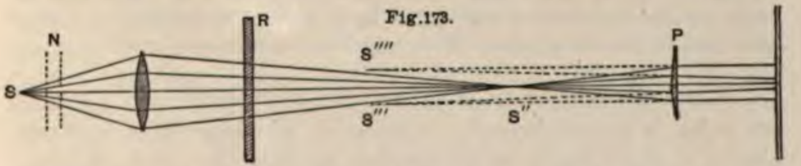


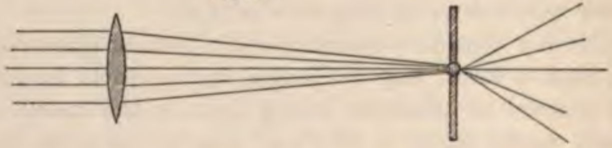
Fig. 173.

alternate fringes of rest and disturbance of figure 75. The two beams apparently travelling from S''' and S'''' are polarised in the same plane, and any irregularity of amplitude characterising the one is participated in by the other. Hence they are in a position to interfere fully and regularly with one another. If, on the other hand, they had been polarised in planes at right angles to one another, they could not have extinguished one another at any point. When common light is used, it may be at once filtered through a piece of red glass and then passed through a convergent lens.

To procure monochromatic light it is better to project a spectrum upon a screen in which there is a slit, and then, behind the screen, to make use of that part of the spectrum whose light falls upon and traverses the slit.

It is very easy to procure a bright spot which may represent a simple luminous point by making a small hole in a metal screen, and in this inserting a drop of glycerine. This acts as a powerfully-convergent lens, and if sunlight be concentrated upon it there will appear on the dark side of the screen an intensely bright little spot of light which may be used as a source of light for many experiments; with such a source of light Fresnel discovered the laws of diffraction. More elaborately, the same result may be better attained by means of the electric light made to converge by an achromatic lens of exceedingly short focus, a high-power microscopic objective.

Fig. 174.

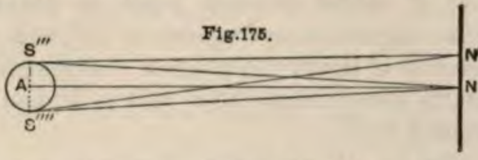


When monochromatic common light, proceeding from a luminous point, is passed through a biprism, its vibrations in each of two planes, at right angles to one another, produce the effects of interference independently of one another, but produce their respective fringes and bands in coincident positions on the screen. When mixed coloured-light or white light is treated in this way, the red fringes do not coincide with the violet fringes; the violet fringes are more numerous than the red fringes, and are closer together. This will be understood from Fig. 75; if the wave-length be increased, the points a', b', c', d', e', must become farther distant from one another. A violet fringe is seen near the axial line of the beam; it is overlapped by a blue, the blue by a green, and so on: each coloured fringe produced by the interference of white light presents a complete spectrum. The number of such spectra is limited; at a little distance from the axial line of the beam the fringes overlap one another so as to produce what appears to the eye to be simply white light, but the spectrum of which shows a series of alternately dark and light bands: all the colours being equally encroached upon by dark bands, the result seems white.

A bent mirror used instead of a biprism produces, by reflexion of white light upon a screen, alternate fringes of white light and darkness.

Measurement of Wave-length.—If $S'''S''''$ be the apparent position of the two images or apparent sources of light, which must be monochromatic; N the position of the central fringe, illuminated by the joint action of S''' and S''''; the angle $S'''NS'''' = 2i$; N' the position of, say, the fourth bright fringe; $S'''N'$ is shorter than $S''''N'$ by four wave-lengths; this difference is very nearly equal to $\{ NN' \times S'''S'''' \div AN \} = NN' \times 2 \tan i$.

Fig.175.



The angle $2i$ can be measured with a theodolite; the distance NN' can be measured with a micrometer; the value of the four wave-lengths, and therefore of one wave-length, can be determined from these data.

Fig. 75 shows that the line of propagation of these fringes in space is hyperbolic; the foci of these hyperbolas being the two apparent sources.

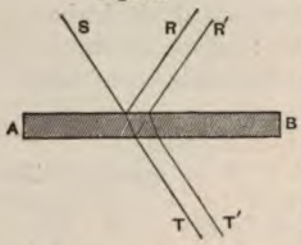
If the light from one of the sources be retarded by being made to pass through a layer of a substance in which light travels more slowly than in air, the whole of the fringes will be shifted somewhat towards the side on which the retardation takes place. From the amount of this shifting may be calculated the amount of retardation; and by means of this the relative velocities of light in (and therefore the refractive indices of) such things as hot air, cold air, hydrogen gas, normal glass, compressed glass, compressed liquids, and so forth, may be estimated.

Colours of thin films.—Thin films of transparent substances, such as oil upon the surface of water, iron oxide upon the surface of tempered steel, oxides deposited upon metals by the galvanic battery, soap bubbles, glass blown out to an extreme tenuity or exfoliating under the influence of slow decomposition, present curious colours when shone upon by a comparatively bright light.

Such films may be rendered permanent; a solution of bitumen and a little caoutchouc in a mixture of benzene and oil of naphtha, dropped upon water, forms films which solidify and may be caused to adhere to a sheet of paper.

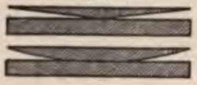
In Fig. 176 light from the source S is incident upon a thin transparent-film AB of uniform thickness. A part of the light is at once reflected to R from the first surface of the film. Another part is refracted to R′ after having undergone one reflexion at the second surface. If the path of the beam in the film be an *even** number of half wave-lengths, the beam travelling to R′ is opposed in phase to that travelling to R, and an eye placed at RR′ (these points being supposed very close together) will receive no impression of light; or, rather, it will receive but a feeble impression, for the ray to R′ cannot be quite equal in intensity to that travelling to R. Again, an eye placed at TT′ will perceive but a feeble impression of light; not absolute darkness, for the ray to T is considerably more intense than that to T′, and is not completely neutralised by it.

Fig. 176.



Let the film be of variable thickness; a film of air between a glass plate and a biprism, or between a convex lens and a plate of glass, varies in thickness with the distance from the centre; in the former case the thickness of the film of air varies as the distance, in the latter approximately as the square of the distance from the central point. Monochromatic light reflected from such a system presents the appearance of alternately dark and bright bands or circles—bright where the directly-reflected light and the light reflected from the second surface of the film are similar in phase—dark where they are opposed. In the case of a lens pressed against a plate they are known as **Newton's rings.** The less the curvature of the lens the greater the distance between two consecutive rings. If mixed coloured or white light be employed, the dark and bright rings of the several components cannot coincide, and the result is a series of circular spectra, in each of which the violet circle is the narrowest. These spectra overlap one another at a little distance from the centre, and blend into what appears to the eye to be white light.

Fig. 177.



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* This seems strange; we might have expected a retardation of an *odd* number of half wave-lengths to produce a difference in phase of half a period; but it will be remembered that the beam reflected at one of the surfaces of the film—that surface, namely, which separates an optically denser from a rarer medium—suffers a loss of half a wave-length, which is independent of the thickness of the film.

A series of dark rings or fringes may be obtained by rubbing a film of soap on black glass, drying it, and breathing gently upon one point of this through a glass tube; this, done in the sunshine, gives rise to bright colours.

It is not possible actually to obtain monochromatic light; even that emitted by incandescent sodium-vapour, in which some five hundred rings can be seen, is not quite monochromatic.

The centre of Newton's rings is dark if there be approximate contact; perfect contact there never can be, for a dustless surface it is impossible to obtain; even when there is no appreciable thickness of film traversed, the fact that one ray is reflected from the upper, and the other from the lower surface, the one at the bounding surface of an optically-denser, the other at the surface of an optically-rarer medium, causes the one to lose, while the other does not lose, half a wave-length on reflexion; they thus become opposed in phase, and the centre is dark. If, however, both reflexions be made to take place from the surface of an optically-denser medium, as in Young's experiment,—in which light travelling through a lens of crown glass was reflected first from the upper surface of a film of oil of sassafras, lying between that lens and a plate of flint glass, sassafras being intermediate in its refractive power between crown and flint glass,—there is no such relative retardation, and the centre of the system of rings is bright.

The Iridescence of mother of pearl and of objects with a finely-grooved or striated surface, such as butterfly's scales, is an effect of interference. Sunlight falls upon their surface; some of this is reflected from the ridges, some from the grooves, and in this way a difference of path is set up among the reflected rays, which causes differences of phase among them, and, in the case of some of them, opposition of phase and extinction. When the incidence of the reflected light is very oblique, the ridges alone may reflect, the differences of phase and of path produced will be very small; there will be little iridescence and very considerable reflexion.

The propagation of light "in straight lines" within the same isotropic medium is itself a result of interference. From it is derived the power of making a geometrical **Shadow.** In Fig. 178 a real focus at F acts as a source of light. It casts a sharply-defined shadow of an object O upon a screen. If the source of light be an extended one, not a mere point, the shadow consists of two regions, a central umbra and a marginal penumbra. In Fig. 179 the sun, S,

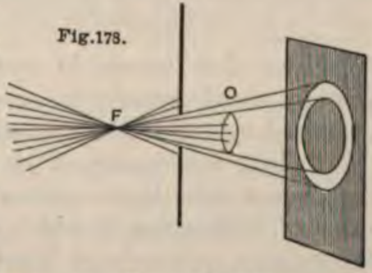
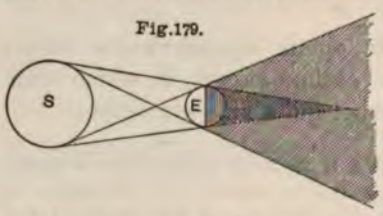


Fig. 178.

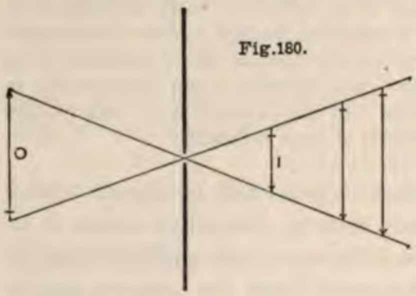
shines upon the earth, E; the earth being smaller than the sun, there is formed a cone of darkness behind the earth; if the moon travel wholly or partially into this cone of shadow, it will be wholly or partly unilluminated, and we have a total or a partial eclipse of the moon.

Fig. 179.



But outside this shadow there is a penumbral region, in which a body, or any point of a body, will be in "half-shadow," not fully illuminated, because able only to see a portion of the illuminating body.

Fig. 180.



When light radiating from an extended object passes through a small aperture, the waves arriving at the aperture from the object traverse the aperture, and there cross each other; they then diverge, and a screen placed on the opposite side of the aperture receives an inverted image of the object, whose size varies with the distance of the screen, as in the well-known **Camera Obscura**.

An aperture of no appreciable breadth would, at whatever distance the screen might be placed, give a perfect image in the natural colours, an image of which no part would be out of focus; one of $\frac{1}{25}$-inch diameter will give on a screen at 40 inches distance an image which, though wanting in brightness, is as perfectly defined an image as any possible lens placed at the aperture can produce: one of $\frac{1}{10}$-inch will produce the same definition at a distance of 250 inches; or, in general (Lord Rayleigh), if λ be the wave-length, r the semi-diameter of the aperture, f the least distance of good definition, $f=(2r^2/\lambda)$. When the screen is nearer than this, each point of the object makes an image on the screen, which has the same shape as the aperture, and the superposition of these makes a blurred image.

Light thus travels in straight lines, and is incapable of passing round corners under ordinary circumstances, and as examined by our ordinary senses.

A closer examination of the subject shows, however, that light does to a certain degree pass even round corners. The phenomena of **Diffraction,** in which this is observed, are explicable on the ordinary principles of interference. Let S (Fig. 77) be the source of light; waves diverge from this as a centre. These waves impinge upon a screen AB. Fig. 77 shows that beyond the screen AB there is a series of fringes within the geometrical shadow; that even in the part directly in view of the source of light there are bands of relative darkness; that the central point of the shadow may be nearly as brightly illuminated as if there had been no screen AB; that the broader the object AB, the narrower will be the fringes; that the forms in space of the regions of approximate darkness are hyperboloids; while if the source of light be removed to an infinite distance, the hyperbolic lines of relative rest in the illuminated region are practically reduced to straight lines, but sweep past the obstacle without touching it.

When the obstacle is circular—a minute circle of tinfoil pasted on a piece of clear glass—the shadow cast upon a screen, or received in the eye directly or by the aid of a lens or telescope focussed on the obstacle, is seen to be surrounded by a series of dark and bright rings; or, if the light from S be mixed-coloured or white light, by a series of spectra; while the shadow is also modified by a series of such bands or spectra, and its centre is bright. A similar construction for a little circular aperture in an opaque screen at AB will show that the bright spot produced on a screen beyond AB will have fringes blurring the sharpness of its edges, and that at certain distances of the second screen from AB the centre of the bright spot will be dark.

When the obstacle or chink is linear and parallel-sided, the fringes or spectra are parallel to one another; when it is not so they assume a curved form; when it is angular the fringes may assume a great variety of remarkable and beautiful forms.

The phenomenon of diffraction can be roughly observed by looking at a distant gas-flame, edge on, with the half-closed eyes; the sun shining on the eye-lashes will also produce a similar effect; the morning sun, shining on twigs of trees situated between the sun and the eye, causes the shadows of some of them to become bright in the centre, and a curious silvery appearance results.

The image of any point seen through a telescope or microscope has its clearness of definition interfered with by the diffraction of rays of light round the edges of the diaphragm, or round the edges of the lens. This effect is generally insignificant in terrestrial telescopes; it is very noticeable in astronomical telescopes, where the source of light, a distant star, ought to appear reduced to a point, but is apparently enlarged into a perceptible disc surrounded by rings; and in the microscope it sets a limit to the powers attainable, for high powers involve small lenses and small apertures, and these bring diffraction in their train.

If a very large number of equidistant lines, some parallel, and others parallel to each other and at right angles to the former, be ruled upon glass, light issuing from a slit or from the image of a slit will, if transmitted through this so-called **Diffraction-grating,** and received upon a screen or in the eye, be found to be resolved into a central bright image of the slit, on each side of which is a dark space, and then a series of successive spectra, separated by dark spaces; these spectra have their violet ends turned towards the central bright image. By multiplying the number of lines in the Diffraction-grating, as in Prof. Rowland's new gratings, which have 43,000 lines to the inch, the spectra may be rendered almost perfectly pure, so that Fraunhofer's lines may be easily seen in them.

A microscopical preparation of muscular tissue will often be found to act as a more or less efficient diffraction-grating; the striations of the muscular fibres take the place of the grooves engraved on the glass. A metallic surface in which grooves have been cut will produce by reflexion a diffraction-spectrum if the image of a slit be reflected from it.

The value of diffraction-spectra is that the deviation in the successive spectra depends directly upon the wave-length; their disadvantage the mechanical difficulties of uniform grooving of the grating.

If any kind of light have, in air, the wave-length λ centimetres, and if n be the average number of lines per centimetre engraved on the grating; and if δ be the angular deviation of any particular coloured light (or, better, of any particular Fraunhofer line),—then $\sin \delta$ is equal to $n\lambda$ for the first spectrum, to $2n\lambda$ for the second spectrum, and so forth; and since n and δ can be measured, λ can be accurately found.

The Twinkling of Stars is another effect of interference: light, coming to the eye from a star so distant as to be practically a single luminous point, arrives in rays which have traversed slightly unequal distances in an irregularly-refracting atmosphere and thus enter the eye in irregularly-unequal phases. Now one colour is extinguished, now another; the eye perceives coloured light complementary to that momentarily lost. No two persons can, as a rule, see any star twinkling in precisely the same manner. The planets twinkle only at their edges: their discs present many points or sources of light, whose scintillations, on the whole, mask one another.

If a planet and a twinkling star—say Jupiter and Sirius—be severally looked at through an opera-glass which is rapidly whirled across the field of view, the image of the planet will appear to be drawn out into a continuous streak, while that of the star will be broken up into a chain of unequally-bright and differently-coloured spots of light.

The colours of light from a bright point twinkling through a dusty haze, or shining through a piece of glass covered with lycopodium; the Corona (red externally) which surrounds the moon as it shines through an atmosphere charged with particles of condensed aqueous vapour; the coloured rings seen when particles float in the vitreous humour of the eye,—these are all different diffractive effects of interference; and the smaller the size of the particles which produce them, the greater the breadth of the coloured rings. Each particle acts as a partially opaque small screen.

The interference of Actinic Rays may be shown by photography; of Dark Heat, by passing a delicate Thermopile, a Tasimeter (p. 582), or a Bolometer (Langley's Thermic Balance, p. 663) through an invisible diffraction-fringe system of dark heat-waves, obtained by treating rays of dark heat with a bent mirror or a biprism; under these circumstances the instrument employed will alternately indicate and cease to indicate the impact of heat-waves.

Double Refraction.

If a transparent medium have the same properties in all directions it is homogeneous, or, optically, **isotropic.** A wave of mechanical disturbance starting from a single point of disturbance in it will be spherical. The properties of the ether-waves within transparent substances are, in some fashion, correlated with the molecular structure of the substance, and thus the ether-waves propagated from centres within homogeneous or isotropic substances are themselves also spherical.

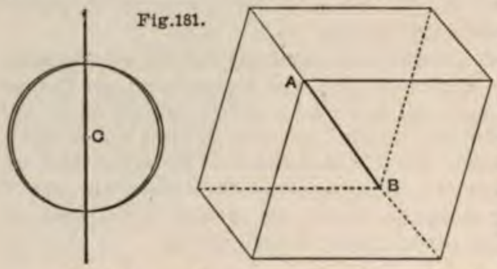
Substances in which the propagation of light is in spherical waves are either amorphous, or else belong to the cubical system of crystals, the system in which the three crystallographic axes of the crystal are equal.

In some crystalline substances one of the crystallographic axes differs from the other two; the crystal is then symmetrical in reference to this axis only, and is said to be **uniaxial.** A mechanical disturbance is propagated in such a crystal in the form of an ellipsoid.

A slice cut out of such a crystal in such a way that its sides are parallel to this principal axis, is said to have been cut parallel to the **Principal Section** of the crystal.

The propagation of an ether-wave in a uniaxial crystal is peculiar. Fig. 181 shows an equal-sided rhombohedron cut out of a crystal of Iceland spar by splitting it along its natural cleavage-planes; its axis AB joins the opposite obtuse-angles. Let a point C on this axis be a centre of optical disturbance. Then two concentric sets of waves are produced; the one spherical, the other ellipsoidal; one of the axes of the ellipsoid coincides with the axis of the crystal, and is equal for a given interval of time to the diameter of the sphere developed in an equal time; the other two axes, which, to avoid circumlocution, we shall here call the **extraordinary** axes, are equal to one another, and are either longer or shorter than the former, according to the nature of the crystal. The next question is,—Which part of a general disturbance at C is propagated in the spherical, and which in the ellipsoidal wave?

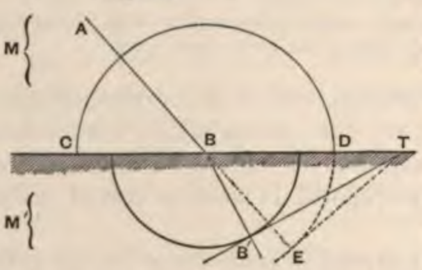
Fig.181.



It may roughly be stated that just as we have seen beams of polarised light differently affected by simple reflexion and refraction according to the plane of their polarisation, so in double refraction the behaviour of a beam of light depends upon its state of polarisation.

On referring to Fig. 59 we find that the construction there given for the course of a refracted plane-fronted wave may be reduced to the following construction (due to Huyghens) for a single ray refracted at the surface of an ordinary isotropic medium. AB is an incident ray travelling through the medium M; CD a circular arc, drawn from centre B, with radius proportionate to the velocity of light in the medium M. Continue the arc CD into the second medium M′; produce AB until it cuts that arc in E; from E draw a tangent line (or plane) cutting the refracting surface in T. From B as centre draw a semicircular arc in the second medium M′, with a radius proportionate to the velocity of light in M′. From

Fig.182.

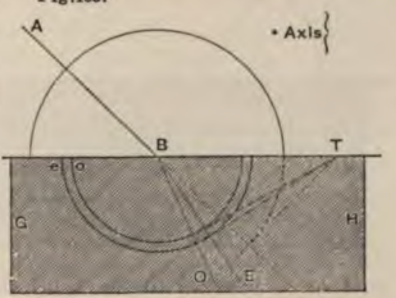


T draw a tangent to this arc; the tangent touches the arc at B′; join BB′. BB′ is the course of the refracted ray (Fig. 182).

A series of somewhat similar constructions will enable us to study a certain number of cases of double refraction.

Suppose a block to be cut out of a doubly-refracting crystal in such a way that one of its cut surfaces is parallel to the axis; and suppose an incident beam to fall upon that surface in a direction at right angles to the axis. Fig. 183 shows that if GH represent such a block, and if the incident beam be in the plane of the paper, the axis is in such a case looked at end-on; and then we find that the incident ray is divided into two parts, which travel at different rates, the slower one, BO, in the central sphere, the more rapid one, BE, in the outer ellipsoid, which, looked at in this aspect, has a circular section; the former, BO, the **Ordinary Ray** (which obeys the ordinary law sin $i = n$ sin r), more refracted than the latter, BE, the **Extraordinary Ray**. Both these rays are in this case in the plane of the paper, like the original incident-ray. The relative radii of the two circles may be found from the respective amounts of refraction of the two rays at this kind of incidence.

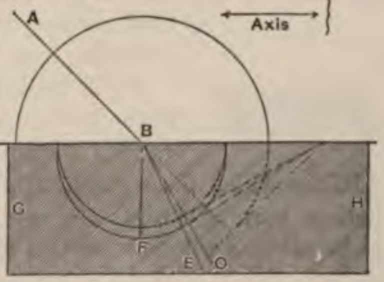
Fig. 183.



For the light emitted by sodium-vapour, the ordinary index and the extraordinary index of Iceland spar are respectively 1·65850 and 1·48635; the reciprocals of these numbers represent the relative velocities of the ordinary and the extraordinary rays in Iceland spar as compared with that of light in air, this being reckoned as unity. In such crystals as those of Iceland spar the ordinary ray is more retarded than the extraordinary.

Let us now turn the block of spar round so that its axis is brought into the plane of the paper—that is, into the same plane with the incident light; the incident light now travels in a principal section of the crystal. One of the extraordinary axes of the ellipsoid, being at right angles to the axis of the crystal, is at right angles to the refracting surface;

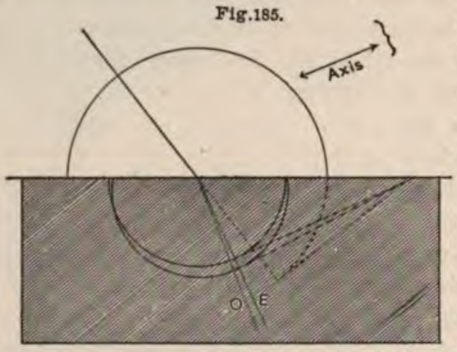
Fig. 184.



its semi-axis, BF in the sectional figure (Fig. 184), bears to the radius of the circle the ratio of 1·65850 to 1·48635 if the light used be that emitted by sodium-vapour.

If the block of spar be cut by a plane at right angles to the principal sections, but not parallel to the axis, we obtain the result shown in Fig. 185. The incident light is in the plane of the paper; the axis of the crystal is also in the plane of the paper.

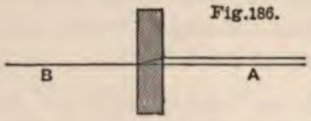
Fig.185.



When the surface which receives the incident beam has been cut at right angles to the axis, and the light falls upon it normally (that is, at right angles to the surface or parallel to the axis), there is no double refraction; the ordinary and the extraordinary rays coincide.

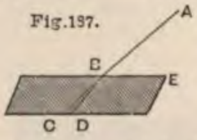
A parallel-sided slice of Iceland spar cut in any other direction than at right angles to the axis will divide an incident ray into an ordinary and an extraordinary ray, as in Fig. 186 (except in the case in which one of the rays is so refracted as to become parallel with the axis, in which case the other ray coincides with it); and thus an observer at A will see two images of a spot at B: while if he turn the slice round, the extraordinary image will rotate round the ordinary one. This can be readily observed with an ordinary crystal of Iceland spar.

Fig.186.



Light striking on a plate or a common crystal of Iceland spar is thus split into two rays, and a single point or a page of print looked at through such a crystal gives a double image. Conversely, a pair of points, C, D, if looked at by an observer at A, will have their images blended, and by finding for various distances between C and D the angle ABE, at which these points appear to blend, the two refractive indices may be found: the rays CB, DB, and BA being caused to lie all in a principal section of the crystal.

Fig.187.



When the incident ray is oblique to the principal section, the extraordinary ray is no longer in the same plane with the incident and the ordinary refracted ray, but is deflected to one or the other side: the tangent plane to the ellipsoid does not touch it in the plane of incidence.

The above figures are drawn for crystals such as Iceland spar, beryl, emerald, mica, ruby, sapphire, tourmaline, the ordinary index of refraction of which is greater than the extraordinary, and in which the ordinary ray travels more slowly than the extraordinary, and lies between the extraordinary ray and the axis; such crystals are called **Negative Crystals.** In others, such as ice, quartz, boracite, the extraordinary ray lies between the ordinary ray and the axis; such crystals are called **Positive Crystals.** In the latter, the extraordinary axes of the ellipsoid are shorter than the diameter of the sphere, which thus encloses the ellipsoid: the extraordinary index of refraction is in them greater than the ordinary index.

The two rays, the ordinary and the extraordinary, are found to be polarised in planes almost exactly at right angles to one another. The **ordinary ray** is **polarised** in a plane containing both the incident ray and the crystalline axis.

If the incidence be that of Fig. 184, the incident ray, the reflected ray, and both refracted rays are in the same plane, the plane of the paper, and the axis is parallel to that plane; the ordinary ray is said to be polarised in that plane; light polarised in such a plane of incidence passes through the spar as an ordinary ray. The definition of the plane of polarisation is a terminological convention; and the question as to the relation of the direction of the vibrations of the ordinary ray, thus said to be polarised in that plane, to the plane itself, we have already seen to be a question still under discussion. The **extraordinary ray,** when the whole three rays thus travel in a principal section of the crystal, is found to be polarised in a plane exactly at right angles with the plane of polarisation of the ordinary ray.

The second face of the block of crystal may be so cut that it receives the ordinary and the extraordinary rays at such an angle as to transmit the one, but totally to reflect the other. In **Nicol's prism** a long rhomb of Iceland spar is cut in this way, and the portions are so cemented by Canada balsam that when common light enters the Nicol it is divided into two rays, of which one, the Ordinary, is totally reflected when it meets the cemented surface, while the Extraordinary ray is transmitted and emerges (the faces of the prism having been, in order to permit this, cut down to the proper angle), in a direction parallel to that of the incident ray. The whole arrangement is thus capable of acting as a **polariser**; and if polarised light be sent through it in one rotational position, the Nicol will transmit it freely; while if the Nicol be rotated through 90° in either direction, on either side of the most favourable position, it will transmit none of it. It can thus serve as a means not only of producing polarised light, but also of detecting polarised light, and of finding in what plane it is polarised; and when it does this duty it is called an **analyser.**

In tourmaline there is double refraction; but one of the rays, the ordinary, is absorbed, and the extraordinary alone passes through. Thus a thin plate of tourmaline acts as a polariser of common light incident upon it, and another plate rotating in front of it may act as an analyser,—a convenient arrangement, were it not that tourmaline is always dark in colour, and absorbs much of the light incident upon it. For this reason Nicol's prisms are commonly used as sources of polarised light.

Crystals of sulphate of iodo-quinine act like tourmaline, but are useless because they are dark, small, and brittle.

We may here recall the different modes of obtaining a beam of plane-polarised light.

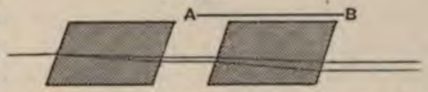
1. Reflexion of ordinary light from glass at the angle of complete polarisation.
2. Transmission through a pile of glass plates with parallel sides; the angle of incidence being the angle of complete polarisation, or an angle approximating to it.
3. Separation of the ordinary from the extraordinary ray produced by double refraction; this being done
   (*a*) by tourmaline, which extinguishes the ordinary ray;
   (*b*) by a Nicol or a Foucault prism, which turns aside the ordinary ray.

Some crystals, such as topaz and arragonite, have two axes, and are called **Biaxial Crystals**: in these the wave-surface is very complex, and they have three indices of refraction.

In general, in these crystals, the wave-front is oblique to the rays, and there is no ray which obeys the ordinary law of refraction that $\sin i = n \sin r$; but that ray which does so most nearly in general, and which does so perfectly when the incidence is in one of the principal sections, is called the ordinary ray; while the other of the two rays, into which a ray of incident light is divided on non-axial incidence, is called the extraordinary ray. In such crystals the positions of the optic axes, which have no necessary relation to the crystallographic axes, are variable; they vary with the temperature of the crystal, and with the kind of light employed; and in some cases a crystal is found to be binaxial for one, uniaxial for another kind of light; Glauberite (native sulphate of soda and lime), for example, being binaxial to red, uniaxial to violet light.

When light has passed through a crystal of Iceland spar and been divided into an ordinary and an extraordinary ray, if it be caused to fall upon a second crystal whose faces are parallel to those of the first, the two rays pass through, suffering no further division; the ordinary ray emerging from the first crystal is still the ordinary ray in the second crystal, which acts like a mere prolongation of the first. If the second crystal be turned 90° round a longitudinal axis parallel to the line AB in Fig. 188, there is still no division of the rays; but the ordinary ray on emergence from the first crystal is an extraordinary ray relative to the second crystal, and is refracted as such in that crystal; and the converse applies to the extraordinary ray emerging from the first crystal. If the second crystal occupy any rotational position intermediate between these, each ray incident on it is decomposed into an ordinary and an extraordinary ray. There are thus, in the ordinary case, four images of a bright point seen through a pair of crystals arranged end to end, at a distance from one another, and these images blend into two when the crystals are, by rotation, placed parallel or at right angles to one another.

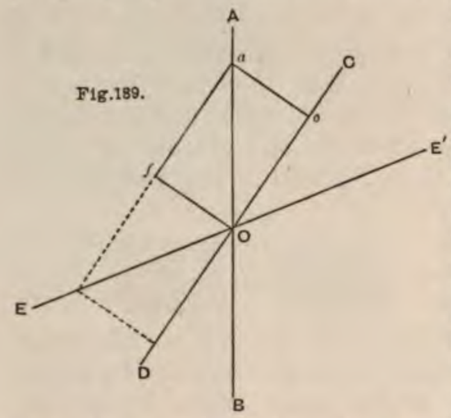
Fig. 188.



Interposed Lamina.—When a polariser and an analyser of any kind are arranged at right angles, so that a plane-fronted beam incident on the system is wholly cut off or deflected by it, an eye placed beyond the analyser can perceive no light; but if a thin film of mica, or other double-refracting substance, uniaxial or biaxial, of uniform thickness, be caused to intervene between the polariser and the analyser, the field may become filled with light, coloured or white, according to the position of the interposed film.

In Fig. 189 the line AB represents a plane vertical to the paper, and cutting the paper in AB: we call this the vertical plane, or the plane AB. Then let us by any convenient means produce a beam of plane-polarised monochromatic light, polarised in the plane AB, and let us suppose this beam to be seen end-on, travelling away from the observer's eye. Interpose a thin plate of some birefringent substance in the path of the beam: let the axis of this lie in the plane CD. The beam AB is broken up by the interposed plate into two: one in which the plane of polarisation is parallel to CD, one in which it is at right angles to that plane. The former is transmitted through the interposed plate as an ordinary ray, the latter as an extraordinary. The lines, Oa, Of, Oc, indicate the relative amplitudes of vibration in the incident polarised beam, in the extraordinary and in the ordinary transmitted beams respectively. The interposed plate may be so thin that although the incident beam is divided into two transmitted beams, these have not perceptibly separated from one another, and on emergence are not only parallel, but are also practically coincident. In a wide-fronted wave-system this coincidence may be held to be absolute except at the edges of the beam. Though the two beams coincide in direction, their undulations do not coincide in phase; in positive crystals the extraordinary, in negative crystals the ordinary, ray is more retarded than its companion. Let us suppose that the more retarded ray has lost one wave-length: then the result of superposition of the two emergent rays will be a **plane-polarised** beam similar to that which had originally fallen upon the interposed plate; if half a wave-length ($=\frac{1}{2}\lambda$) be lost, the result will again be an equal **plane-polarised** beam, polarised in the plane EE′ (Fig. 189).

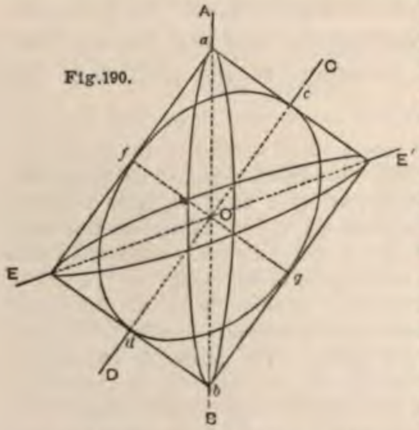
Fig.189.



If, again, CD coincide with AB—that is, if the principal plane of the interposed crystalline plate be parallel to the plane of polarisation of the incident light—there is no extraordinary beam, O*f*; and the light, having been transmitted through the interposed film as an ordinary ray, emerges **plane-polarised**, as it entered. If, again, CD be at right angles to AB, the incident beam is wholly transmitted as an extraordinary ray, and emerges polarised in the original plane.

Let us now suppose that CD is inclined to AB at an angle of 45°: if one of the rays be retarded by some even multiple of $\frac{1}{4}\lambda$, the result is plane-polarised light, either polarised in the original plane (when the retardation may be measured in whole wave-lengths), or in one at right angles to it (when the retardation is some odd number of half wave-lengths), for EE′ is at right angles to AB when CD makes 45° with it. Again, if the retardation be some odd multiple of $\frac{1}{4}\lambda$, the extraordinary and ordinary rays are compounded into a **circularly-polarised** ray of light; and if the retardation be of any value other than some multiple of a quarter wave-length, the result is an **elliptically-polarised** beam, the ellipse being, according to the amount of retardation, some one of those indefinitely numerous ellipses which may be described within the rectangle E*a*E′*b* (Fig. 190).

Fig.190.



In the general case, AB (Fig. 190) being the plane of the incident beam, CD the principal section of the interposed plate, the angle AOC having any value, and O*a*, O*c*, and O*g* being respectively the relative amplitudes of the incident ray polarised in the plane of AB, and of the ordinary and extraordinary rays emergent from the interposed plate; compounded, their result is an **Elliptically-Polarised** beam, of which the limits are :—

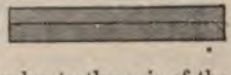
- (*a*) A **plane-polarised** beam, whose plane of polarisation is AB and whose amplitude is represented by O*a*.
  - (1) When CD coincides with AB.
  - (2) When CD is at right angles to AB.
  - (3) When the relative retardation of *cd* and *fg* is 0, or an even multiple of $\frac{1}{2}\lambda$.
- (*b*) An equal **plane-polarised** beam whose plane of polarisation passes through EE′; the angle AOE′ being equal to twice AOC: this is the result when the relative retardation is an odd number of half wave-lengths.
- (*c*) A **circularly-polarised** beam when the angle AOC is one of 45°, and the relative retardation is some odd multiple of $\frac{1}{4}\lambda$.
  - (1) Right-handed (rotation contrary to hands of a watch) when the component polarised in O*f* loses (together with any number of whole wave-lengths) one quarter wave-length or gains three quarters relatively to O*c*.
  - (2) Left-handed when it relatively gains one quarter or loses three.

Elliptically-polarised light is produced in every other relative position of CD. This is right-handed if the relative retardation of the extraordinary ray *fg* transmitted through CD lie between 0 and $\frac{1}{2}\lambda$ or between $n\lambda$ and $n\lambda + \frac{1}{2}\lambda$; left-handed if its relative retardation lie between $\frac{1}{2}\lambda$ and λ, or between $n\lambda + \frac{1}{2}\lambda$ and $(n+1)\ \lambda$. If the plane CD lie so that the angle AOC lies to the left of AB (the observer being, as hitherto, supposed to be stationed near the source of light), these conditions of left- and right- handedness respectively are reversed.

A plate of refracting substance of such a thickness, that when it is interposed in the path of a beam of plane-polarised light of a particular colour, with its principal section at an angle of ± 45° to the plane of polarisation, it converts that plane-polarised light into circularly-polarised light, is called a **quarter-undulation plate.**

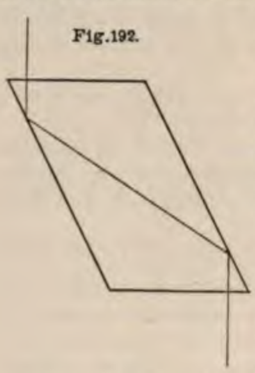
Quarter-undulation plates are of two kinds: (*a*) Where the thickness is just such as to cause a relative retardation of quarter wave-lengths; (*b*) Where the plates are thicker, but are opposed in their action. In Fig. 191 two plates cut out of a doubly-refracting crystal are shown fitted together; the one is cut so that its axis is parallel to the plane of the paper; the other has its axis at right angles to the paper. Incident light arrives already polarised; it is divided by double refraction into two rays, an ordinary and an extraordinary; then, since the second plate has its axis at right angles to the axis of the first, the ordinary ray of the first plate is refracted in this as an extraordinary ray, while the extraordinary ray of the former passes through as an ordinary ray. On emergence both rays are parallel and practically coincident; and the amount of relative retardation is equal to that produced by a thin plate equal in thickness to the difference between the thicknesses of the two plates.

Fig. 191.



When light, plane-polarised, is totally reflected from glass, it is found to be elliptically polarised, unless it had been originally polarised in the plane of incidence, or in a plane at right angles to this. Reflexion from metals presents this peculiarity at all angles of incidence. The vibratory movement actually extends beyond the surface of the glass into the rarer medium beyond, as may be proved on bringing a second piece of glass close to the totally reflecting surface, when interference-colours will be seen. As a result of this, a difference of phases is set up between the two components (polarised in and at right angles to the plane of incidence) into which the incident light may be resolved. A similar result occurs in metallic reflexion, for some of the light penetrates to a slight depth below the reflecting surface. A wave cannot have its direction abruptly changed: and during the gradual change of its direction, its phase becomes altered to a slight extent: and this effect differs in amount according to the direction of vibration of the incident waves. When the angle of incidence is such that the difference of phase set up is $\frac{1}{8}\lambda$, two such total reflexions would convert a plane-polarised ray into a circularly-polarised one. If a rhomb of glass be cut in such a form that a ray of light may pass normally through one surface, strike a second surface at the appropriate angle of incidence, and be there totally reflected, strike the third surface at an equal angle, and pass out normally through a fourth surface, a ray so travelling through it will, on emergence, be found to be circularly-polarised. Such a rhomb is known as a **Fresnel's Rhomb**, and acts as a quarter-undulation plate for every kind of light, while a film of mica, real or virtual, can only act as such towards light of one kind.

Fig.192.



If plane-polarised light pass successively through two similar quarter-undulation plates, similarly placed, the emergent light is plane-polarised in a plane at right angles to the original plane of polarisation; whereas, if the two quarter-undulation plates be opposed in their action, the light is restored by the second to its original plane-polarised state. A second quarter-undulation plate of known action affords us a means of distinguishing right from left-handed elliptically or circularly-polarised light.

The two vibrations which make up the circular or elliptic vibration of the ether in a circularly or elliptically-polarised beam of light are not in a condition to interfere with one another on account of their difference of phase, because they are executed in planes at right angles to one another. If a beam circularly or elliptically polarised by an interposed lamina be received upon a birefringent prism, it is split into two parts, one an ordinary ray, the other an extraordinary ray, and each of these is plane-polarised. In Fig. 193 AB is the plane of original polarisation, CD a principal section of the interposed lamina, EE′ a principal section of the analysing crystal. Then a plane-polarised ray whose amplitude is represented in magnitude by the line Oa, and whose plane of polarisation is AB, is resolved by the interposed lamina into two, Oc and Of, which are upon emergence compounded into a plane or an elliptically or circularly-polarised beam, according to their relative retardations. When this strikes the analyser, its components Oc and Of are themselves resolved each into a pair of components parallel and at right angles to EE′; these are respectively Oe' and Oh from Oc, and $-Oe$ and Og from Of.

In the plane EE′ we have therefore two vibrations, Oe' and $-$ Oe; in the plane at right angles to EE′ we have the vibrations Og and Oh. But Oe' and $-$ Oe differ in phase; so do Og and Oh. These are therefore in a condition for interference. The ordinary ray, passing through the analyser, is made up of the mutually-interfering components Oe' and $-$ Oe, and the extraordinary of Og and Oh; the effect of interference is to cause a distribution of energy such that the ordinary ray gains or loses as much energy as the extraordinary loses or gains, and thus the energies of the ordinary and the extraordinary rays are, taken together, equal to the energy of the incident plane-polarised ray. The amount of relative retardation caused by the interposition of the doubly-refracting plate, when measured in wave-lengths, depends upon the particular kind of light employed. Hence when the original plane-polarised light is a white light, each colour obeys its own law; each colour, if strong in the ordinary, is weak in the extraordinary ray, and *vice versâ;* thus the extraordinary ray and the ordinary are coloured, and their colours are complementary.

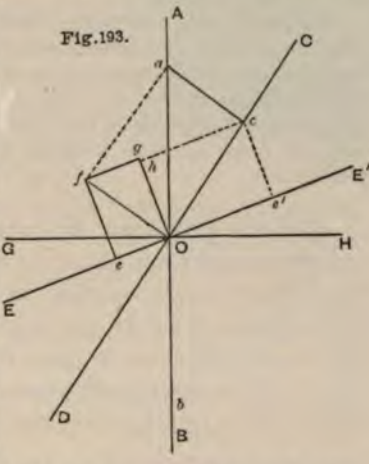


Fig. 193.

The following are the limiting cases:—

1. There is no extraordinary ray when—
    (*a*) AB, CD, and EE′ (Fig. 193) coincide.
    (*b*) AB and EE′ coincide, and CD is at right angles to them
2. There is no ordinary image when—
    (*a*) AB and CD coincide, and EE′ is at right angles to them.
    (*b*) CD and EE′ coincide, and AB is at right angles to them.
3. The two images are equal for every colour, and are therefore white—
    (*a*) When AB and CD coincide, and the angle AOE′ = ± 45°.
    (*b*) When AB and CD are at right angles, and the angle AOE′ = ± 45°.
    (*c*) When CD and EE′ coincide, both being at an angle of ± 45° with AB.
    (*d*) When AB and CD are at right angles, and EE′ makes an angle of ± 45° with either.

In every other position the two images are complementarily coloured.

Determination of the character of a Beam of Light.—A crystal of Iceland spar capable of rotation round a longitudinal axis may be used as an analyser, and will enable one, with the intervention of a doubly-refracting lamina, to determine the character of a beam of light falling upon it.

Plane-polarised light: as the prism is rotated, the ordinary and the extraordinary images appear and alternately wax and wane, disappearing and reappearing. In this instance the doubly-refracting lamina is dispensed with.

Elliptically-polarised light and partially-polarised common light: the two images never entirely disappear, though they become alternately brighter and dimmer.

Circularly-polarised light, and natural light: the two images do not vary in their relative intensity with the rotation of the prism; they continue nearly equal.

Elliptically and circularly-polarised light on the one hand, and common light unpolarised or partially polarised on the other, are distinguished by the respective actions upon them of a quarter-undulation plate, interposed between the source and the analyser; the former are converted by this plate into plane-polarised light, the latter are not; and the former then produce only one image in some positions of the analyser, while the latter always produce two.

Colours produced by interposed film.—When a polariser and an analyser are placed so that the latter quenches the light which the former transmits, the interposition between them of a plate of mica or selenite, or any other doubly refracting substance, will cause light again to reach the eye, provided that the principal section of the interposed substance is neither parallel nor at right angles to the principal sections either of the polariser or analyser.

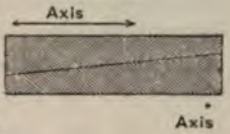
In Fig. 193 above let the angle AOE′ be made a right angle; O*g* and O*h* come to coincide in direction with AB; O*e* and O*e*′ with GH, at right angles to AB. The polariser allows *ab* to pass: the analyser cuts off all components polarised in the plane AB; whence crossed prisms produce perfect darkness.

But the intervention of the doubly-refracting substance resolves the light which cannot traverse the analyser into two rays, of each of which there is some part that can traverse that obstruction. If the doubly-refracting substance interposed be uniform in thickness, the whole field under crossed prisms becomes filled with uniform coloured light; if the polariser, or the analyser, or the interposed film, be turned round, the light first becomes white, and then passes into the complementary colour.

The colours produced by a given film depend upon the amount of relative retardation produced by it in light of each kind. This depends upon (*a*) the substance of the film and its refractive indices; (*b*) its thickness; (*c*) the inclination at which the ray traversing it strikes it; (*d*) the relation of its optic axis or axes to the plane of its surface.

When an irregular film of mica or selenite, flaked off with a penknife from a large mass, is interposed between crossed prisms, the eye, looking through the analyser, sees the darkness of crossed prisms transformed by the interposition into a series of gorgeously brilliant colours; and as the analyser is turned round these fade away into white light, and reappear in complementary tints. If the film be a very thin wedge, each thickness of it produces its own colour, and a kind of spectrum is thus produced. A double wedge, known under the name of **Babinet's compensator**, and shown in Fig. 194, acts as a virtual film of graded thickness, and gives a series of fringes or spectra. This chromatic property of a doubly-refracting film and an analyser may be made use of to detect polarised light: if the light looked at through such a system be wholly or even partially polarised, the phenomena of polarisation-colours come into view; and while, for example, natural light in such a case gives two nearly equal white images when a crystal of Iceland spar is used as the analyser, circularly-polarised light, on the other hand, gives two complementary coloured-images of almost exactly equal intensity—equal, that is, from the physical point of view, though to the eye these coloured images may not seem equally bright.

Fig.194.



When a divergent or a convergent beam of white light passes normally through an interposed film cut at right angles to its axis, the centre of the ordinary image is, when the analyser is parallel to the polariser, found to be bright and colourless, while round this there is a series of annular fringes or spectra, the local tints of which depend upon the local relative retardations; the whole being traversed by a colourless cross, whose branches are parallel, and at right angles to the plane of polarisation. At the same time, the extraordinary image presents the complementary appearances—a black centre, a black cross, and complementary colours. When the analyser is turned round through 90°, so that the ordinary image becomes an extraordinary one, it reverses its appearance.

This cross is really a coincidence of two crosses, one parallel and at right angles to the primitive plane of polarisation, and the other parallel and at right angles to the principal section of the analyser.

When a lamina is interposed whose axis is not at right angles to its surface, the coloured (or isochromatic) lines are modified into hyperbolic curves, or even into lines nearly straight; but if the axis be parallel to the surface of the lamina, and if the axis be also parallel or at right angles to the original plane of polarisation of the incident light, or again, if the principal section of the analyser be parallel or at right angles to that of the lamina, there is no coloration produced.

When the lamina used has been cut from a biaxial crystal, the isochromatic lines are converted into a series of curves known as lemniscates, and the dark or colourless crosses are represented by a pair of hyperbolic curves.

The doubly-refracting power of a body may be detected when it is placed between crossed prisms, and by this means it is found that substances which are ordinarily isotropic become doubly refracting when they are exposed to compression, or to dilatation, or flexure, or torsion, or vibration (especially at the nodes), or to molecular stress, as where they are heated and then suddenly cooled, or to electrical stress; and crystals ordinarily isotropic become double-refracting when exposed to mechanical stress, or when they crystallise irregularly or are not homogeneous. Organic tissues are by this means for the most part found to be double-refracting, and they seem, when placed between crossed prisms, to shine by their own light against a dark background—a circumstance favourable to definition, for there is no diffraction of light round the fibres, but practically of little utility, for it is difficult to get prisms of Iceland spar sufficiently clear to be interposed in the path of the rays coming from a high-power objective.

It has been proposed to make use of a dynamometer which measures forces by the compressions exerted on glass which is interposed between crossed prisms, these compressions being estimated by the colours produced: the greater the compression, the greater the differences of phase set up between the ordinary and the extraordinary rays, and the longer the wave of that colour which is cut out of the emergent light.

It has also been found that slices of different minerals placed between crossed prisms act in very characteristic manners, and are thus, in many cases, easily identified.

Andrews proposed as a test for sodium to make sodium-platinum chloride, which produces, when placed between crossed prisms, colours so vivid and characteristic that the millionth part of a grain of sodium can be detected by this means.

ROTATORY POLARISATION.

When natural white-light is passed through a polariser, then through a film of mica or selenite cut parallel to the axis, and lastly, through an analysing prism of Iceland spar, it gives, as we have seen, two colourless images of the source of light. If now we replace the mica or selenite by a slice of **quartz** cut parallel to the axis, the two images produced are complementarily coloured.

If their light be examined with a prism, it is found that the spectrum of the light of the extraordinary image is lacking in a particular region, which presents a dark band, while that particular region is bright in the spectrum of the ordinary image. Further, as the analyser is turned round, the dark band in the spectrum of the extraordinary ray seems to travel up or down the spectrum; and if the piece of quartz used be very thin, this dark band may traverse the whole spectrum while the analyser is rotated through an angle of less than 180°. That particular kind of light which is absent in the extraordinary ray leaves the quartz plate in a condition of polarisation in a plane parallel to the principal section of the analyser.

Each position of the analyser cuts off a distinct kind of light in the extraordinary ray: hence light of each colour must have become polarised in a special plane, and the plane of polarisation of the light incident upon the quartz has been rotated, that of each component colour to a specific extent.

Biot found that α, the amount of angular rotation of the plane of polarisation of each colour, was, very roughly, proportional to the square of its wave-frequency, or inversely proportional to the square of λ, the wave-length. Boltzmann showed that the true law is that $\alpha = (A \div \lambda^2) + (B \div \lambda^4)$: in quartz, for example (Stefan), $\alpha = \left[(7{\cdot}07018/10^6) \div \lambda^2\right] + \left[(0{\cdot}14983/10^{12}) \div \lambda^4\right]$, where λ is the wave-length in mm., and α the rotation produced by a slice 1 mm. thick.

We have seen that a plane-polarised beam is equivalent to two equal and opposite circularly-polarised beams; but quartz allows a right-handed circularly-polarised beam to travel faster through it than a left-handed one; at any given point the right-handed component is therefore not so advanced in its phase as its left-handed companion: this is equivalent to a relative gain of phase by the so-called left-handed component (see definition, p. 477); this causes the plane of the plane-polarised ray gradually to turn to the right, in the same direction as the hands of a watch when the ray is looked at from behind, from polariser towards analyser.

A piece of quartz 1 mm. thick thus turns the plane of polarisation of yellow rays about 22°; a piece about 16·36 mm. thick will turn them through 360°, for the amount of rotation is proportional to the thickness of the rotating medium. For the Fraunhofer line B the specific rotatory power of quartz (1 mm. thick) is 15°·55; for line D, 21°·67; for line H_1, 50°·98.

A substance which acts in the same sense as quartz is said to be **dextro-rotatory** or positive; one which, causing a relatively-slow propagation of right-handed circularly-polarised light, rotates the plane of polarisation to the left, is **lævo-rotatory** or negative. This property is not confined to crystals. The following list comprises a few examples of bodies of each kind:—

Dextro-rotatory.—Some samples of quartz; cane sugar, grape sugar, camphor; many essential oils, such as oil of orange, oil of caraway; cinchonine, quinidine; castor-oil.

Lævo-rotatory.—Some samples of quartz; oil of anise, oil of mint, oil of turpentine; quinine; sugar of fruits, starch; albumin.

The rotatory powers of different substances are compared by means of two constants. (*a*) The real specific rotatory power; the rotation for a given colour or Fraunhofer line produced by a layer 1 mm. thick of the substance itself. The symbol α_D denotes the real rotation for the Fraunhofer line D.

(*b*) The apparent specific rotatory power, $[\alpha]$, for a given line or colour ($[\alpha]_D$, that for the line D); the rotation produced by a substance in a state of dilution. It is equal to $\alpha/\epsilon l\rho$, where α is the observed rotation, ϵ the quantity of active substance per gramme of solution, l the length of the column employed, and ρ its density.

The apparent specific rotatory power is slightly increased by rise of temperature and modified by the nature and proportion of the diluent substance.

With these variations, for cane sugar $[\alpha]_D$ is about 67°; for milk-sugar—α-lactose 80°, β-lactose 54°·5; for crystallised grape-sugar in 7·68% solution $[\alpha]_D = 52°{\cdot}89$, in 82·6% solution $[\alpha]_D = 57°{\cdot}8$.

A column 20 cm. in length of solution of cane sugar, containing in each 100 cubic cm. 16·350 grms. of cane sugar, is equivalent in rotatory power to a plate of right-handed quartz 1 mm. thick. This fact, coupled with the fortunate circumstance that the rotatory dispersion for quartz is the same as that for cane sugar and glucose, enables the strength of solutions of sugars to be approximately determined by means of a Saccharimeter.

Essential oils are found to retain their rotatory power unimpaired (due allowance being made for proportionate dilution) when in dilute solution, or even when in the state of vapour, provided that they undergo no chemical change. When substances lævo- or dextro- rotatory are mixed with each other or with indifferent substances, and if there be no chemical change, the rotatory effect of the whole is found by multiplying the rotatory index of each substance by the proportion in which it is present, and finding the joint effect of the components of the mixture by a process of simple addition. If a rotatory substance assume the crystalline form, its rotatory action is very often masked by double refraction: whence solids, such as camphor, are generally best examined in solution; exceptions to this being found in some cases, such as those of benzile and chlorate of soda, where the rotatory power depends upon the crystalline structure, and in which the crystals are generally hemihedric, or, as it were, distorted towards one side.

Rotatory polarisation is thus due either to crystalline arrangement of molecules or to the structure of the molecules themselves; and it has been shown (van 't Hoff) that bodies gifted with the molecular power of rotation have, in their chemical graphic formulæ, a marked want of symmetry.

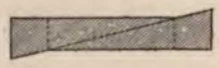
We are now in a position to understand the pieces which make up a **Soleil's saccharimeter.** 1. A Nicol's prism, achromatised by a properly shaped prism of glass through which the transmitted extraordinary ray passes: the achromatic prism thus acting as a polariser is so placed that the light transmitted by it is polarised in a vertical plane.

2. A double-quartz plate, or Biquartz; two semicircular plates of quartz joined by a vertical cement-line, and thus forming a circular disc of uniform thickness: the two halves have opposite rotatory power, and their thickness is so adjusted that they respectively deviate through 90° in opposite directions the plane of polarisation of incident plane-polarised yellow light; they therefore both deviate yellow light, incident upon them and polarised in a vertical plane, into the same horizontal plane.

3. A Liquid-holder; a tube fitted with clear glass at each end, in which is placed a layer of the liquid to be examined, 10 centimetres in length, such being the distance between the terminal glass-plates.

4. A Compensator. This is in its effect a quartz plate of variable thickness. It consists of two pieces of quartz of a wedge shape. One of these can be made to slip over the other; the central thickness is thus variable at will. The amount of movement can be measured by means of a vernier connected with the one wedge, and a scale connected with the other. When the zero of the vernier coincides with the zero of the scale, the thickness of the joint system is such that it exactly neutralises the rotatory effect of one of the halves of the biquartz, while it doubles that of the other, the effect being, in both cases, to bring the light back to the original vertical plane of polarisation.

Fig.195.



5. An Analyser: this is generally a Nicol's prism.

6. A Lens to be focussed on the biquartz.

To use the instrument:—Fill the liquid-holder with water, and put it in position; focus the lens 6 so as to obtain a clear image of the biquartz; make the vernier and the scale of the compensator to coincide; turn the analyser round until there is observed to fill the field a particular hue, lying between the red and the blue, and called the *teinte de passage*; this hue being chosen out of the many which will successively come into view, on the ground that as the instrument is constructed, the appearance of this colour denotes that the principal section of the analyser is parallel to the plane of polarisation of the yellow; if the yellow could have passed through the analyser it would have been transmitted as an ordinary ray: but a Nicol prism cuts off the ordinary ray: the yellow is therefore cut off; the extraordinary image, which alone passes through the analyser, is thus represented in colour by white daylight, minus its bright yellow: the remainder produces in the eye the effect of a dim lavender gray, which, with great sensitiveness, merges into red on the one hand, or into blue on the other, when the analyser is slightly rotated. Both halves of the quartz plate appear of the same colour, because, from them both, the yellow light issues polarised in the same horizontal plane.

If now the water in the liquid-holder be replaced by the liquid to be tested, and if that liquid have rotatory power, the two halves of the quartz will cease to appear of the same colour: the liquid aids the rotatory effect of the one, and is opposed to that of the other. The effective thickness of the compensator is now varied until the rotatory effect of the liquid is neutralised: the vernier shows by its displacement how much the thickness has been increased or diminished: the graduation of the vernier is arbitrary, but a displacement of one step on the scale amounts generally to a difference of one-tenth of a millimetre in the thickness of the quartz; and as the vernier reads to tenths, the positive or negative alteration of thickness of the quartz found necessary to restore the uniform coloration of the field may be measured to the hundredth of a millimetre. If the thickness of the compensator have to be diminished, the liquid has rotatory power similar to that of the quartz used in making the compensator; if it have to be increased, its action is contrary to that of the quartz. It is necessary to know of what kind this quartz is; this being known, it can be stated that 100 mm. of the liquid are equal, positively or negatively, to so many millimetres of dextro-rotatory or lævo-rotatory quartz, as the case may be; and thus the rotatory power of the liquid can be specified with precision.

Thus a layer of water 10 cm. thick, containing diabetic sugar in solution in the proportion of 10 grammes per litre, is equivalent to a thickness of 3·42 mm. of right-handed quartz: the thickness of a dextro-rotatory compensator of quartz would have to be diminished by an amount corresponding to 34·2 divisions of the scale on the interposition of a solution of that substance of the given thickness and the given strength; while if the solution were weaker or stronger, the amount of change of thickness of the quartz, as shown by the amount of displacement of the vernier, would be approximately proportional to the strength.

For other Saccharimeters in use, see Watt's *Dictionary of Chemistry*, Suppt. iii. p. 1198.

TRANSFORMATIONS OF THE ENERGY OF ETHER-WAVES.

We have already seen this energy transformed into molecular work in the processes of photography, and it is now merely necessary to remark that whatever increases the absorption of light by a set of molecules, increases the chemical work done by the incident ether-waves; if, for example, a spectrum be cast upon a photographic plate prepared with collodion in which chlorophyll has been dissolved, the *local* of the chlorophyll absorption-bands comes out most strongly in the resultant photograph of the spectrum.

The impact of ether-waves upon some substances gives their molecules a new arrangement: selenium is thus so acted upon by light that it becomes a better conductor of electricity than it is in the dark; and hard rubber is superficially acted upon by light, so that when the incident beam is intermittent or harmonically variable in intensity, the rubber emits a sound which reproduces in its pitch or its complexity the peculiarities of the incident light.

It has been proposed to call the last-mentioned property of hard rubber the sonorescence of that substance.

As to the mechanical or molar work, the pressure exerted by the impinging ether-waves, though small, is definite. The energy in one cub. cm. of sunlight at the earth's surface is about 41/1,000,000 erg, and the pressure per square cm. is about 41/1,000,000 dyne, or roughly speaking, about the weight of a threepenny-piece on each acre of ground.

The mechanical effect of ether-waves is rather to be looked for in their heating effect than in direct pressure. They may heat absorbent gases, such as ammonia, and cause them to do mechanical work, or to produce sound, if the incident beam be intermittent or harmonically variable.

OPTICAL INSTRUMENTS.

The Eye, considered as a simple lens, brings parallel rays incident upon the cornea to a focus upon the retina. Hence, when it is at rest, as when one meditatively contemplates space, it is adapted for vision of infinitely-distant objects, or, as the phrase goes, it is accommodated for infinity. To look at nearer objects requires an effort for each—an effort of accommodation. This is effected by increasing the convexity of that part of the eye called the crystalline lens, which is normally flattened.

The range of accommodation provided by our power of varying the form of the crystalline lens is the same as if we were provided with a set of lenses of all focal lengths between infinity and about ten centimetres.

The eye presents several faults, as we find when we expose it to severe tests. Its several parts are not truly centred. Its surfaces are never truly symmetrical round an axis. It is often too long in the bulb, so that rays are brought to a focus before arriving at the retina, and produce, instead of clear images of the several points of an object, a number of overlapping diffusion-circles, and there is consequently produced a blurred image of the whole; this condition requires the use in front of the eye of thick-edged lenses, in order somewhat to diverge the incident beam. The bulb may, on the other hand, be too short, so that converging lenses are necessary. Spherical aberration is always present; and the images of equally-distant coloured objects can never appear equally distinct, for they are not in focus at the same time. The front of the cornea has frequently a somewhat cylindrical form, in consequence of which horizontal and vertical objects do not come to the same focus. The field of vision is extremely limited, and the most sensitive part of the retina is excentric. Yet for all this we are for the most part insensible to these defects; we have the power of adjusting the eye with extreme rapidity for all the parts of an extended object, and we have been educated by experience to use both our eyes, and thus, by blending the separate pictures provided by the two eyes, to form judgments as to the solid form and distance of remote objects,—a power which we discover to have depended greatly upon binocular vision when we try, shutting one eye, suddenly to touch any given object at arm's length, though it can be cultivated even with one eye; just as microscopists who have long used a monocular, and cultivated a habit of keeping the fine adjustment in action, find no perspective advantage in the use of a binocular microscope. With age the power of accommodation wanes; for near objects the image cannot be brought to a focus on the retina; and then, in order clearly to see near objects, the aid of convergent lenses must be sought.

The Microscope.—An ordinary thin-edged lens is called a simple microscope or magnifying glass. The compound microscope is formed of an objective—a combination of lenses which converges the rays divergent from the object into an inverted real image, achromatic and aplanatic (*i.e.*, devoid of the effects of spherical aberration), in a plane in space between itself and the eyepiece—and of a convergent eyepiece, also compound, and corrected for spherical and chromatic aberration, which magnifies this inverted real image, and produces an inverted virtual image at an apparent distance from the eye, not less than that of the nearest distinct vision. The real image formed by the objective must be at the focus of the eyepiece; hence, when a more highly-magnifying eyepiece is used, in order to throw the real image back to the focus of the eyepiece the objective must more closely approach the object examined.

The rays from the real image, instead of being received by an eyepiece may, the eyepiece being removed, be allowed to diverge and fall upon a screen; they will there form an image of any size, which may be traced by hand, if the illumination be sufficient; if the screen be a sensitised photographic plate, a photograph may be produced.

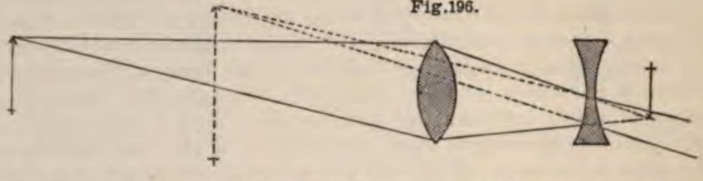
In the **astronomical telescope** parallel rays from a distant star are made to converge and form a small real image; this is examined by a simple achromatic eyepiece. The image is inverted like that in the microscope.

In the **terrestrial telescope** rays nearly parallel are made to converge and form a small inverted real image; this small image is magnified and reinverted by an arrangement of lenses equivalent to a compound microscope.

The minute virtual image of surrounding objects produced by reflexion from a globule of mercury is one of the most trying tests for an ordinary microscope.

In the **opera glass** a convergent lens directs incident rays towards an inverted real image, but before this is formed the rays meet a divergent lens, which causes them, instead of converging towards a real inverted image, to diverge as if from a virtual erect image, as is shown in Fig. 196. This

Fig. 196.



combination of lenses—Galileo's doublet—is one of the simplest and most useful.

In the **ophthalmoscope**, as used for the observation of an erect image of the fundus of the eye, the principle of Galileo's doublet is *sometimes* utilised. In the first place, light is made to fall upon the fundus of the eye by means of a concave mirror held in the hand or fixed upon the forehead of the observer. The fundus is thus illuminated and becomes a source of light. Rays from it pass towards the eye of the observer through a central aperture in the mirror, placed opposite the eye of the observer. These rays from the fundus are, if the eye observed be myopic, too long in the bulb, rendered convergent by the media of the observed eye itself, and a thick-edged lens placed near the eye observed causes them to enter the eye of the observer as if they had proceeded from an enlarged erect virtual image. The convergent lens of Galileo's doublet is thus represented by the observed eye itself, while the biconcave lens employed makes up the pair of lenses. If the eye observed be normal, and accommodated for an infinite distance, rays proceeding from any point of its retina emerge parallel, and a second lens is not absolutely necessary if the observing eye be normal, for the rays come to a focus on the observing retina, if the observing eye be also accommodated for infinity; if the observed eye be too short in the bulb the rays are, on emergence from it, still divergent, and in this case a convex lens is necessary.

The ophthalmoscope may also be used in such a way as to give an inverted image, not so much magnified as in the preceding case, but more extensive in its field, brighter, and more easy of attainment. A beam of light reflected from the mirror converges upon and passes through a focus; it then diverges on its way towards the eye, but encounters a thin-edged lens which causes it rapidly to converge into and then to pass through a focus within the eye, and, after traversing this focus, to illuminate a wide area of the fundus of the eye. Light from the illuminated fundus is collected by the biconvex lens before mentioned, which forms a real image; the rays from this image pass on through the aperture in the mirror into the eye of the observer, who then perceives an inverted and magnified image of the fundus of the eye,—an image which may be still further enlarged by means of a second convergent-lens placed behind the aperture of the mirror.

VISUAL PERCEPTION.

The retina is not a uniform surface, but is made up of elements whose average distance from one another, in the yellow spot, is about ·005 mm. Distant points whose angular distance is such that their images on the retina are less than ·005 mm. from one another, seem to blend into one, and thus two stars, whose angular distance is less than 70″, appear to the eye as a single star.

The stimulation of nerves is associated with chemical work in the nerve-ends; and this with absorption. In this respect it is interesting to find that the retina, which is particularly sensitive to yellow and green light, absorbs green and yellow light, and in white light appears purple. It has been pointed out that the blindness of the eye to heat-waves and actinic waves is of advantage: for the energy of heat-radiation is relatively so great that everything would appear intensely bright, and our ordinary vision of objects would be impossible if the rays of dark heat were visible; while if the ultra-violet rays were visible, the image of every point would be shrouded in a haze due to chromatic aberration.

It is now the received view that each element of the eye which is broad enough to perceive white light consists of three ultimate elements, each of which is capable of perceiving one of three physiologically-primary colours. All colour-perceptions, infinite as these may be in intensity and in hue, are due to the simultaneous excitation of the three sets of nerve-ends by stimuli of given absolute or relative amount. As to what the physiologically-primary colours are, opinion is still somewhat divided: red, green, and violet (Young and Helmholtz); vermilion, emerald green, and artificial ultramarine blue (Maxwell); red, green, and blue (Fick). Any three such colours may be made the basis of a systematic classification of colours. When orange light, monochromatic or compound, affects the eye, the nerve-ends sensitive to red are affected; those sensitive to green are simultaneously affected, but less so; while those sensitive to violet are very feebly affected. When the red and the green nerve-ends (as we may call them) are equally affected, the resultant impression is one of yellow; red light and green light together make yellow light. This may be shown by several methods of mixture of colours.

1. A source of light: a prism: a screen upon which a spectrum is formed: two slits in the screen, so placed as to admit the passage of two selected colours of the spectrum: achromatic lenses behind the screen converge the two coloured beams towards a common crossing point: a screen there placed indicates the mixed colour.

2. A V-shaped slit in a screen (Helmholtz); a prism behind this: the two spectra produced overlap each other and produce a very extensive series of combination-colours.

3. By **Maxwell's discs**: a disc of red and one of green-painted cardboard: each disc slit down to the centre, and cut out at the centre so as to be fitted upon a rotating top: the one disc being slipped through the other, the relative proportions of red and green in view can be modified at will: the whole is rotated at such a rate that the successive impressions of red and green enter the eye at least from twenty-five to fifty times per second: each local impression of red in the retina is still vivid while that of green has already commenced, and *vice versâ*; the colours blend in the eye, and various shades of orange-red, orange, yellow, or yellowish-green are produced, according to the relative proportion of the colours blended.

4. Parallel rays are caused, by a lens, to converge upon a focal point; the light traversing different portions of the lens is, by the interposition of transparent coloured screens, diversely coloured; all comes to the same focus; the eye, placed axially at the focus, receives mixed rays; the colours blend in the eye (Aitken).

Red and yellow make orange; yellow and green, yellowish green; green and blue, a bluish green; or in general, colours near one another in the spectrum give rise, when compounded, to an average or intermediate sensation. Red and green in different proportions may produce all the colours of the spectrum between red and green: green and violet all the colours between green and the violet colour employed.

When blue and yellow lights are mixed the impression produced on the eye is that of a white light; blue and yellow are complementary colours. This is contrary to the general impression that yellow and blue make green: when yellow and blue *pigments* are mixed, the yellow and the blue lights reflected from the mixture destroy one another, forming white light; and the residual green, never absent from the purest blue or yellow pigment-reflected light, is perceived, somewhat wanting in brightness, and diluted by the white light produced by the complementary colours.

The phenomena of double refraction enable us to produce an indefinite number of pairs of complementary colours.

Some peculiarities of perception of colour are readily explicable on this theory of three kinds of nerve-ends, which is due to Young. A spectrum formed by light travelling from a waning source is found to modify its tints as the light fades; the orange-red seems to become more purely red, the yellow-green more purely green, and so on; at length the faint spectrum is approximately restricted to red, green, and violet, or violet-blue; of each triplet of nerve-ends, one is feebly stimulated by a given colour, the other two are inappreciably so, though if one be stimulated the others can never remain wholly unaffected.

On the other hand, if a coloured light be rendered exceedingly bright, the other nerve-ends participate in the excitement: very bright red seems somewhat orange; violet very easily passes over into whiteness when its brilliancy is excessive.

A black colour is due to the absence of stimulation of any of the nerve-ends; and between bright white and black there is a gradation of weak whites which are called grays.

Fatigue of the retina causes it to become insensible to a colour long looked at: when white light is then looked at, it appears of a hue complementary to that colour, the sense for which has been temporarily exhausted.

When some of the nerve-ends of the retina are stimulated, the stimulation spreads to some degree: a very narrow white-hot wire appears, especially from a little distance, to be much wider than it really is; this phenomenon being named **Irradiation.** In consequence of this the crescent moon appears larger than that part of the moon which is illuminated by light reflected from the earth; a candle- or gas- flame appears continuous, though its incandescent particles are by no means in contact with one another; and the glowing filament of an electric incandescent lamp appears much thicker than it really is.

Perception of Form.—The two eyes receive images of different form; these are blended by a mental operation into a compound image, which education has taught us to associate with the distance of the several parts of the object. This is applied in the Stereoscope: two pictures of images taken from different photographic standpoints are formed, one in each eye, and the effect is that of outstanding relief. This may be exaggerated with singular effect where the photographs are taken from standpoints situated at a mutual distance of several feet: mountain scenery is thus brought into perspective. The same exaggerated effect may be observed when a landscape is looked at through a pair of telescopes, parallel but at several inches' distance from one another, the light traversing each being brought into the corresponding eye by an arrangement of reflecting prisms.

The images in the two eyes may often differ in brightness: when this is the case, there is a struggle between the two fields of view, which causes the impression known to us as that of **Lustre**; this effect being especially well marked in the case of metals.

CHAPTER XVI.

ELECTRICITY AND MAGNETISM.

ELECTRICITY and MAGNETISM are not forms of Energy; neither are they forms of Matter.

They may perhaps be provisionally defined as properties or Conditions of Matter; but whether this Matter be the ordinary matter, or whether it be, on the other hand, that all-pervading Ether by which ordinary matter is everywhere surrounded, is a question which has been under discussion, and which may now be fairly held to be settled in favour of the latter view.

At first sight it would appear that the electricity of an electrified body is a condition of that body itself. When a small piece of resin and a small piece of glass are rubbed together, it is found that after they are pulled asunder, the resin and the glass are in such a condition that they attract one another with a definite and measurable force; and that this force varies inversely as the square of the distance between them. This attraction across an intervening space is by some held to be due to a so-called Mutual Action at a Distance; but when the bodies are pulled away from one another, work is done upon them which will be restored when they are allowed to approach one another, and it seems probable that this work has been done not upon two isolated bodies mutually acting at a distance, but upon a system which consists of the two bodies together with the Ether between them, which has been stressed by their separation; the tendency of the two bodies to approach one another is the elastic tendency of the Ether to recover its original condition; and phenomena of electric attraction and repulsion may be explained as phenomena of **Ether-stress.**

Two masses of resin rubbed on glass are found to repel one another; two masses of glass which have been rubbed with resin also repel one another; in other words, two masses in a similar electric condition generally repel one another.

According to the nature, the size, the dryness, of the pieces of material exposed to mutual friction, and according to some other circumstances, it is found that after friction and separation the force of mutual attraction or repulsion of two electrified bodies varies. One body may thus be more or less highly electrified than another; it is said to possess or be **charged** with a greater or a less **quantity** of electricity.

Two bodies are said to be **equally charged** or to be charged with equal quantities of electricity when (being of the same size) they can precisely replace one another in their action upon other electrified bodies.

When two equally electrified bodies, at a mutual distance of one centimetre, attract or repel one another with a force which balances one dyne, they are each said to be charged with a quantity equal to one C.G.S. Electrostatic **Unit of Electricity**. If one of these bodies, thus said to be charged with a unit of electricity, be brought to an exact centimetre's distance from a body charged with an unknown quantity of electricity, the force between the two electrified bodies may be measured directly; and if it be equal to n dynes, the body tested is shown to bear a charge of n units of electricity. Further, if a body bearing m units be brought to the same distance from a body charged with n units, the force between them will be equal to $m \times n = mn$ dynes.

A piece of glass, after being rubbed with resin, is said to bear a charge of **vitreous** electricity; the resin, on the other hand, is said to be charged with **resinous** electricity. If any body become electrified in any way, it must become either vitreously or resinously electrified.

Similarly-electrified bodies **repel** one another; **dissimilarly**-electrified bodies **attract** one another; these statements being, when the bodies are very near one another, subject to an exception hereafter to appear (p. 550).

When a jet of water issues from a metallic nozzle connected with an electric machine, the particles of the issuing stream, being similarly electrified, repel one another, and the jet is broken up into spray. When the nozzle has a capillary orifice, the surface-tension at the aperture is overcome by the electric self-repulsion, and the liquid rapidly issues as if its viscosity were greatly diminished.

If a body charged with resinous electricity and one equally charged with vitreous be brought into contact, the charges of both apparently disappear and the bodies resume a **neutral state.** Vitreous and resinous electricities are thus found to bear to one another the same relation as positive and negative quantities in algebra, and by a purely arbitrary convention charges of *vitreous* electricity are said to be **positive,** and *resinous* **negative.**

The above statements are comprised within the statement that if f be the force of repulsion between two charges of electricity, these charges being m units in one body and m' units in another, and d the distance between them, $f = k.mm'/d^2$; and if our units of quantity be so chosen that $k = 1$, $f = mm'/d^2$. If m and m' be both positive or both negative, the product mm' is positive, and the stress is expansive or repulsive; while, if one of the charges be resinous and the other vitreous, mm' is negative, and the stress is such that the bodies appear to attract one another.

When an electrified body presents a charge of m units uniformly distributed over a superficial area of s sq. cm., its charge per sq. cm. is $m/s = \sigma$, the so-called **Superficial Density** of the Electric Charge.

If the distribution be not uniform, the density over any minute area may be expressed as the ratio of the charge borne by that area to the area itself.

The density of a charge may be increased by diminishing the free surface of the charged conductor.

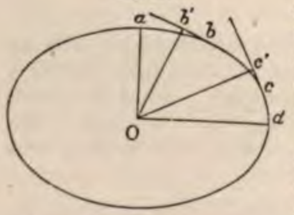
A piece of tinfoil charged and connected with a gold-leaf electroscope (p. 553) will cause a divergence in the leaves of that electroscope, which increases when the tinfoil is partly rolled up. Such an arrangement is called a Condenser.

When charged aqueous particles coalesce to form raindrops, their free surface diminishes, and the density of their charge increases.

The superficial density of a charge borne by a conductor varies from point to point, according to the form, but independently of the material, of the conductor. Only upon a sphere is it uniform.

A charge borne by an ellipsoid assumes at the points a, b, c, d (Fig. 197) densities proportional to Oa, Ob', Oc', Od; bb', cc' being tangents at b, c, and Ob', Oc' at right angles to these. The density at the extremities is thus greater than it is elsewhere. A needle-point resembles the extremity of a very elongated ellipsoid, and the density of a charge borne by a needle tends to be extremely great at the apex.

Fig. 197.



The density varies over a surface of any form in the same way as the thickness of a hollow shell of the same form would vary if that thickness were so adjusted as to produce, under the law of inverse squares, no interior effect.

A body cannot bear an indefinite charge of electricity; if the density be very great over the surface or at any part of the surface of a conductor, sparks will fly, generally from the point at which the density is greatest, either to surrounding objects or into the surrounding air.

When a body is charged with electricity, there is always an equal charge of the opposite kind of electricity somewhere: every distribution of electricity has a corresponding **complementary distribution** of an **equal** amount of electricity of the **opposite** kind. In the case of the mutually-rubbed pieces of glass and resin previously adverted to, the charges borne by the two masses are equal and opposite: when a single object, electrified by friction, stands within a room, the walls of the room are over their whole inner surface oppositely electrified, and bear a charge numerically equal to that of the electrified body. When an object is electrified in the open air, the earth itself (together with the heavenly bodies) takes up an equal and opposite charge; and thus the **algebraical sum** of the positive and negative electricities in the universe is constantly equal to **zero.**

This doctrine is by some writers (Lippmann, Silvanus Thompson) called the Law of the Conservation of Electricity.

If two bodies equally and strongly charged with opposite electricities be brought sufficiently near one another, a **spark** will pass between them, their electricities will combine, and they will be discharged and return to the neutral state. Sparks will pass, as a rule, when two bodies differing greatly in their electrical condition are brought sufficiently near one another.

The thickness of air across which a spark can leap is known as the Striking Distance in air; and it depends in general upon the nature of the substance through which the spark passes, as well as, in particular cases, upon the density of the charges at the points from which the sparks leap. The hotter the charged bodies, the greater the striking distance.

An electric spark is disruptive in its effect; it heats air; it produces sound and light; in water it may jar the liquid and shatter the containing vessel; it can pierce glass, and will scatter but not ignite gunpowder.

For many purposes of calculation it is convenient to feign a distribution of positive or of negative **imaginary electric matter** in the place where the electric condition manifests itself. For example: a metallic sphere or hollow globe, when electrified, presents no electric phenomena within its substance or its cavity; its electrical condition is manifest at its surface and is there uniform; whence it may be imagined that a uniform film of electric matter covers or coincides with the surface of the metallic body, and this imaginary film attracts or repels equally imaginary films of matter distributed over the surfaces of neighbouring electrified bodies, and does so with forces whose amount may be calculated in accordance with the propositions of the section on Attraction, p. 176.

The law of the resultant force resembles that of Gravitation. Every particle of this imaginary electric matter in the Universe repels every other existent, at any given moment, with a force proportional to the product of the masses, and varying inversely as the square of the distance between them.

From this ensue the following propositions:—(1.) An electrified metallic sphere acts upon all external electrified particles as if its charge were concentrated at its centre.

The density of its surface distribution, $\sigma = \text{charge} \div \text{area} = \text{charge} \div 4\pi r^2$.

Its attraction for or repulsion of a Unit of Electricity placed just outside it is equal to $4\pi\sigma$; this is F, the **Resultant Electric Force**; and the resultant electric force, at a point just outside an electrified sphere over which the density is σ, is equal to $4\pi\sigma$.

(2.) A superficial distribution of electricity presents from point to point such variations of density that it has no action upon particles within it. A metallic chamber of any form may be electrified until sparks fly from its outer surface; yet no electrical effect will be perceived internally. This is the strongest proof of the law of the inverse square that can be imagined: no other law of attraction could result in a force imperceptible internally, the distributions being such as those actually observed.

(3.) Every element of the imaginary superficial shell or film itself is subject to repulsion from its fellow-elements. This repulsion amounts, per unit of area, to $p = 2\pi\sigma^2$. A soap-bubble when electrified expands; the atmospheric pressure is resisted by an electric self-repulsion or so-called Electric Tension over the surface, whose outward resultant is equal to $2\pi\sigma^2$ dynes per sq. centimetre. A soap-bubble may be electrified by blowing it on a metallic pipe, and connecting the pipe with an electric machine.

From this we must conclude that the surface of every electrified body is in a state of expansive tension, and that the film of air in contact with it is subject to a disruptive tendency, which varies as the square of the superficial density.

When a body is very highly charged, the air in its immediate neighbourhood becomes similarly charged, is repelled, and masses of it are torn off and repelled in constant succession, an electrical wind or stream of electrified air being produced.

Sparks fly from the surface of a charged conductor into the surrounding air, when the density becomes so great that the absolute diminution of air-pressure due to expansive tension is about 66,708 dynes per sq. cm.

Since $p = 2\pi\sigma^2$, and the Resultant Force just outside a charged conductor is $F = 4\pi\sigma$, it follows that $p = F^2/8\pi$.

Dust floating in the air between two charged surfaces finds its way to one or the other, and if sticky, agglomerates (Lodge). "Thunder clears the air."

If an electrified body be "insulated" by being placed on a dry glass stand within a room, the walls of the room are oppositely electrified, and bear a complementary charge numerically equal in the aggregate to the charge of the insulated body. The space comprised between the electrified body and the oppositely-electrified walls of the room is a **Field of Force,** permeated by **Lines of Force** and **Equipotential Surfaces.** The lines of force traversing such a field quit the free surface of the insulated body at right angles, and strike the walls of the room, again at right angles. They are, in general, of a curved form. A certain number of lines of force may be grouped within a bundle or **Tube of Force,** whose cross-sectional area increases as the lines of force diverge from one another, or diminishes as they converge; and F, the resultant force on a unit of electric quantity placed within any such tube, must vary inversely as the local cross-sectional area of the tube. If the tubes of force be constant in cross-sectional area, the lines of force are parallel to one another and the equipotential surfaces are equidistant and plane; the field is then a **Uniform Field of Force.**

Such a field we find in the central part of the space between two parallel plates insulated from one another and brought to different potentials.

The conception of **Potential** is one of the highest importance in the theory of Electricity.

The **Absolute Electrical Potential** at a point is a mathematical expression, possessing a numerical value: it measures the tendency which the existing electric forces would have to drive an electrified particle away from or to prevent its approach to the point in question, if such a particle, one unit in quantity, were situated at that point or were brought up to that point; and it is numerically equal to the number of ergs of work that must be done in order to bring a positive unit of electricity from a region where there is absolutely no electric force—*i.e.*, from a region at an infinite distance from all electrified bodies—up to the point in question; provided always that the transfer of the positive unit of electricity be supposed to have no effect whatsoever upon the distribution of the electricity of other bodies in the neighbourhood of that point.

Difference of potential between two points.—If m ergs of work must be done in order to remove a unit of electricity from the point A to the point B against electric repulsion, then the two points A and B are at potentials which, considered absolutely, may be unknown, but which differ numerically by m: and B is at a higher potential than A by m units of potential.

When there is a difference of potentials between any two points in space, a body bearing a charge of positive electricity, and placed at the point at which the potential is greater, is driven towards the point of less potential, just as in the corresponding gravitation-problem, a mass tends to fall towards a lower level; and if free to move it will follow the track of the lines of force, travelling thus from each equipotential surface to the next one, infinitely near it, by the shortest path. The path between the two points is not necessarily the shortest, for the lines of force are often curved (see Fig. 241).

A positively-charged particle placed in a region of positive potential will be repelled along the lines of force into a region of less or of zero or of negative potential: a negatively-charged body under the same circumstances travels in the opposite direction.

The mean force acting upon a unit-charge of electricity within an electrical field is equal to the difference between the potentials of two points within that field, and situated at a mutual distance of one centimetre, that distance being measured along the lines of force: for if $V_{,}$ and $V_{,,}$ be the potentials of two points whose mutual distance is s, the work done in moving a unit of electricity from the point of lower to the point of higher potential is $V_{,} - V_{,,}$; but it is also equal to Fs, where F is the mean force resisting the transfer; whence $Fs = V_{,} - V_{,,}$ and $F = (V_{,} - V_{,,}) \div s$. When $s = 1$ cm., $F = V_{,} - V_{,,}$.

We have now two expressions for the force acting upon a unit-charge placed within a field of force. These are $F = (V_{,} - V_{,,}) \div s$ anywhere within the field, s being the distance (measured along a line of force) between two equipotential surfaces whose potential differs by $(V_{,} - V_{,,})$; and $F = 4\pi\sigma$ near the surface of either of the charged conductors whose surfaces bound the field. In a uniform field the potential diminishes equably, and F is constant throughout the field. In a non-uniform field the force, which at the smaller bounding surface is $4\pi\sigma$, diminishes as A, the local cross-sectional area of any tube of force, increases (for $F \times A = F_{,} \times A_{,,}$); but the divisor in the other expression—s, the distance between successive equipotential surfaces—increases in the same proportion; thus at any point the local force $F_{,} = 4\pi\sigma . A/A_{,} = (V_{,} - V_{,,})/s$.

The local force may also be defined as the reciprocal of the distance (measured along the lines of force) between two surfaces whose potential differs by unity; for $F = (V_{,} - V_{,,}) \div s$; but when $(V_{,} - V_{,,}) = 1$, $F = 1/s$.

Since under the law of inverse squares the potential due to repelling mass Q at distance r is Q/r, and at distance r' is Q/r', the difference of potentials at points situated at distances r and r' respectively from the repelling mass Q is $\left(\frac{Q}{r}-\frac{Q}{r'}\right)=\frac{Q\,(r'-r)}{rr'}$: and since the difference of potentials of any two equipotential surfaces is numerically equal to the work done in transferring a unit-charge from the surface of lower to the surface of higher potential, it follows that the work done by or on a unit-charge, on its moving or being moved from distance r to distance r' from a charge Q, is $Q\,(r'-r)/rr'$, positive or negative as the case may be.

If a body charged with electricity be not free itself to move along the lines of force, we find this most remarkable phenomenon —that in a field of force, the points of which corresponding to the extremities of the body are at different potentials, the **electrical condition** of the body tends to **travel**: one aspect of the charged body—the aspect, namely, which looks towards that direction in which the charged body would itself travel if it were free to do so—tends to become more strongly charged or to acquire a greater density; the opposite aspect tends to become less strongly, or, it may be, even oppositely charged. This redistribution of the electric charge, if it take place, has the effect of equalising the potential throughout the body placed within the field of force; and it reminds us of the readjustment of **level** and accumulation of **water** towards the lower end of a tank laid on a sloping surface, during which readjustment a difference of level produces a flow of water. The new distribution once assumed is permanent so long as the field of force which immediately surrounds the body, and which tends to determine a difference of potential between its opposite aspects, remains unchanged; but while that distribution is being assumed we have a brief **Current** of electricity.

A difference of potential, in whatever way it may be set up within a body, produces a tendency to prompt equalisation of potential throughout that body, and thus to the establishment of a momentary current of Electricity; a permanent difference of potential, in whatever way kept up, tends to produce a continuous current.

A lightning conductor is the seat of a continuous current so long as the earth at its base and the air at its apex are at different potentials.

Difference of potential is analogous to difference of level or Head of Water in hydraulics; and when it determines a flow of electricity, it is often called electromotive force or E.M.F. —a term which might with advantage be abandoned, and instead of which we shall use the phrase electromotive difference of potential or E.M.D.P. In place of this phrase the reader who has any reason for doing so will easily read the words Electromotive Force, the objection to which is simply that a difference of potential, like a difference of water-level, is not itself a force, and does not even completely specify the force which determines an electric flow: to determine this the form and the dimensions of the electric conductor must be known, as well as the difference of electric potential between its extremities, just as the dimensions of a water-pipe must be known, as well as the available head of water, before we can calculate the local falls of pressure and the forces producing flow.

We have already seen that $F = (V_{\prime} - V_{\prime\prime}) \div s$; and this is Clerk Maxwell's "Electromotive Intensity," the Force acting upon a unit charge of electricity.

If two bodies be at different potentials, when they are connected by a metallic wire the charge over them will be readjusted by a momentary current through the wire, and they will come to the same potential.

Two bodies are said to be at the same potential when electricity has no tendency to travel from one to the other, even though they be brought into communication by a metallic wire. Difference of potential is thus also analogous to difference of temperature.

The **earth** itself is arbitrarily assumed to be at **zero** potential: and bodies in such a condition that when they are placed in contact or in metallic communication with the earth their electric condition is unaltered, have a potential whose value is equal to this arbitrary zero.

The arbitrary or conventional potential—or, briefly, **The Potential** of a point in an electric field of force—is, numerically, the number of ergs of work necessary to bring a unit of electricity up to the point in question from a region of nominal zero-potential —*i.e.*, from the surface of the earth.

Between a positively-charged body within a room and the negatively-charged wall of the room there must lie, in the intervening field of force, one equipotential surface which has a Zero Potential, its potential being the same as that of the earth outside the room. Within this equipotential closed surface there is a region of Positive Potential; exterior to it there is a region of Negative Potential. The potential of the inner region is greatest at the surface of the electrified body; the potential in the negative region is most negative on the surface of the walls.

The potential cannot be a maximum or a minimum at any point within a field of force, if that point be not upon the surface of one of the conductors whose surfaces bound the field.

Conductors and Non-conductors.—In the familiar case of a lightning-conductor we see a marked distinction between the conductive copper in which a continuous current of electricity can flow, and the air or an unprotected building which can only be traversed by a disruptive discharge. A **conductor** is, when a charge is borne by it and retained by it in equilibrium, a substance throughout the whole volume and over the whole surface of which the potential is uniform; while if inequalities of potential were set up within it, the conducting material of a perfect conductor would offer no resistance to the readjustment of potential by means of a current. A perfect non-conductor or **dielectric** would, on the other hand, be a substance the different parts of which may, after an electric disturbance, remain, without any process of readjustment and for an indefinite period of time, at potentials differing to any extent. There are no bodies which are absolute non-conductors; all conduct electricity more or less slowly. There are no bodies which are perfect conductors; all offer more or less resistance to the flow of electricity. Bodies which conduct extremely badly are called Non-conductors or **insulators**: bodies which offer comparatively small resistance to the passage of electricity through them are in practice called Conductors.

When a charged body is placed upon an insulator, such as ebonite, guttapercha, indiarubber, dry glass, sealing-wax, it is said to be **insulated**; its potential cannot become equal to that of the earth for a long period of time; it is said to retain its charge for a long period.

Air at a high pressure is almost an absolute insulator: cold air, damp or dry, at the ordinary pressure is one of the best insulators: but even within cold air bodies charged with electricity gradually lose their charge; a partial vacuum is a good conductor; a good vacuum is again a good insulator. Ice insulates, water is a bad conductor; obsidians and lavas insulate when hot; glass when dry is an insulator, but when very hot is a conductor. A body charged and supported upon a dry-glass stem within a vacuum or a very dry cold atmosphere will retain its charge for a very long period; but if the air be damp, so that the insulating glass stem condenses upon its surface a film of moisture from the air, that film will slowly conduct the charge to earth.

A sufficient difference of potential will cause a spark to fly between two charged conductors across the intervening dielectric: in the case of turpentine, paraffin, and olive oil, the striking distance is, when the discharge is continuous regularly, when the discharge is interrupted irregularly proportional to the difference of potential; in air the striking distance increases faster than the difference of potential, and the curve indicating the ratios of striking distances to differences of potential is a parabola.

The phenomena of electricity present themselves within a conductor only while a current is actually passing through it; for then only are there any differences of potential within the conductor. At other times—that is, when there is no current, but a more or less permanent condition of Statical Equilibrium of the charge—electrical phenomena are restricted to the Field of Force—that is, to the non-conductor or Dielectric between two complementary charges; for within non-conductors alone, not in conductors, can any electrical stress or difference of potential, permanently or for any length of time, be maintained.

If the air had been as good a conductor as copper we would probably never have known anything about electricity, for our attention would never have been directed to any electrical phenomena.

Phenomena of electricity in a state of equilibrium, associated with more or less permanent differences of potential and evinced within a dielectric, are said to be **electrostatic**; those evinced during adjustment of electric potential by the passage of a current in a conductor are said to be **electrokinetic.**

If electrostatic phenomena be due to stresses in the ether, electrokinetic are due to movements of the same; and a momentary current of electricity in a copper wire is a throb due to release from stress of that part of the all-pervading ether which pervades that wire—a throb which is propagated not without influence, as we shall see, upon the ether external to the wire.

"Free" and "Bound" Charges.—A distinction is frequently made between a free and a bound charge of electricity. The former is understood to be a charge borne by an insulated body, and independent of surrounding objects, while the latter is such a charge as is held in position by the presence and attraction of a charge of the opposite character upon a neighbouring body. In truth, however, all charges are bound charges; the complementary distribution must be somewhere; the field of force may be great or small, but it must have its limits. It may be small, as when a little electrified body is suspended within a metal flask which is not insulated; it may be great, like the field of force between a thundercloud and the earth: in the former case the complementary charge is distributed over the inner surface of the flask; in the latter it travels about, and is at its densest upon the surface of the earth beneath the travelling thundercloud, or else upon adjacent clouds. Even when an electrified body is placed at an extremely great distance from all surrounding objects, it cannot be held to have a free charge, for its charge is bound by the complementary distribution upon the far-distant objects; and a particle isolated in otherwise vacuous infinite space, if such a thing were possible, could not become charged with electricity at all, for the complementary charge could, in such a case, have no locus.

If the ether be stretched or compressed, it must be stretched or compressed between at least two points, which may be near or far from one another. Bearing this in mind, however, it is undoubtedly convenient in many respects to permit ourselves the use of such expressions as "a body freely charged with m units of + electricity," and in so doing to omit, provisionally, all consideration of the complementary charge, which is supposed sufficiently distant.

Division of Charge.—When a conductor charged with electricity is brought into contact with another at a different potential, the electric potentials of the two conductors become equalised; and if the two bodies be of the same form, size, temperature, and chemical nature, and if they be symmetrically arranged, they will, after separation, each bear a charge equal to one-half of the algebraical sum of the original charges of the two bodies.

This change of distribution involves a readjustment of the lines of force and of the equipotential surfaces throughout the surrounding dielectric, and an alteration of the distribution of the complementary charge over its opposite boundary.

When the bodies are unequal in size, etc., or are unsymmetrically arranged, the division of the charge between them is not equal. Two similar but unequal spheres are found, after being brought into communication by a long thin wire, which is then removed, to bear charges proportioned to their radii.

Capacity.—When a conductor has a charge of electricity imparted to it, the potential of its surface and of its whole volume is raised, positively or negatively (*i.e.*, lowered), as the case may be. When an insulated body requires a charge of C units of electricity to be imparted to it in order to raise its potential by one unit—that is, from zero to unity, or from V to V + 1—it is said to have a Capacity of C units. When a body whose capacity is C has its potential raised by the amount V, the Charge of electricity imparted to the conductor is $Q = VC$ units.

When a series of conductors—whose capacities are $C_{,}$, $C_{,,}$, $C_{,,,}$, etc., and which are charged to potentials $V_{,}$, $V_{,,}$, $V_{,,,}$, etc., so that their several charges are respectively $C_{,}V_{,}$, $C_{,,}V_{,,}$, $C_{,,,}V_{,,,}$, etc.—are connected by a wire, the potential thereupon assumed by the whole system is equal to—

$$\frac{\text{The whole charge}}{\text{The whole capacity}} = \frac{V_{,}C_{,} + V_{,,}C_{,,} + V_{,,,}C_{,,,} \dots}{C_{,} + \quad C_{,,} + \quad C_{,,,} \dots} = \frac{\Sigma VC}{\Sigma C.}.$$

The capacity of a conductor is the same whether it be solid or hollow: the merest film of gold leaf supported on a wooden ball has as great a capacity as a solid metallic sphere.

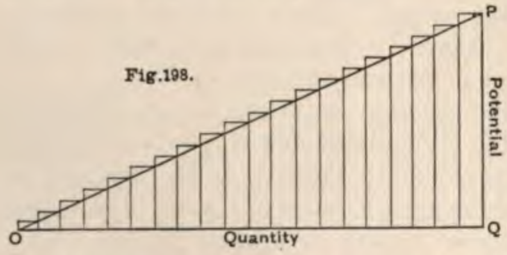
Electrostatic stress can only persist within the field of force, the dielectric, which is limited by the surface of the conductor; beneath this surface it is a matter of indifference what the metallic thickness may be, since within a conductor there can be no permanent difference of potential, no permanent electrostatic stress.

The Capacity of a Sphere.—A sphere of radius r, within an unlimited space containing no other charged bodies, is charged with quantity Q; this quantity, uniformly distributed over the surface, acts as if it were gathered at the centre, and therefore at a distance r from the surface. The potential over the surface of the sphere must therefore be $V = Q/r$. The capacity $C = Q/V$; this is $Q/\overline{Q/r} = r$; the electrostatic Capacity of a Sphere is therefore equal to its Radius.

The **Work** spent in charging any conductor is equal to half the product of Q, the Charge imparted to it, into V, the Rise of Potential produced in it.

To bring a quantity, Q, of electricity from a place of zero potential to a place of constant potential, V, involves the expenditure of QV units of work, by our definition of Potential. To bring a charge Q by successive instalments into a region whose potential is at first zero, but steadily rises as the successive instalments arrive, work must be done which is equal to **half** the product of the final rise of potential into the whole charge brought up. In Fig. 198 the small rectangles represent the work done in bringing each successive instalment up to the corresponding potential; these rectangles increase, and their sum, which represents the total work done in bringing up the whole charge OQ to the final potential OP, is represented by the triangle OQP, whose area $= \frac{1}{2}OQ \times OP = \frac{1}{2}$ whole quantity × final rise of potential $= \frac{1}{2}QV$.

Fig.198.



The work done in charging a conductor is equal to the electrical energy stored up in that conductor; and since this is equal to $\frac{1}{2}QV$, we see that the **energy** of an electrified body depends not only upon the Quantity of electricity borne by that body, but also upon the Potential; just as the potential energy of a mill-pond depends not only upon the quantity of water contained in it, but also upon the average elevation of that water above surrounding objects. For which reason a mere quantity of Electricity is not a quantity of Energy, and Electricity is not a form of Energy.

The energy of a charged conductor of any kind is measured by $\frac{1}{2}QV$; but this is equal (since $Q = CV$, where C is the capacity of the conductor) to $\frac{1}{2}CV^2$ or to $\frac{1}{2}Q^2/C$.

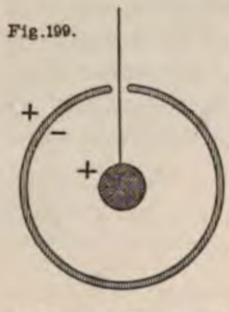
The energy of a system of connected conductors is equal to $\frac{1}{2}V^2 . \Sigma C$, or to $\frac{1}{2}Q^2 \div \Sigma C$, where ΣC is the aggregate capacity of the whole system.

Suppose now that two conductors, of which the one is charged to potential V, while the other is at zero potential, and of which the respective capacities are $C_{,}$ and $C_{,,}$, are placed in metallic communication; on contact they form a joint conductor whose capacity is $(C_{,} + C_{,,})$. The energy of the single charged conductor was $\frac{1}{2}Q^2/C_{,}$; that of both taken together is $\frac{1}{2}Q^2/\overline{C_{,} + C_{,,}}$, a smaller quantity. There is therefore an apparent Loss of Energy equal to $\left\{\frac{1}{2}Q^2/C_{,} - \frac{1}{2}Q^2/\overline{C_{,} + C_{,,}}\right\} = \left\{(\frac{1}{2}Q^2/C_{,})\ (C_{,,}/\overline{C_{,} + C_{,,}})\right\}$, or $(C_{,,}/\overline{C_{,} + C_{,,}})$ times the energy of the original charge. If $C_{,} = C_{,,}$, half the energy of the charged conductor is apparently lost by partial discharge.

Wherever there is a readjustment of electricity in the form of a running-down of electricity from a place of high potential to a place of low potential, there is a loss of energy of electrification; just as when a full pond is allowed partly to discharge itself into an empty one, the average level of the whole is lowered, and the energy of position partly disappears, to reappear in the form of heat. In general, where electrified conductors are connected by metallic wires, if there be a current, the potential energy of the system sinks to a minimum; heat and—if a spark pass—light, sound, and mechanical effect being produced. Where the components of an electrified and insulated system are allowed to approach or to recede from one another in obedience to the electric forces, the energy of electrification becomes in part converted into mechanical work, and therefore falls in amount; while if they be pulled asunder or made to approach against the electric forces, the mechanical work done upon the insulated system from without is converted into the energy of a higher electrification. In the former case the energy is that of the same charge at a lower potential; in the latter case it is that of the same charge at a higher potential.

Induction.—When an electrified body, or a system of such

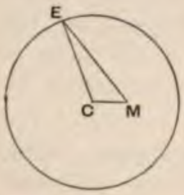
Fig. 199.



bodies, is placed within a hollow metallic shell (Fig. 199) with which there is no communication except through non-conductors, the shell becomes charged by Induction across the intervening dielectric. If the bodies placed within the shell be positively charged, the inner surface of the shell becomes negatively, the outer positively electrified. The opposite charge thus induced on the inner surface of the shell, the similar charge induced upon its outer surface, and the original inducing charge on the internally-suspended system, are all equal in amount, if the shell completely or even if it very largely surround the electrified body suspended within it. Thus the positive and the negative charges called into existence by Induction are together algebraically equal to zero.

The distribution of the induced charge on the interior surface of a completely-surrounding shell is such that on external points it produces an effect equal and opposite to that of the interior insulated charge; the two inner charges therefore produce together no effect upon external bodies, and the induced charge on the outer surface is the only charge which can affect particles situated in the outer air. The two interior charges are bound to each other, for they are of opposite character, and there is a field of force between them; the outer charge is said to be free.

Fig. 200.



The distribution of electricity over the inner and outer surfaces of the shell is, if the shell be spherical, governed by the law that the superficial density σ at any point E is $\sigma = (\mathrm{CE}^2 \sim \mathrm{CM}^2) \,.\, \mathrm{Q}/\overline{4\pi . \mathrm{CE} . \mathrm{ME}^3}$, where C is the geometrical centre of the shell, and M the point at which the charge Q is situated. Such being the law of distribution, the inner charge Q at M, and the opposite charge − Q on the surface (the inner surface of the shell), produce together no effect on surrounding particles.

If the outer surface of the shell be connected with the surface of the earth, the shell and the earth become one extended conductor, and the positive charge on the outer surface of the shell is repelled to the earth's surface; it now blends with and neutralises the negative charge previously borne by the earth in consequence of the original positive electrification of the inducing body. As a result of this we have, within the shell, a purely local field of force, restricted to the space between the internally-suspended body and the interior surface of the shell, and giving rise to no phenomena outside that cavity.

If, on the other hand, the insulated body within be made to touch the enveloping shell, the internal field of force will be destroyed; but the outer induced charge will remain, distributed over the outer surface of the shell.

Any quantity of electricity may thus be wholly transferred to the surface of a hollow insulated conductor, if a charged body be made to touch its internal surface.

A sheet of tinfoil charged, and separated from a second sheet by an intervening layer of air or glass or mica or waxed paper, will act inductively across the dielectric. The nearer surface of the second sheet is oppositely, the farther surface similarly charged; and if the second sheet of tinfoil be connected to earth, the similar charge escapes and the field of force is now almost wholly limited to the thin space between the two sheets or plates.

Two oppositely-charged conducting surfaces thus separated by a dielectric or limited field of force constitute an **Accumulator,** or, as it is often termed, a Condenser. The capacity of an accumulator is greater than that of either or both of the conducting surfaces of which it is made up.

Suppose an accumulator to be made up of an inner conducting sphere (solid or hollow) of radius R, and, concentric with the sphere, an outer shell whose inner spherical surface has a radius r. Let the inner electrified sphere bear a charge Q; this charge acts as if it were concentrated at the centre and produces a potential, $\mathrm{V} = \mathrm{Q}/\mathrm{R}$, which is the same throughout and over the surface of the inner sphere. Similarly, the potential at any point within the outer shell, due to the charge $-\mathrm{Q}$ upon its inner surface, is $-\mathrm{Q}/r$. The outer surface of the shell may or may not have a free charge. In any case, the potential of the joint system is the excess of its internal over its external potential. If there be no charge on the external surface, the external potential is zero. The total internal-potential is then simply the algebraic sum of the several potentials $+\mathrm{Q}/\mathrm{R}$ and $-\mathrm{Q}/r$. This sum is equal to $(r-\mathrm{R})\,\mathrm{Q}/\mathrm{R}r$. The Capacity of the Accumulator is $\{\text{Quantity} \div \text{Potential}\} = \mathrm{Q} \div \{(r-\mathrm{R})\,\mathrm{Q}/\mathrm{R}r\} = \mathrm{R}r/\overline{r-\mathrm{R}}$; this is greater than the capacities of either of the two component spherical-surfaces, these capacities being respectively R and r.

The smaller the difference between r and R—that is, the thinner the dielectric between the two metallic surfaces—the greater is the capacity of the accumulator, and correspondingly, the less will be the potential to which a given charge will raise it.

The **nature of the dielectric** between the plates of the accumulator is not a matter of indifference. It is found—and this proves that in the phenomena of electric attraction the dielectric plays an important part—that the capacity of an accumulator varies with the nature of the interposed dielectric, and is proportional to a constant special to each substance, and called th **Specific Inductive Capacity** of that substance. A thickness c 3·2 mm. of sulphur is thus equivalent as a dielectric to a thicknes of 1 mm. of air, and the sp. ind. cap. of air being taken as standard and equal to unity, that of sulphur is 3·2. Sometime the sp. ind. cap. of a vacuum, which differs very little from tha of air, is taken as unity, in which case the dielectric is the ethe itself. The sp. ind. cap. of glass rises slightly when the temperature is increased (between 12° and 83° C., a rise of 2½ per cent All gases have very nearly the same inductive capacity, whatever their chemical constitution, their temperature, or their density. If, however, their pressure be increased or diminishe the minute difference between their sp. ind. cap. and that of vacuum is also increased or diminished in the same proportion and conversely, when a gas is employed as a dielectric, inductio across it diminishes its pressure, the gas then adjusting itself s as to become rarer and consequently less inductive.

If a given charge will raise an accumulator in which air is the dielectri to a potential V, it will only raise a similar accumulator whose dielectric h sp. ind. cap. $= k$ to the potential V/k; and since in the special case of a co ducting material used in the place of a dielectric the difference between th inner and outer coats is zero, the sp. ind. cap. of a conductor may be considered infinite, for $0 = 1/\infty$.

The sp. ind. cap. of a dielectric diminishes with the time, and is therefor difficult to measure directly; and when an accumulator is discharged b metallic communication set up between its two coatings, its charge does n at once completely vanish, but the condition of the dielectric is apparentl very similar to that of a body which, being imperfectly elastic, recover slowly and irregularly its primitive form and condition after deformation and it is curious that the same means—vibration, shaking, jarring, etc.— which facilitate the return of such a body to its normal condition after strain, facilitate the prompt and complete discharge of an accumulator whos two coatings are kept in metallic connection. On sending alternate charg into a condenser, the residual discharge liberates them in the reverse ord (Hopkinson); a result strikingly like that of Boltzmann with reference successive torsions. Quartz employed as a dielectric has one-ninth the res dual capacity of glass; Iceland spar seems to have no residual capacity at a and permits prompt discharge.

The dielectric of an accumulator may become double-refracting under th influence of Electric Stress, which tends to dilate it at right angles to th lines of force: its optical axis is parallel to the lines of force. Solids slowl liquids instantly, acquire or lose this condition of stress.

The capacity of a spherical accumulator in which the dielectric has a s ind. cap. $= k$ is $k . Rr/\overline{r - R}$. If the two radii be very nearly equal, this may taken as equal to $k . r^2/t$ or $(k/t) . r^2$, where t is the thickness of the dielectri and this is equal to (k/t).(Surface$/4\pi$). This last is the general formula f the capacity of a Leyden jar.

The form of accumulator known as a **Leyden jar** usually consists of a glass vessel lined internally and externally with tinfoil. The inner coating communicates by wire with a smooth metallic knob projecting externally and insulated from the outer coating. By contact between the knob and a charged conductor the inner coating is charged. By induction through the glass there is produced an electrical separation in the external tinfoil. The external surface of this is temporarily connected with the earth. Thereafter there remains a field of force in the glass between the two tinfoil coatings. This may be discharged by establishing a metallic communication between the two coatings, the outer tinfoil being first touched, then the inner.

A Leyden jar when charged dilates somewhat, and as it expands its capacity increases; the potential, to which a given charge is competent to raise the jar, sinks to a corresponding degree. When discharged, the jar makes a dull sound, and the glass glows at the edges of the tinfoil, while the internal air becomes warm.

A submarine telegraph cable is, in effect, a very long Leyden jar. The copper core is the inner coating; the guttapercha or other insulator represents the glass; the outer coating of tinfoil is represented by the protecting iron wire or by the bounding surface of the sea-water. When a charge of electricity is passed into a deep-sea cable, the cable takes some time to become fully charged: it then bears an electrostatic charge upon the surface of its copper core.

An ordinary aërial telegraph-wire is again, but to a less marked degree, a Leyden jar. The inner coating is the surface of the wire itself; the dielectric is the air; the outer coating is the surface of the earth. The electrostatic capacity of an aërial wire is small in comparison with that of a submarine cable; but it is not insignificant.

If the two coatings of a Leyden jar be slid past one another so as to diminish the opposed surfaces, the capacity diminishes and the potential due to the actual charge of the jar increases: the potential may thus be adjusted (Sliding Condenser).

Batteries of Leyden Jars.—When a Leyden jar has its inner coating placed in simultaneous metallic communication with the inner coats of a series of uninsulated jars, the whole becomes in effect one great Leyden jar of increased surface, and the jars are said to form a battery connected in Surface. The charge of the first jar, being then distributed throughout an enlarged conductor, brings it to a reduced potential; and energy is lost in the production of sparks when the battery is charged by the first jar.

A series or battery of Leyden jars is said to be charged in Cascade when the outer coat of one jar is connected by metal with the inner coat of the next, and so forth, while a charge is imparted to the inner coating of the first. The difference of potential between the inner coating of the first jar and the outer coating of the last is distributed between the jars of the battery, and thus the risk is diminished of any of the jars being destroyed through an excessive difference of potential in any one jar causing a spark to pass and perforate the glass. The charge of the whole system is only equal to that of a single jar, and the difference of potential in each of n jars is $(V_i - V_o)/n$, where $V_{,}$ and V_{0} are the potentials of the first and the last coatings ; whence the energy of the whole (=charge × potential-difference) is the *n*th part of the energy of a single jar loaded with the same charge as the battery.

If the conductors surrounding an inducing charge do not completely enclose it, the charges induced upon them are each numerically less than the inducing charge, and the sum of those of each kind is also numerically less than that charge. In no case can the induced charges exceed the inducing charge.

Coefficient of Mutual Induction.—The Coefficient of Induction of a conductor A on a conductor B is the ratio of the Charge (or change of charge) developed in B to the Potential of A. It can be proved that the coefficient of induction of A on B is always equal to the converse coefficient of B on A ; and this reciprocally valid coefficient is called the Coefficient of Mutual Induction. It depends upon the relative positions of A and B.

Inverting the statement, a unit charge on either body will, by induction, alter the potential of the other by an amount equal in both cases.

The effect of induction is seen when an electrified body—such as a glass rod rubbed with a dry silk-handkerchief—is brought into the neighbourhood of light bodies suspended or floating in the air. Over each of these bodies there is a separation of electricities; the aspect nearer to the inducing body is charged with electricity of the opposite kind, and is attracted; the farther aspect is charged with electricity of the same kind, and is repelled, but to a less extent, because it is more distant; its charge not being "bound" is, besides, more readily dispersed into the surrounding air. On the whole, these light bodies are attracted. If they come in contact with the inducing body, they acquire a part of its charge, and are thereupon repelled.

As another effect of induction we find that while two similarly-charged bodies at the same potential within the same field will always attract one another, yet if they be not at precisely the same potential, the one of higher potential will, by its presence, alter the distribution of electricity over the other, the weaker, in such a sense that the weaker one may even become oppositely charged over the nearer aspect, and the attraction of the more highly-charged body for this side of the weaker may prevail over its repulsion of the farther side ; and on the whole, two such bodies will, if they be placed at a sufficiently small distance and if no spark pass between them, attract one another. In a certain intermediate position there will be unstable equilibrium, and at all greater distances there will be repulsion.

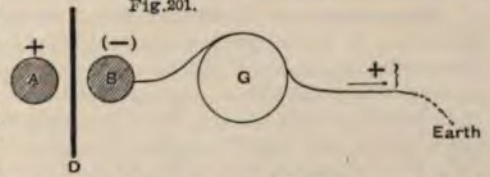
When a conducting body is brought into the neighbourhood of a system of insulated and charged conductors, the energy of that system falls, for the interposed body causes by its presence a redistribution of the charge of the system; and such a redistribution of the charge causes a fall of the potential and therefore of the energy of the system. If the body introduced be a dielectric, the effect produced is similar but smaller.

Electric Screens.—A conducting sphere surrounding an insulated electrified body and connected with the earth will, as we have seen, shelter an external particle from the inductive action of the enclosed electrified body; and conversely it will shelter the internal electrified body from the distribution-disturbing and potential-lowering influence of the outside particle. A screen of perforated tinfoil or a cage of wire gauze has nearly an equal effect: such screens are used to protect delicate instruments from the inductive effect of external electrified bodies.

The place of an enveloping sphere may be taken by a plate of metal connected with the earth. If the diameter of this be infinite—or practically, if it be very great as compared with the distance between the electrified and the protected particle—the screening action will be perfect.

In Fig. 201, A is an insulated body positively charged by a galvanic battery or a frictional electric machine; D is a large metallic screen; B is a metallic body connected by a wire with the earth; this wire passes round the magnetic needle of a Galvanometer, G; the screen D is suddenly removed: there is a sudden separation of electricities in B: a positive charge escapes round the galvanometer and deflects its needle by an instantaneous twitch.

Fig. 201.



The phenomena of electricity in equilibrium are very similar in their mathematical aspect to those of the steady flow of Heat; equipotential surfaces represent isothermal surfaces; lines of force represent lines of flow of heat; specific inductive capacity takes the place of conductivity.

Again, the calculation of the variation of the force throughout a dielectric field resembles very closely that of the distribution of the flow in a steadily-flowing mass of fluid; just as the stream lines in a field of liquid-flow may be held to exert lateral pressure upon one another, so do the lines of force in an electric field laterally repel one another, as is specially manifest at the surface of a conductor, where the elements of charge repel one another; in each of the tubes of force or tubes of flow the product of the force or of the flow into the cross-sectional area is constant (Law of Continuity in Hydrodynamics): and the energy per unit of volume in a field of force or of flow is at each point numerically equal to the electrostatic or hydrostatic pressure per unit of area at that point.

In Faraday and Maxwell's theory of Ether-stress, the flow of charge across an electrified surface is insisted on: this flow occurs whenever a separation of electricities takes place. When a thin insulated sheet of tinfoil is exposed to the inductive influence of a charged conductor, there is a separation of positive from negative charge across the conductor influenced, and on each side there is a charge induced whose density is σ. The quantity of electricity thus induced to flow in either direction is, over any given area A, equal to $A.\sigma$: this quantity is equal to $A.F/4\pi$, since $F = 4\pi\sigma$. If any other dielectric than air intervene between the inducing charge and the conductor acted upon, the quantity of flow, the so-called **Electric Displacement,** is $k.A.F/4\pi$. A conductor offers no permanent resistance to the displacement of an electric charge through it, and as long as there is maintained between the extremities of a conductor a permanent difference of potential, so long will the electric displacement produced be continuously relieved by an electric flow or Current; but in a non-conductor the extremities may remain under a permanent difference of potential, a permanent stress or state of polarisation, for the Electric Displacement, the flow set up in the first instant of exposure to electric stress, is arrested by a certain Electric Elasticity of the Dielectric, which, being represented by the fraction $\frac{\text{electric stress}}{\text{electric strain}} = \frac{\text{force acting across each } \textit{unit} \text{ of area}}{\text{quantity of flow across each } \textit{unit} \text{ of area}} = F \div (k.F/4\pi) = 4\pi/k$, is inversely proportional to k, the specific inductive capacity of the dielectric. The energy of unit volume of the dielectric is the product of half the displacing force into the displacement at the bounding surface; *i.e.*, it $= \frac{1}{2}F(k.FA/4\pi) = k.F^2/8\pi$ when $A = 1$; and this is numerically equal to p, the pressure across unit area of the bounding surface of the dielectric.

Dimensions of Electrostatic Measures.—The Absolute unit of Quantity of electricity is a quantity which, placed at a certain distance from a similar and equal charge, repels it with a certain force. The force between two quantities at a given distance is therefore equal to (Product of Quantities) ÷ (Distance²). The Dimensions of this expression are $[Q] \times [Q] \div [L^2]$; but the dimensions of a force are otherwise known to be $[ML/T^2]$; whence $[Q^2/L^2] = [ML/T^2]$ and $[Q] = [M^{\frac{1}{2}}L^{\frac{3}{2}}/T]$.

Density σ; quantity of electricity on a given area: its dimensions are those of (Quantity) ÷ (Area), or $[\sigma] = [Q] \div [L^2] = [M^{\frac{1}{2}}/L^{\frac{1}{2}}T]$.

Difference of Potential, E: quantity of Work required to move a quantity of electricity from one point to another: its dimensions are those of (Work done) ÷ (Quantity moved); whence $[E] = [ML^2/T^2] \div [M^{\frac{1}{2}}L^{\frac{3}{2}}/T] = [M^{\frac{1}{2}}L^{\frac{1}{2}}/T]$.

Electric Force, or "Electromotive Intensity," F: the Electric Force at any point in a field is the mechanical force acting upon a unit quantity of electricity placed there: it is therefore equal to (Quantity × unity) ÷ (Distance²), and its dimensions are $[F] = [Q/L^2] = [M^{\frac{1}{2}}L^{\frac{3}{2}}/T] \div [L^2] = [M^{\frac{1}{2}}/L^{\frac{1}{2}}T]$.

Capacity, C: the Quantity necessary to produce a certain rise of Potential: its dimensions are those of (Quantity) ÷ (Potential-difference); $[C] = [M^{\frac{1}{2}}L^{\frac{3}{2}}/T] \div [M^{\frac{1}{2}}L^{\frac{1}{2}}/T] = [L]$, a Length simply. The relative capacities of conductors of similar form are simply proportional to their diameters.

Specific Inductive Capacity, *k*: the ratio of two quantities of electricity developed under similar circumstances: it is therefore simply a Number: its dimensions are accordingly 0.

Coefficient of Induction; the ratio of a Charge developed to a Potential inducing; Quantity ÷ Potential; $[M^{\frac{1}{2}}L^{\frac{3}{2}}/T] \div [M^{\frac{1}{2}}L^{\frac{1}{2}}/T] = [L]$.

Observation of Differences of Potential.

Observation of Differences of Potential is effected by means of instruments called Electroscopes and Electrometers; the former indicate the nature, the latter measure the amount, of differences of potential.

Gold-Leaf Electroscope.—A glass flask with a vulcanite stopper: through the stopper passes a metal rod surmounted by a metallic sphere or plate and terminated below by a pair of freely-suspended strips of gold leaf. If the metallic part of the electroscope be charged by contact with an electrified body, the gold leaves, becoming similarly charged, repel one another, and diverge, slightly if the charge be feeble, widely if it be great. The electroscope may also be temporarily charged by induction: a + electrified body brought into the neighbourhood of the sphere or plate causes that sphere or plate to become negatively, while the more distant gold leaves within the flask are positively charged. If, while the electroscope is electrified by induction, its upper extremity be momentarily touched by the experimenter, the gold leaves collapse, for their charge escapes to the earth: the plate or sphere, however, retains its charge, and when the inducing body is removed, the opposite charge borne by the sphere or plate becomes free to distribute itself over all the metal of the electroscope, and the leaves again diverge, for the instrument is now permanently charged.

If the electroscope be permanently charged, the approach of a body similarly charged will cause a further divergence of the leaves: the approach of a body oppositely charged will cause the leaves to repel each other with less force: whence the nature of the electrification of a given charged-body can be ascertained.

The deficiencies of the electroscope are: that its indications are qualitative, not accurately quantitative; and that the glass does not thoroughly screen the gold leaf from the direct inductive action of external charged-bodies. In order to obviate the latter defect, the inner surface of the flask is sometimes lined with perforated tinfoil, or the whole is surrounded by a cage of wire gauze.

The gold-leaf electroscope is a development of earlier instruments, in which straws, plain balls of elder-pith, or gilt pith-balls, were employed.

In the discharging electroscope the gold leaves, when they diverge, come in contact with two metallic uprights which communicate with the earth: they are thus discharged and collapse, again to be charged: the number of oscillations of the gold leaves affords a rough measure of the quantity of electricity borne by a conductor which is discharged to earth through such an electroscope.

Peltier's Electroscope.—A vertical brass ring, insulated; attached to its inner circumference, at the lowest point, a vertical pointed rod; on the pointed rod is poised either a magnetic needle or else a metallic rod whose direction is determined by a small magnetic needle attached to it. The whole is turned round a vertical axis until the ring and the poised metallic rod lie in the same plane. If the ring be charged, the charge is shared with the poised metallic mass, and the ring and the poised mass repel one another; the latter swings round until the force of electrical repulsion is balanced by the tendency of the magnet to point to the magnetic north and south. This instrument may, by imparting to it a series of successive known charges, be so adjusted as to act as an electrometer.

Bohnenberger's Electroscope.—Two vertical dry piles (p. 569), the one with its + pole, the other with its — pole uppermost; between these oppositely-charged uppermost poles there is a field of force, within which a strip of gold leaf is suspended. If uncharged, this strip hangs vertically; if charged, it is repelled by one pole and attracted towards the other.

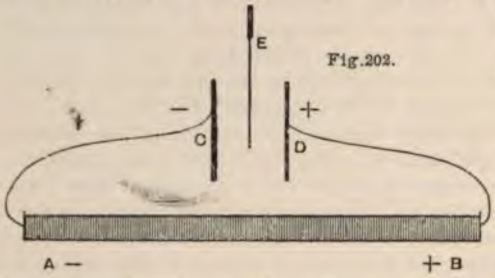


Fig.202.

Instead of two piles, the two extremities of one and the same dry pile may be used to make such an electroscope. In Fig. 202 AB is a dry pile whose poles are connected with the metallic plates C and D, between which there is thus formed a field of force, in which the gold leaf E is suspended.

On the same principle the **Quadrant Electrometer** of Sir William Thomson is based. In Fig. 203 the two opposite quadrants A and D are connected with one another by wire, but are insulated from B and C. A and D are thus at the same potential, while B and C are also at the same potential,—a potential which may differ from that of A and D. A and D may

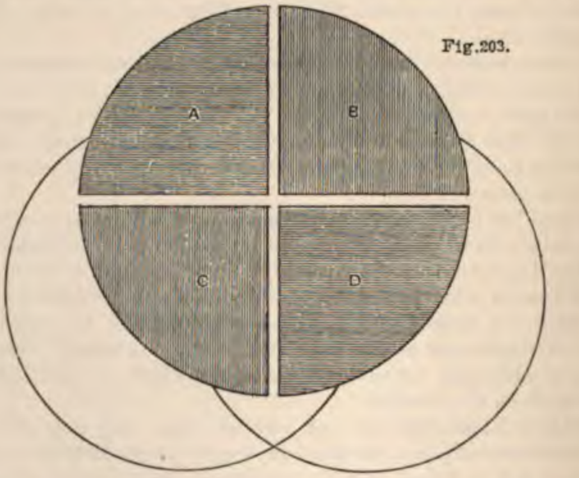


Fig.203.

be brought to the potential of the earth by means of a wire connected with gas or water pipes; B and C may be brought to the potential of any given object by connecting them with it by means of a wire. The quadrants A, D, and B, C, are thus at different potentials, and a metallic needle—an aluminium needle of a flat dumb-bell shape—will, if it be suspended symmetrically over the quadrants by means of two threads arranged parallel to one another, and if it be kept charged by constant connection with one coating of a Leyden jar, which may be replenished when necessary, impose a certain amount of torsion upon those two suspending parallel threads; the amount of this torsion will indicate the nature and—approximately—the amount of the difference of potential between the two pairs of quadrants, and therefore between the earth and the object whose Potential is to be measured.

If the quadrants be made hollow, and the needle suspended within them, the arrangement is better adapted for electrometric purposes.

The whole arrangement is well adapted for testing the adjustment to equality of the potentials of two bodies.

The amount of deflection of the suspended needle may be observed by connecting with it a very light mirror, upon which a very narrow beam of light shines; as the needle is deflected the beam of light reflected from the mirror is deflected through an angle twice as great as that of the deflection of the mirror; and the beam of light, if received upon a distant scale, thus acts as a weightless pointer.

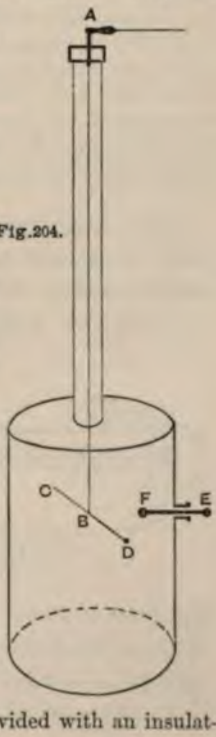


Fig. 204.

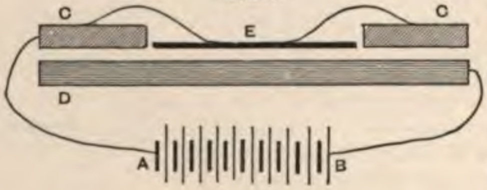
Upon the scale the deflection of the spot of light may be read off; that deflection is proportional to the tangent of twice the angle of deflection of the mirror: for small angles it is nearly proportional directly to twice the angle (Fig. 161).

Coulomb's Torsion Balance.—A long vertical slender hard wire or silk fibre, AB, Fig. 204, by which there is suspended in a horizontal position a thin counterpoised rod of glass or shellac, CD, which bears at one of its extremities a little gilt sphere D. In one position of the suspending wire the gilt sphere D comes into contact with a sphere-ended metal rod EF: this rod projects through the walls of the glass case in which the whole is encaged, and is therefore insulated. This metal rod terminates externally in a sphere E, which may be charged by contact with an electrified body, such as a proof-plane. A proof-plane is a small metallic disc provided with an insulating glass or ebonite handle. It is used by laying the disc upon the surface of an electrified body: when the disc is withdrawn it bears with it a charge proportional to the charge previously borne by that part of the surface of the electrified body with which it had been placed in contact: it is then made to touch the sphere E of the torsion balance. EF being charged, the two spheres, F and D, when they come in contact, become charged with electricity of the same kind and repel one another: they do this until there is equilibrium between the electric repulsion and the torsion of the suspending wire AB. The proof-plane may be used directly in the place of EF; and instead of a proof-plane a proof-sphere may be used when the curvature of the body whose charge is to be examined is but small.

Different charges may be compared by comparing the amounts of torsion necessary to bring the two mutually-repellent bodies, D and F, to equal distances. A preliminary charge is given to the ball D; a charge Q of the same kind is imparted to F, or brought in by a proof-plane or proof-sphere. Let the repulsion between Q and the charge on D be such that the suspended horizontal fibre makes an angle FBD of 10° with that position in which D is in contact with F, while the upper end A is twisted in the contrary direction—so as, as it were, to tend to force F and D together—through an angle of 410°; the total torsion of the wire AB is 420°. Now remove the charge Q and substitute a charge Q′; the index at A indicates 95° of rotation there when D is in its former position: the total torsion of the wire is now 105°. The charges Q and Q′ are proportional to the torsions which their repulsions balance; and $Q : Q' :: 4 : 1$.

Coulomb also made use of the method of oscillations (p. 38): he swung an electrified needle in presence of an electrified ball; the duration of the oscillations varied as the distance; but the duration varies inversely as the square root of the force acting: therefore the force acting varies inversely as the square of the distance. When the distance is kept fixed, the charges of the needle or ball being varied, the durations of the oscillations vary inversely as the square root of the varied charge.

Fig.205.



The **Absolute Difference of Potential** between two bodies may be ascertained by measuring the attraction between two metallic plates which are respectively connected by metallic wires with the two bodies in question. In Fig. 205 AB is a galvanic battery, the extremities of which are permanently at different potentials: it is desired to find the difference between these potentials. Connect A and B with the plates C and D. The field of force between C and D is uniform at its centre. D is fixed. But E, the central part of C, is movable. The attraction between E and D may be measured by observing the distortion of a spring which tends to pull E upwards while the electrical attraction tends to pull E downwards, this observation being made when the distance of D is so adjusted that the lower surface of E is flush with that of C. It is sometimes found advantageous in the use of instruments of this kind to connect D alternately with B and with the earth: the spring tends to become differently distorted in the two cases, and in order to render its distortion equal in both cases the distance of D must be varied. The amount of approximation or retraction of D may be measured by a micrometer-screw.

The spring which keeps up E against the attraction of D may be replaced by transforming E into one pan of a delicate balance, of which the other pan may be loaded with known weights.

The potential of E is V_E; that of D is V_D: the difference of potential to be measured is $V_E - V_D$: the force in the uniform field of force between E and D is uniformly equal to $(V_E - V_D)/t$, where t is the thickness of the dielectric; it is also uniformly equal to $4\pi\sigma$. The total attraction between the two plates E and D is equal to the sum of the attractions of the plate D for every little element of the surface of E. The attraction of a large plate of superficial density σ for a unit of quantity placed opposite its centre is $2\pi\sigma$ (p. 177, (9)); for the whole surface of E, a surface of area S and superficial density σ, it is $2\pi\sigma \times S\sigma = 2\pi S\sigma^2$. But since $4\pi\sigma = (V_E - V_D)/t$, as above, $\sigma = (V_E - V_D)/4\pi t$; and therefore the Attraction $= 2\pi S\sigma^2 = S\,(V_E - V_D)^2/8\pi t^2$, which is equal to T, the Tension of the spring. The Difference of Potential in absolute measurement is thus $V_E - V_D = t\sqrt{8\pi T/S}$, in which expression t, S, and T can be directly measured.

Since $\sigma = (V_E - V_D)/4\pi t$ per unit area, the charge on the attracted circular disc of radius r is $(V_E - V_D)\,r^2/4t$: the capacity of the system is therefore $r^2/4t$, and can thus be measured absolutely. Standards of Electrostatic capacity can thus be constructed.

When D is connected with B, the respective potentials of E and D are V_A and V_B; and $V_A - V_B$, the difference of potential between the extremities of the pile, is equal to $t\sqrt{8\pi T/S}$. When D, instead of being connected with B, is connected with the earth, its potential becomes zero; and when D is brought by its micrometer-screw into such a position that the plate E again assumes a position flush with the fixed guard-ring C, the tension T of the spring is the same as before, and $V_A - V_{\text{Earth}} = V_A - 0 = t_{,}\sqrt{8\pi T/S}$, where $t_{,}$ is the new distance between the plates. Hence $V_B = (t_{,} - t)\sqrt{8\pi T/S}$; and the Potential of B is easily measurable, for $(t_{,} - t)$, the change of distance between C and D, is much more easily measurable than t, the absolute distance between them.

PRODUCTION OF DIFFERENCE OF POTENTIAL.

The principal source of Difference of Potential is the **Contact of dissimilar surfaces**—that is, either of different substances, or of two pieces of the same substance whose surfaces are in different conditions. A piece of resin and a piece of glass will, after contact, be more difficult to pull asunder than two pieces of resin or two pieces of glass: and if they be rubbed together, so as to multiply the points of contact, the effect is multiplied. When pulled asunder, two such bodies are found to be charged equally and oppositely: across the surface of contact there has been a Separation of positive from negative electricity. The development of electrical condition is thus necessarily a phenomenon of continual recurrence: and it greatly influences the adhesion of one body to another. In all probability, wherever there is friction, the energy ultimately converted into heat is, in the first place, converted into the energy of electrical separation.

When two substances have different molecular velocities at their common surface of mutual contact, the molecules hamper one another and energy is lost: this energy, formerly that of molecular motion, now takes the form of the energy of electrical displacement. Within the interior of a homogeneous body the same thing must happen between colliding molecules whose velocities are different; but, all being alike, and the average molecular velocity being the same throughout the mass, there is on the whole no effect.

When two **metals** come in contact they at once assume different potentials: at the surface of contact there is an electrical separation, but each metal is equipotential throughout its volume. The difference of potential produced varies (1) with the nature of the metals, (2) with their temperature at the surface of contact, and (3) with the nature of the intervening or surrounding gas, if there be any.

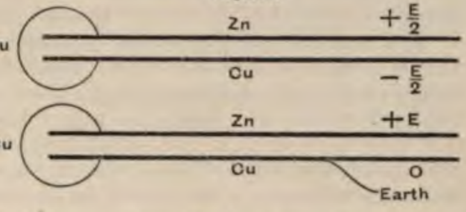
If a metallic disc be composed of four quadrants, soldered together and consisting alternately of zinc and copper respectively, the zincs are permanently at a positive, the coppers at a negative potential. If the disc be arranged horizontally, a needle suspended horizontally over the centre of the disc will, if it be charged with positive electricity, be repelled by the positive zincs and attracted by the negative coppers, and it will therefore swing round so as to lie over the copper quadrants; while if it be charged negatively it will come to lie over the zinc quadrants. The needle may be so suspended by two threads that, when uncharged, it lies along a diameter of the disc, a diameter coinciding with a line of junction between quadrants.

Take an electroscope surmounted by a copper plate, varnished on its upper side; upon this plate lay a zinc plate varnished on its lower side: these plates, separated by the varnish, act as an accumulator. Bring a copper and a zinc plate, both of which are unvarnished and insulated, into contact: separate them; with the zinc touch the zinc, with the copper the copper of the accumulator. Repeat this operation several times: then remove the zinc plate of the accumulator: the copper is found to be strongly charged with negative electricity, while the zinc plate removed is positively charged.

Copper filings falling through an insulated zinc funnel, as they leave that funnel, carry with them a negative charge.

In Fig. 206 a zinc plate Zn and a copper plate Cu are connected by a copper wire. They assume different potentials, $+\frac{1}{2}E$ and $-\frac{1}{2}E$, and the difference of potentials is constant $= E$. The point of electrical separation is the point of contact of the copper wire with the zinc plate. If the copper plate be connected to earth, its potential becomes $= 0$; but the difference of potential remains constant, and the potential of the zinc rises from $+\frac{1}{2}E$ to $+E$.

Fig. 206.



Within the field of force between such plates arranged with an intervening dielectric, $F = 4\pi\sigma = \text{const.} = (V_{Zn} - V_{Cu})/kt$, where t is the thickness of the dielectric, and V_{Cu}, V_{Zn}, the potentials of the copper and the zinc respectively. Hence the superficial density $\sigma = (V_{Zn} - V_{Cu})/4\pi kt$; but if the numerator of this fraction, the difference of potential, be constant, as it is between two metals, $\sigma = \text{const.} \times (1/t)$, or $\sigma \propto (1/t)$. The nearer the plates are to one another the greater is their tendency to discharge themselves by spark. The striking distance varies with the density σ; this varies inversely as the distance between the plates; and if we suppose the plates, nominally in contact, to be at a mean molecular distance of about $\frac{1}{200000000}$ cm., the density is so great that if the copper and the zinc could be separated from one another before their charges are allowed to recombine, they would then spark across 20 feet of air. They cannot be so removed, for they almost wholly discharge themselves when, after being placed in contact, they are pulled asunder, and there then remains in them only a residual charge of small density, which varies very slightly in amount, according to the mode in which they are pulled asunder. The moving molecules must therefore, even though the masses in contact seem to be at rest, be constantly discharging and renewing the separation of electricities.

Non-conductors in contact also become electrified; but only on their surfaces of contact. When they are separated their final discharge is incomplete, and the residual charges—their superficial distribution being restricted to those parts of the surfaces which have been most nearly in actual contact—are small in quantity but of great density, and therefore of high potential; and as these charges are not diffused by conduction over the whole surface, their potentials remain high after separation.

When sulphur is melted in a glass test-tube, after cooling the sulphur is found to bear permanently a negative, the glass a positive charge.

In the following series, due to Faraday, each member becomes positively charged when rubbed on one following it, negatively when rubbed on one preceding it: Cat and Bearskin—Flannel—Ivory—Feathers—Rock Crystal—Flint Glass—Cotton—Linen—Canvas—White Silk—the Hand—Wood—Shellac—the Metals (+ Fe, Cu, Brass, Sn, Ag, Pt)—Sulphur. There are certain irregularities here to be observed: for example, a feather lightly drawn over a piece of canvas becomes negatively electrified, whereas if it be drawn through a pressed fold of canvas it becomes positively charged.

The separation of electricities by contact and friction is utilised in the various forms of electric **frictional machines**, which range in complexity from a simple piece of sealing-wax or a glass rod rubbed with a catskin or a silk handkerchief, or a stout glass tube rubbed with a piece of dry flannel, to a machine in which a glass or vulcanite disc or cylinder, set in rotation, rubs against silk rubbers: these rubbers, whose conductivity is improved by anointing them with a mixture of fat and mercury, communicate with the ground, and their negative electricity is thus carried off to the earth; the positive charge, borne by the rotating glass or vulcanite, blends with a negative charge developed by induction in the tips of a comb-like series of sharp metallic points which come almost in contact with the rotating glass; while the complementary positive-charge is conveyed either to a large insulated Conductor connected with these points by a metallic chain or wire, or to the surface of a large insulated hollow conductor which surrounds the rubbing parts of the machine, or to the inner coat of a Leyden jar, or to the inner coat of one of the constituent members of a battery of Leyden jars. A charge of positive electricity may be thus accumulated. If, on the other hand, the positive charge of the glass be conveyed to the earth, while the insulated conductor is metallically connected with the rubbers, a charge of negative electricity may be accumulated in the conductor. If the conductor, in which positive electricity is being accumulated, be connected by wire with the negatively-charged rubbers, a current of electricity will pass through the connecting wire as long as the machine is worked, and that wire will be heated. If this current be sent through a second electric machine it will tend to cause in it a reversed rotation. It is possible (Gaugain) thus to produce continuous currents by the friction even of dissimilar metals.

When the plates of copper and zinc of Fig. 206 are connected, not by a copper but by an iron wire, we have between the two plates precisely the same difference of potential as before. But we have now three metals, two surfaces of contact; and the three metals are at the three different potentials, V_{Zn}, V_{Fe}, V_{Cu}.

$$\text{Hence } (V_{Cu} - V_{Fe}) + (V_{Fe} - V_{Zn}) = V_{Cu} - V_{Zn};$$

or as we may otherwise write it,

$$\mathrm{Cu}|\mathrm{Fe} + \mathrm{Fe}|\mathrm{Zn} = \mathrm{Cu}|\mathrm{Zn}.$$

This looks like an algebraical truism; it is, however, the representation of an experimental fact. Further,

$$\mathrm{Cu}|\mathrm{Fe} + \mathrm{Fe}|\mathrm{Pt} + \mathrm{Pt}|\mathrm{Sn} + \mathrm{Sn}|\mathrm{Zn} = \mathrm{Cu}|\mathrm{Zn};$$

any number of metals arranged in linear series or Open Circuit gives a permanent difference of potential between the extremities of the series, equal to the difference of potential which would have been developed between the terminal metals, had those terminal metals themselves come into contact.

Now let us recur to the copper-iron-zinc circuit and close it by bringing the terminal metals into contact. The electromotive difference of potential is now—

$$\mathrm{Cu}|\mathrm{Fe} + \mathrm{Fe}|\mathrm{Zn} + \mathrm{Zn}|\mathrm{Cu} = \mathrm{Cu}|\mathrm{Cu} = 0.$$

A closed circuit, composed of different metals, remains in electrostatic equilibrium: there is apparently no current, although there is an apparently constant difference of potentials between the different metals of the circuit. Each metal is equipotential throughout.

Here, as before, we must suppose a continual discharge and recharge at each surface of contact.

No continuous current can therefore be obtained from a closed metallic circuit, or indeed from any closed circuit of conductors in which the material of the circuit suffers no alteration. If, on the other hand, one of the conductors of the circuit suffer a chemical change, energy may be liberated, which may take the form of the Energy of a Continuous Current. Let us consider a circuit consisting of copper—hydrochloric acid—zinc—connecting wire—copper. Since it does not matter what the material of the connecting wire may be, we may use copper; the circuit is then $\mathrm{Cu} — \mathrm{HCl} — \mathrm{Zn} — \mathrm{Cu}$. If all the members of this series were mere conductors, the differences of potential within the circuit, which are independent of chemical action, would together amount to zero, for in that case $\mathrm{Cu}|\mathrm{HCl} + \mathrm{HCl}|\mathrm{Zn} + \mathrm{Zn}|\mathrm{Cu} = 0$. But they are not all mere conductors; at the junction of zinc and hydrochloric acid we have **Chemical Action.**

If a piece of zinc and a piece of copper be placed in hydrochloric acid, but not in contact with one another, the **zinc** (which, when it is placed in mere contact with copper, becomes positively charged, and is therefore said to be electropositive to copper) is now **negatively charged**; and conversely, the electronegative **copper** becomes **positively charged.**

This difference of potential may be explained, but not quite satisfactorily, in the following manner:—Each molecule of hydrochloric acid may be considered as composed of an atom of hydrogen and an atom of chlorine; these atoms, being in contact within the molecule, are permanently charged, the hydrogen positively and the chlorine negatively, just as masses of zinc and copper become charged on contact. The negatively-charged chlorine is, somehow, more attracted by the zinc than it is by the copper; it unites with a part of the zinc, forming chloride of zinc; and its charge is communicated to the remainder of the metallic zinc, which thus acquires a negative electrification. The hydrogen of the decomposed molecule either travels direct to the copper, or else it assumes the chlorine of a neighbouring molecule of hydrochloric acid, whose hydrogen in its turn takes the chlorine of the next molecule, and so on; ultimately a hydrogen atom, positively charged, is liberated upon the surface of the copper, and charges it positively.

The difference of potential between the copper and the zinc cannot be discharged through the decomposed fluid or Electrolyte, and as long as the circuit is open there will be electrostatic equilibrium; but if for an instant the copper and the zinc be connected by a conducting wire a momentary current will run in that wire. When the connection is broken the metals become charged as before. Repeated metallic contacts between the copper and the zinc will produce repeated momentary currents. If the connecting wire be kept permanently in position, there is a constant charge and discharge; a constant flow of electricity **along the wire** in the direction **copper to zinc** is kept up until there is either no more hydrochloric acid to be decomposed or no more zinc to be dissolved; and the Energy of that flow or current is equal to the Heat which would have been evolved if the zinc had been directly dissolved in the hydrochloric acid.

When the copper and zinc are not connected, that is, when the **circuit is open,** the condition assumed is, as we have seen, one of electrostatic equilibrium: in the language of the above explanation, the zinc becomes negatively charged until its negative charge electrically repels the negative chlorine-atoms as much as the metallic zinc chemically attracts them; and then all chemical action ceases if the zinc be pure, or if it be homogeneous, as it is when amalgamated. When this state of equilibrium has been assumed, the free terminals, which must be both of the same metal, will continuously present a definite difference of potentials.

Confusion is multiplied in this subject, which is at best in an unsatisfactory state, in this way: sometimes the copper end of a battery is said to be negative, perhaps because copper itself is electronegative to zinc in contact with it;* sometimes it is said to be positive, because it is positively charged relatively to the zinc end, and because the current flows from it through the wire.

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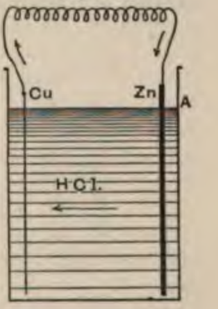
* Also because in the earliest forms of Volta's pile there was a superfluous zinc at the copper end, and *vice versâ*. The current then flowed from the apparent zinc end to the apparent copper end.

The reader will please clearly understand that in this volume the latter of these expressions is employed.

When the circuit is closed the metallic connection between the copper and the zinc tends to cause the zinc to become positively charged; but this condition tends to be continuously discharged through the hydrochloric acid, —a condition which tends to aid the current set up in consequence of the chemical action.

A circuit of the kind just described is a **Galvanic Circuit.** In Fig. 207 A is a glass vessel containing hydrochloric acid; Cu is a plate of copper, Zn a plate of zinc. The two metals are connected by a wire of any conducting material: the current runs in the direction copper—conducting wire—zinc—acid—copper. Excluding the connecting wire, such an arrangement is called a Galvanic Element or Cell: while a number of such cells may be arranged so as to form a Galvanic Battery or Pile.

Fig. 207.



The E.M.D.P. within a galvanic circuit or battery is measured by the electrostatic difference of potential between the free extremities of an open circuit, with terminals of the same metal at the same temperature; such an open circuit as might be obtained by cutting through the conducting wire of Fig. 207.

Different metals have different chemical affinities for different chemical fluids; and consequently the amount and even the direction of the electromotive difference of potential within a galvanic circuit depends not only upon the nature of the metals, but also upon the nature of the fluid or electrolyte employed. Copper and iron in dilute sulphuric acid give a current running along the conducting wire from copper to iron, and the iron is attacked, not the copper: in a solution of sulphide of potassium the copper is attacked, and the current runs in the wire from iron to copper. In the presence of facts of this order the theory must as yet be considered wholly incomplete, for chemical affinity remains unexplained.

For each liquid it is possible to make up a table of relative potentials: in dilute sulphuric acid the series is, commencing with the most negative:—Amalgamated zinc—ordinary zinc—cadmium—iron—tin—lead—aluminium—nickel—antimony—bismuth—copper—silver—platinum.

If two equal galvanic cells be set against one another, as in

Fig. 208, no current is produced: the aggregate E.M.D.P. within the entire circuit is equal to zero.

If a cell containing copper and zinc in dilute sulphuric acid be set in this way against one containing platinum and zinc in the same liquid, the E.M.D.P. is the same as that of a single cell containing platinum and copper in dilute sulphuric acid.

If two cells be coupled side by side, copper to copper, zinc to zinc (*i.e.*, "in Surface"), and if the conducting wire be led from any one of the coppers to any one of the zincs, the whole acts like one cell of double surface, and the E.M.D.P. within the circuit is not increased. So also for *n* cells so arranged.

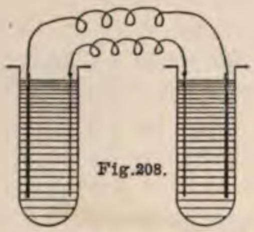


Fig. 208.

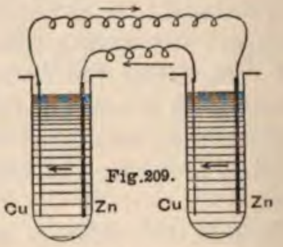


Fig. 209.

If two cells be set behind one another, as in Fig. 209, copper being connected with zinc, the difference of potential between the first copper and the last zinc is twice as great as that between the copper and the zinc in a single cell: or if *n* cells be arranged one behind the other in Indian file or "in Series," the copper of each being connected with the zinc of the next in regular succession, the effective difference of potential is *n* times that of a single cell.

Principal Forms of Galvanic Cells and Batteries.—These may be divided into two principal classes: (1) those which have in each cell one fluid; (2) those which have in each cell two fluids.

One-fluid cells and batteries.—Copper, sulphuric acid (diluted), and zinc, form the most commonly used triad of materials. Volta's pile; a number of repetitions of the sequence:—Copper plate, cloth dipped in water or acid, zinc plate: the terminal copper is positively, the terminal zinc negatively, charged. Volta's corona di tazze: a number of cups containing dilute sulphuric acid, in each of which are placed a plate of copper and a plate of zinc, not in contact with one another: each copper is connected with the zinc of the preceding cup. For practical purposes this is made in guttapercha-lined boxes divided into cells by partitions which are themselves made of copper on one side, zinc on the other; and to avoid spilling, the whole may be filled up with sand or stuffed with asbestos. The form of a single cell may vary; a cylinder of zinc placed within an open-ended hollow cylinder of copper, but not in contact with it, the whole being immersed in acid: a copper cylinder within a similar hollow-cylinder of zinc (Oersted);

a sheet of copper and a sheet of zinc separated by flannel, rolled up and immersed in acid (Hare's Deflagrator); a larger piece of copper, bent so as to face both sides of a smaller sheet of zinc, and thereby to diminish the "resistance" within the cell (Wollaston). In all these cases the difference of potentials between the free extremities of an *open circuit* with similar terminations, but containing a battery of n cells, is, when we employ pure copper, pure zinc, and a 2 % solution of pure sulphuric acid in water, about $\frac{921}{300000}$ n C.G.S. electrostatic units, or ·921 n "Volts;" if dilute hydrochloric acid of the same strength be used, about $\frac{753}{300000}$ n C.G.S. units or ·753 n Volts.*

Instead of ordinary zinc, zinc whose surface is amalgamated may be employed: it is not corroded by the acid except while the current is passing, and the difference of potential within the circuit is raised by about ·13 Volts for each cell of the battery. Ordinary zinc wastes away when left in acid, because it is not homogeneous; local differences of potential are set up in it, and local circuits are formed. Zinc may be amalgamated by setting it to stand in contact partly with mercury, partly with dilute hydrochloric acid, or by rubbing mercury into it with a rag dipped in acid, or by dipping it in a liquid prepared by dissolving 200 grms. Hg in a mixture of 250 grms. of nitric and 750 grms. of hydrochloric acid, and, when the solution is clear, adding 1000 grms. of hydrochloric acid. Iron or platinum can be wetted by mercury containing sodium.

The difference of potential set up by single-fluid batteries diminishes seriously when their action is prolonged, in consequence of their so-called Polarisation. Hydrogen is liberated at the copper or positive plate, and remains there as a film; this hydrogen is positively charged, and tends to repel all other atoms of hydrogen, and to attract the negative components of the molecules of the fluid. In consequence of this there is a certain tendency towards the production of a current opposed to the main galvanic current: and if a copper-zinc couple, which has been for some time in action, be taken out of sulphuric acid and immersed in water, a reverse current, comparatively feeble, will run for some time from the zinc to the copper through the conducting wire. In order to minimise this polarisation various devices have been resorted to: the hydrogen has been swept off the positive plate by air from a bellows (Grenet), or by shaking the cell, or by rapidly rotating the positive plate in the fluid (Mocenigo), or it has been removed by covering the positive plate with a film of oxide of copper, which is reduced by the hydrogen (Becquerel), or by covering the positive plate with a film of clay (Pulvermacher), or by otherwise roughening its surface so that bubbles of hydrogen may readily form and rise; this was done by Poggendorff, who electrolytically deposited a rough film of copper on the positive copper plate, and by Smee, who used a similarly-platinised platinum or silver or lead plate as the positive plate. Platinised iron (Paterson), amalgamated iron (Münnich), and platinised charcoal (Walker), have been recommended as positive plates. Bunsen used gas coke with dilute sulphuric acid and amalgamated zinc.

For the negative plate zinc is used, because it is very readily oxidisable, convenient, and moderately cheap. Magnesium would give a higher effective difference of potential, but is too expensive: iron gives with copper too feeble a current, but may be in some cases advantageous as compared with the more expensive zinc, although to obtain a given current by its aid a greater number of cells is required.

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* 1 **Volt** = $\frac{1}{300}$ C.G.S. Electrostatic Unit of Difference of Potential.

For the intervening fluid or electrolyte, instead of sulphuric or hydrochloric acid other liquids may be employed, which oxidise the hydrogen liberated at the positive plate. Nitric acid oxidises hydrogen, being itself reduced to nitrous acid: a solution of iodine with iodide of potassium in water (Laurie) forms with it hydriodic acid: chromic acid is reduced by the hydrogen to chromic oxide: instead of chromic acid a mixture of bichromate of potash and sulphuric acid may be employed, and the reaction then is

$$3Zn + K_2Cr_2O_7 + 7H_2SO_4 = 3ZnSO_4 + \underset{\text{Chrome-alum.}}{2KCr\,(SO_4)_2} + 7H_2O$$

A common bichromate-cell, in which gas-carbon, bichromate-mixture—1000 water, $100K_2Cr_2O_7$, 300 pts. by wt. H_2SO_4 (Grenet); 6·182 grms. $K_2Cr_2O_7$, 6·282 c. cm. strongest H_2SO_4, 60·47 c. cm. water (Bunsen); with the addition (Ducretet) to each litre of about 2½ grms. of $HgSO_4$ in order to keep the zinc well amalgamated—and zinc are employed, gives a difference of potential of about 2 Volts, which remains fairly constant when the circuit is closed for about three-quarters of an hour, but which in an hour and a half sinks to about 1 Volt.

Chloride of ammonium, chloride of zinc, used as exciting fluids, also tend to check polarisation. In Leclanché's cell the materials are zinc, a solution of chloride of ammonium, and a positive plate: this plate consists in the older Leclanché cells of a mixture of moistened binoxide of manganese and crushed gas-coke surrounding a central rod of carbon, or, in the newer, of a mixture of 55 sulphur, 40 gas-coke powder, and 5 shellac, pressed upon a carbon core; and the whole of this positive plate is surrounded by a hollow cylinder of porous earthenware. The zincs of Leclanché batteries are very little corroded when they are not in use; hence they are much used for occasional telegraphy or bell-ringing; and for therapeutic purposes a large number of such elements, each the size of a small test-tube, can (Beetz) be packed within a very small space.

In some other cells the electrolyte is somewhat complex, and there are differences of potential, due to chemical action, set up even within it. A cell whose metals are silver and zinc, separated by an intervening mass of chloride of silver moistened with or lying within a solution of alkaline chloride (Warren de la Rue), chloride of zinc (Gaiffe), or alkali (Scrivanoff), gives a very constant and, relatively to the bulk of the cell, a powerful current. The difference of potential in a silver—salt-water-and-chloride-of-silver—zinc cell is (Warren de la Rue) 1·065 Volts. Becquerel used salt water and sulphate of lead between zinc and lead.

Two-fluid cells. Daniell's cell:—A hollow cylinder or cell of porous ware, as thin as practicable, and containing dilute sulphuric acid and a rod of zinc, is surrounded by a saturated solution of sulphate of copper and a larger cylinder of copper. The current runs through the fluid in the direction $Zn - H_2SO_4 \parallel CuSO_4 - Cu$ (where the symbol ‖ is used to indicate the porous cell), and through the conducting wire as usual from copper to zinc. The chemical action, which may be expressed by the diagram

$$- \;\Big|\;\overline{Zn \quad SO_4}\;\overline{H_2 \parallel SO_4}\;\overline{Cu \quad Cu}\;\Big|\; +$$

results in the formation of sulphate of zinc within the porous cell, sulphuric acid within the walls of the porous cell, and the deposition of copper upon the inner surface of the copper cylinder. There is thus no evolution of hydrogen, and no polarisation. The effective difference of potentials is 1·124 Volts when the liquids employed are a neutral saturated solution of sulphate of zinc and a saturated solution of copper sulphate, and when the zinc is amalgamated and the copper electrolytically deposited ; and this does not vary very much with the strength of the solution of sulphate of zinc, nor does it do so to any great extent though the temperature of the cell rise from 3° to 70° C. ; and it is equal to almost exactly 1 Volt when the liquids used are—the one a solution of sulphuric acid, 1 vol. to water 22 vols., and the other a saturated solution of nitrate of copper. The "internal resistance" sinks (Preece) to one-third when the cell is heated to 100° C. Batteries of this construction were originally due to Becquerel ; and they are very constant, lasting even for months if the resistance in the circuit be kept very great ; but if the external resistance be very small, as where the copper and the zinc are connected by a short piece of wire, the current produced rapidly falls off.

The solution of sulphate of copper is kept saturated by crystals placed in it. Any metal can be used as a positive plate, for it soon becomes covered with copper.

In some forms of Daniell's cell the porous cell, which is fragile, and which tends to have its pores blocked up, is dispensed with : in gravity batteries—*e.g.*, Callaud's—a stratum of acidulated water or of a solution of sulphate of zinc floats upon a denser solution of sulphate of copper : in the former stratum the zinc is suspended ; in the latter the copper lies. Sometimes, as in Minotto's battery, the copper is protected by sand or sawdust, beneath which a layer of copper-sulphate-crystals rests upon the copper.

In Maidinger's cell the crystals lie in a special inverted flask filled with zinc-sulphate solution ; the heavy solution in this flask sinks down whenever the density of the lowest layer, the solution of sulphate of copper, diminishes in consequence of the deposition of its copper upon the positive plate.

These cells without porous diaphragms are liable to diffusion of the copper-sulphate-solution upwards into the upper layer, the solution of sulphate of zinc : the zinc suspended in this is attacked, and a film of copper is deposited on it, which interferes with the efficiency of the cell. Cells of this kind are therefore good only for frequent use, such as tends to exhaust the layer of sulphate of copper solution.

The copper plates submerged in the lower layer of liquid are connected with the external circuit by wires passing down through the whole liquid, and protected by an insulating covering of guttapercha.

In Remak's portable form of Daniell's battery, discs are arranged in the following sequence :—Copper plate, cloth dipped in solution of copper sulphate, porous earthenware disc, cloth dipped in dilute sulphuric acid, zinc plate, copper plate, etc.

In Beetz's dry Daniell-cell, which is exceedingly constant even when the circuit is kept closed, a U-tube has one limb filled with plaster-of-Paris made up with sat. $ZnSO_4$ soln. and containing a Zn wire ; the other limb similarly with $CuSO_4$ and Cu wire. The E.M.D.P. is (corrected) 1·04 Volts.

In Grove's cell the current passes through $Zn - H_2SO_4 \parallel HNO_3 - Pt$. The nitric acid dissolves the hydrogen liberated by the sulphuric acid, and is itself reduced to nitrous acid, which, if it be not too abundant, is dissolved by the remaining nitric acid. The difference of potential maintained by a Grove's cell is equal to about 1·92 Volts. This is 1·708 × that of a Daniell, and the internal resistance of a Grove is much less ; for a short time, and against a small resistance, a Grove can produce a much stronger current than a Daniell of the same size; but its fumes are unwholesome, noxious in a laboratory, and destructive to the binding-screws of the Grove cell itself.

Grove's cell, like Daniell's, may be made either cylindrical or flat-plated: the former is preferable, because cylindrical porous-cells are not so liable to break as flat ones.

The difference of potential maintained by a Grove mounts from 170·8 to 240 (Daniell = 100), when the dilute sulphuric acid surrounding the zinc is replaced by a concentrated solution of caustic potash.

The nitric acid surrounding the platinum is often mixed with strong sulphuric acid, which exercises a dehydrating action, takes water to itself, and keeps the nitric acid concentrated.

Instead of platinum, carbon may be used, as in that modification of Grove's cell known as **Bunsen's** cell, originally due to Grove; or iron, which becomes positive, and is not dissolved by strong nitric acid; or, as in **Callan's** cell, platinised lead.

The nitric acid of Grove's cell may be replaced by bichromate-of-potash-and-sulphuric-acid mixture. In the place of nitric acid a saturated solution of ferric chloride, to which 4 per cent of nitric acid has been added, forms an excellent liquid: when it is used, the total difference of potential kept up by the cell is about midway between that of a Daniell and that of an ordinary Grove: this liquid is readily renovated by boiling it with a little nitric and hydrochloric acid.

In **Marié-Davy's** cell the current runs through Zn – pure water || paste of Hg_2SO_4 with water – carbon. Any mercury passing through the porous cell merely amalgamates the zinc and does no harm. Polarisation is great in this cell, but it is very convenient because very portable.

In **Latimer Clark's Standard Cell** the current runs through Zn – pure concent. $ZnSO_4$ soln. || Hg_2SO_4-and-water-paste – Hg. This cell is very constant, and its difference of potential = 1·457 Volts: it may be used as a standard for comparative electrostatic measurements of difference of potential, but is greatly lacking in constancy if it be kept in action for some time. Lord Rayleigh finds the D. P. to be 1·453 Volts; but correcting for the error of the Standard Ohm (p. 581), this is 1·434 Volts. Such cells differ among themselves by about ± 0·2 per cent. If the paste be made of subsulphate of mercury and a concentrated solution of sulphate of zinc, the constancy is even greater, and the difference of potential is 1·465 Volts, or corrected for the true Ohm, 1·4455 Volts.

The greatest difference of potentials yet observed as having been produced and maintained by a single cell is that of a combination devised by **Goodman** in 1847. The current in this runs successively through potassium-amalgam —inner porous cell—solution of caustic potash—outer porous cell—solution of permanganate of potash—and lastly, as a positive plate, stick-sulphur. The difference of potential is (Beetz) 302·3 (Daniell = 100).

Two fluids or melted substances separated by a porous diaphragm will give a current even though plates of the same metal be immersed in both. Iron in nitric acid and iron in sulphuric acid (Grove), or copper in dilute sulphuric acid and copper in dilute nitric acid (Louis Napoléon), aluminium in dilute caustic soda and aluminium in dilute hydrochloric acid (Wöhler), or platinum in caustic-potash-solution and platinum in nitric acid (Becquerel), will give a current, and as the one metallic plate is dissolved away the other is thickened. In a flask set aside and containing a lower layer of solution of sulphate of copper and an upper layer of acidulated water, together with a copper wire set to stand in the fluid, it will be found that that part of the copper wire which is within the acidulated water becomes thinned away, while that part which is within the solution of sulphate of copper becomes thickened. Further, two plates of the same metal immersed in acids or alkalies of different degrees of concentration will give a current which, in the case of sulphuric and hydrochloric acids, flows from the stronger through the porous diaphragm into the weaker acid, but which, in the case of caustic alkalies, flows towards the stronger solution.

Two metals in the same gas, or two gases covering different parts of the surface of the same mass of metal, are again at different potentials. From this it follows that the presence of the air exercises a disturbing influence when zinc and copper are placed in contact and then separated; and, as an extreme case, we see this disturbing influence of chemical action actually predominate over the contact-effect when zinc and copper are brought in contact and then separated in an atmosphere of sulphuretted hydrogen: the zinc becomes negatively, the copper positively, charged.

Dry piles may be constructed as follows:—Pieces of "gold paper" and of "silver paper" may be pasted back to back and cut into small discs: these discs are then piled up and pressed into a glass tube, or, better, strung upon a silk thread, their similar faces all looking in the same direction. Such a pile develops a considerable difference between the potentials of its extremities, and it remains thus charged for apparently indefinite periods. In principle a dry pile resembles a Volta's pile, the discs of wet cloth in which have almost dried up. Paper is never perfectly dry: the paper between the metallic faces of each disc takes the place of the moist discs of cloth; and besides this, the air acts more on the one metallic face of each disc than on the other. In consequence the chemical action is not *nil*; and a definite difference of potential is set up by which chemical change, otherwise too feeble to be detected within any reasonably short period of time, is rendered strikingly manifest. The quantity of energy liberated by a dry pile is very small, and little work can be done by it; but one extremity of a dry pile can keep a charged gold-leaf steadily repelled for a long time. If the two ends or poles of a dry pile be brought near one another, an insulated strip of gold leaf suspended between them and alternately attracted by, coming in contact with, and repelled from, each pole, may oscillate between the poles for a very long time, but only so long as the chemical decompositions going on within the pile can furnish the energy requisite to overcome the friction of the air and the rigidity of the gold leaf.

Difference of potentials is the most delicate test that we possess for chemical action.

The chemical action set up under the influence of actinic rays also produces difference of potential, which may serve (Becquerel) to measure the chemical energy of sunlight.

Difference of potential is also produced by friction of water against steam or air, as where a jet of partly-condensed steam or of suddenly-expanding undried air is driven through a conical nozzle of metal or glass or wood: the steam or air becomes positively, the vessel from which it is driven becomes negatively, charged. If the nozzle be of ivory there is no charge. If the vessel contain some turpentine-oil the charges are reversed.

When a liquid is brought into the spheroidal state it assumes an electrical condition which varies with the nature of the liquid and with that of the hot surface on which it lies.

When saline solutions are evaporated, the vapour and the liquid assume different electrical conditions if there be either friction of the crystals on the vessel, as when the crystals crackle, or friction of the heated water upon the salt. In the evaporation of water there is no difference of potential set up unless there be friction between the water and the vapour: if there be friction, the steam becomes positively charged.

Pressure or traction applied to tourmaline crystals, if the force applied have a component parallel to the crystallographic axis, causes a separation of electricities; opposite extremities of the axis become oppositely electrified, and the amount of difference of potential produced depends only on the amount of force applied. The same result follows whether the alteration of form of the crystal affected be the result of force or of the application of heat or cold.

Electro-capillarity.—Mercury standing under water has a convex surface and a definite surface-tension. The water and the mercury, being two conductors in contact, are at different potentials. The surface of the water and the surface of the mercury, though nominally in contact, are at a mean distance of about one twenty-millionth of a centimetre. The two surfaces therefore act as an accumulator which has a definite capacity. The surface of contact between mercury and water has thus three properties—Surface Tension, Difference of Potential, and Electrostatic Capacity; and these depend upon one another, so that if one be varied the other two will vary (Lippmann). Thus if we vary the surface-curvature of mercury, as by setting it in vibration in a conical tube, and thus altering the area and the amount of tension of the surface; or if we heat the mercury or the water, and thus again alter the surface-tension, the capacity and the difference of potential will also vary. Part of the work done upon the mercury in setting it in vibration, or of the heat supplied, is spent in setting up a difference of potential, the very existence of which causes a tendency to restitution of the original surface-tension; for if we vary the difference between the potentials of the water and the mercury by charging either the one or the other, the surface-tension, and consequently the surface-form, of the mercury varies also.

Thermo-electricity.—The difference of potential set up between two metals by their mere contact depends upon their temperature as well as upon their chemical nature and state of purity or their physical state—their hardness, their tension, and so forth. If bismuth and antimony (in the form of commercial pressed wire) develop, on contact at 19° C., a difference of potential of V volts, the same materials develop, on contact at 20° C., a difference of potential of $(V + \cdot 000103)$ volts. If a semicircle of bismuth wire and one of antimony wire be joined so as to form a circle, and if one of the two junctions be maintained at 19° C., while the other is kept at 20° C., then, since the hotter junction presents a greater difference of potential than the colder, the aggregate difference of potential within the circuit is not zero, but is equal to $\{(V + \cdot 000103) - V\}$ volts $= \cdot 000103$ volts, or 103 microvolts (millionths of a volt). This difference of potential within the circuit is maintained as an electromotive difference of potential, and there is therefore a constant current round the circuit, so long as the junctions are kept at these fixed temperatures; and the energy of this current is partly (but in part only) derived from the heat supplied at the hotter thermo-electric junction, and is due to transformation of the energy of molecular motion. The current runs from **bismuth to antimony** through the **hotter** junction.

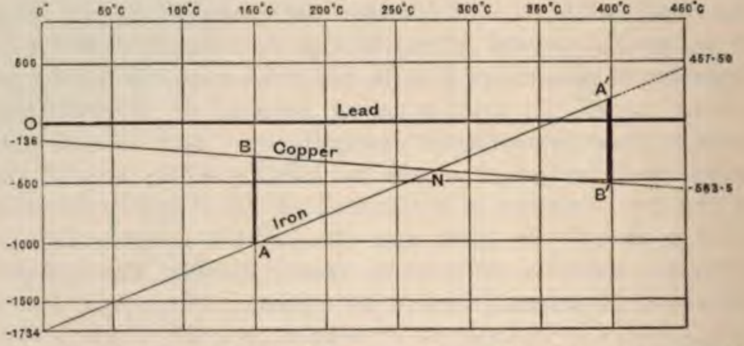
Antimony and bismuth are the extremes of a thermoelectric series, which is, according to Seebeck, the following :—Sb, As, Fe, Steel, Cd, Wo, Zn, Ag, Au, Mo, Sn, Pb, Hg, Mn, Cu, Pt, Pd, Co, Ni, Bi.

The electromotive difference of potential produced and maintained within the closed circuit is approximately proportional to the difference between the temperatures of the two junctions, if this difference be very small; and it is therefore, when measured in microvolts, equal to the product of the difference of temperatures into a number: this number is called the **thermo-electric power** between the two given metals at the given mean temperature. For Bismuth and Antimony, at a mean temperature of $19\frac{1}{2}°$ C., it is 103; for E.M.D.P. $= 103$ microvolts $= 103 \times (20° \text{ C.} - 19° \text{ C.})$ If the E.M.D.P. be measured in C.G.S. electro-magnetic units, of which 100 make a microvolt, the thermo-electric power in this case is 10,300.

The thermo-electric power between any two metals is not a constant number, but varies with the temperature. In

Fig. 210 (after Tait) it may be seen that near the freezing-point of water a difference of one degree between the temperatures of two junctions of a lead-iron circuit makes between the two junctions a potential-difference of 17·34 microvolts, or 1734 electromagnetic units, while at higher mean-temperatures the thermo-electric power is progressively less, becomes *nil,* and ultimately changes its sense. The thermo-electric power between copper and lead, on the other hand, increases.

Fig.210.



The lead is nominally positive to the copper, for the current passes from lead to copper across the hotter junction.

A diagram of this kind is called a **Thermo-electric Diagram,** and indicates the thermo-electric power between its metals at any mean temperature within its range.

The lines of iron and copper cross one another at 274°·5 C. An iron-copper couple, one of whose junctions is at a temperature slightly over, the other at a temperature equally under 274°·5, will develop within its circuit no electromotive difference of potential. That mean temperature, 274°·5 C., is for iron and copper the so-called **neutral point.**

For every temperature on the one side of the neutral point it is possible to find one on the other side such that a copper-iron circuit whose junctions are at these respective temperatures will develop no current.

If one copper-iron junction be at 150° C., at what temperature must the other be in order that there may be no current? The temperature required is 399° C., which lies as far beyond the neutral point, 274°·5 C., as 274°·5 does beyond 150° C. The triangle ABN (Fig. 210) represents the total E.M.D.P. when the junctions are respectively at 100° C. and 274°·5 C.,* the triangle NA′B′ represents the total and opposite E.M.D.P. which would be developed if the junctions were at 274°·5 and 399° respectively: these triangles are equal: their sum is *nil:* the total electromotive potential-difference between 150° and 399° is *nil:* there is consequently no current.

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* Since for a small difference of temperature, E.M.D.P. = Therm.-elect. power × diffce. of temp. measured in °C., each step in temperature multiplied by its

If one copper-iron junction be maintained at the constant temperature of 100° C., and the other be successively exposed to temperatures 101°, 102°, 103°, and so forth, each step in the temperature of the hotter junction produces an increment of the E.M.D.P. developed within the circuit; but each successive increment is smaller than its predecessor: as the temperature of the hotter junction nears 274°·5 the successive increments of E.M.D.P. become less and less: when the hotter junction is at 274°·5, the **neutral point,** the increment is *nil*, and the electromotive difference of potential and the current which it causes to run round the circuit are at their **maximum.** Thereafter, as the hotter junction is still more strongly heated, the E.M.D.P. at first gradually and then more rapidly sinks. When at length the hotter junction is at 449° (the colder one still remaining at 100°) there is no E.M.D.P., and no current round the circuit: and when the temperature of the hotter junction exceeds 449°, the direction of the current is **reversed,** being now from iron to copper across the hotter junction; and thereafter successive increasing differences of temperature develop successive numerically greater negative E.M.D.P.'s. The Neutral Point is thus a fixed temperature for each pair of metals; and the temperature of the colder junction on the one hand (whatever that temperature may be), and the corresponding **Temperature of Reversal** on the other, are equidistant on either side of it.

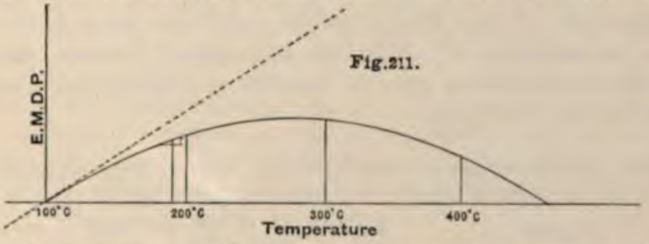

Fig. 211.

Curves indicating the relation between the differences of Temperature between two junctions and the electromotive differences of Potential developed

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corresponding thermo-electric power forms in the thermo-electric diagram a small rectangle, which represents the E.M.D.P. developed by each difference of temperature: the sum of all these rectangles between 100° and 274°·5 represents the total E.M. difference of potential set up when these are the temperatures of the two junctions: this sum is equal to the triangle ABN.

in consequence of them (sometimes called Gaugain's curves), have a form which, for most pairs of metals, is that of a parabola: and the numerical value of the tangent of the angle made by this curve with a line parallel to the axis of x, and cutting the curve at that point of it which corresponds to any given temperature, $x°$ C., is a numerical measure of the thermo-electric power at that mean-temperature: for both the tangent and the thermo-electric power are numerically equal to the fraction

$$\frac{\text{Increment of E.M.D.P.}}{\text{Increment of mean temperature}} = \frac{\text{Change of ordinate}}{\text{Change of abscissa}} .$$

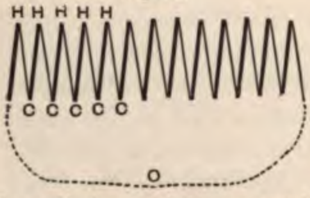
Even within one and the same bar, differences of potential are set up when a bar is unequally heated, and some of the heat supplied is expended in setting up this electrically-stressed condition; but in a homogeneous metallic ring, however irregularly heated it may be, there is no current. The metal on either side of a hot or cold junction is, on the other hand, like a single bar, and differences of potential are set up within it, which modify the amount of the effective difference of potential within the whole circuit, and are found to supply an explanation of the phenomena of inversion.

Metals interpolated in the circuit produce no effect on the amount of the effective difference of potential within the circuit, unless, indeed, their junctions be at different temperatures. If that be the case, their thermo-electric effects form a part of the general thermo-electric effect of the circuit.

When a number of pieces of bismuth and antimony are arranged end to end, their alternate junctions being hotter and colder respectively, the E.M.D.P. maintained between the extremities of a pile of n pairs of elements is n times that found to be due to one such pair.

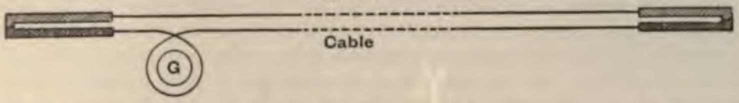
This principle is utilised in the Thermo-electric Pile, which consists of a number of pieces of bismuth and antimony (or, better, of an alloy of 10 parts by wt. of bismuth and 1 of antimony, and an alloy of 15 parts of antimony and 7 of cadmium) arranged after the fashion of Fig. 212. When the face of this pile marked HHH is exposed to heat, the junctions H, H, H become warmer than the junctions C, C, C, and a current passes through the circuit O. If it be placed opposite a piece of ice, the face HHH will cool itself by radiation, and the current is now in the reverse direction.

Fig. 212.



When a junction of two metals is connected by a pair of copper wires with a similar junction at the same temperature, no current passes, whatever be the length of the intervening cable or its local variations of temperature, but if one of the junctions assume a different temperature from the other, then a current passes, and the temperature of the distant junction may be inferred from the strength of the current which passes, this being measured by a galvanometer, or directly determined by heating or cooling the similar junction situated under the observer's control until its temperature becomes the same as that of the distant one: this is known to have occurred when the current through the galvanometer ceases. A couple of junctions of this kind, with an intervening double wire and galvanometer, form a differential thermometer, the indications of which must be interpreted with reference to the thermo-electric diagram of the two metals used.

Fig. 213.



As sources of electricity, thermo-electric piles are not much in use. Becquerel's thermo-electric piles, made of thirty pairs of blocks or rods of artificial sulphide of copper (which fuses only at about 1000° C.) and of German-silver, can decompose water when the differences of temperature employed are from 250° to 300°. In Řebiček's form of Noë's thermo-electric pile twenty-five pairs of plates of German-silver and of an alloy of zinc and antimony are arranged round a Bunsen gas-burner: each such pile maintains an effective difference of potential of from 2 to 2·75 Volts as long as the Bunsen burner is kept lighted, while the internal resistance is 0·75 Ohms. In Clamond's pile about 6000 couples of iron and of bismuth-antimony alloy are ranged round a coke fire, and the E.M.D.P. produced is, if the couples be arranged in file, about 218 Volts. The disadvantage of thermic piles as sources of electricity is that, in general, the E.M.D.P. produced is so extremely small that moderately-slight external resistances make the current extremely weak; and even in Clamond's pile, which is able to keep a pair of electric arc-lamps in action, about 95 or 96 per cent of the heat of the fire is not converted into the energy of a current, and is thereby practically wasted. For many purposes, such as electroplating on the small scale, Noë's batteries, three of which produce an E.M.D.P. nearly equal to that produced by seven Daniell's cells, are very useful, for when they are once built up their current can be produced or arrested at will. An arrangement like that of Fig. 213 has been used as a self-acting source of electrical currents, and therefore of energy sufficient to maintain in action a self-winding clock.

The most important source of electricity is the transformation of the energy of work into that of electrical separation by means of magneto-electric and dynamo-electric machines, the action of which will be explained in the sequel.

Atmospheric Electricity.—The atmosphere in different regions is often found to be at different local potentials, which differ from that of the earth sometimes even by as much as 3000 Volts within 100 feet. This is possibly (Tait) due to a contact-effect between air and aqueous vapour. A conductor insulated from the earth may be brought to the same potential as any point in the air, by leading to that point a metallic wire, and by furnishing this exploring wire with an extremely fine point, or, better, by fixing at its extremity a sponge dipped in spirit and set on fire, or a little cistern from which a quantity of water is allowed to drop. In the former case the flame continuously conveys masses of gas away from the end of the exploring wire; and as long as there is any difference of potential between the region of the air explored and the conducting system of which the exploring wire forms a part, there will be a current along the wire, and finally the whole conducting system will come to the same potential as the air around the flame. Similarly, waterdrops, on falling from an insulated cistern, bring the cistern to the same potential as the air around it : each drop, just before falling off, becomes electrified with a charge opposite, while the nozzle, the cistern, and the main mass of water are electrified with a charge similar to that of the air in the neighbourhood of the falling drop. As the drop is in the act of falling off, it is attracted by the cistern : it is held back as it falls : it falls down with less speed than it would have assumed if it had fallen from an uninsulated cistern ; and when it reaches the ground it produces less heat. The energy of the electrification acquired by the cistern is equal to the missing kinetic energy of the falling drops.

In an analogous way the air within a room may be strongly electrified ; connect a flame with the conductor of an electrical machine, and work the machine : in one minute a Holtz machine will raise the potential of the air of a room by 2000 Volts.

Differences of potential which have been once originated may be increased when work is done from without against the electric force.

Charge a plate to potential V ; its free capacity being C, its charge is CV. Bring up to it a second plate, parallel and at a distance *t* ; the two plates now form a condenser. The capacity of this condenser is $C_{,} = k \times$ surface$/4\pi t$ (p. 548). The charge will now be divided into two portions, and the potential will sink to $V_{,}$. The one portion of charge is "free charge" $= (C/\overline{C + C_{,}})\, V_{,}$; the other is bound charge $(C_{,}/\overline{C + C_{,}})\, V_{,}$. On the second plate there are induced an attracted charge $-(C_{,}/\overline{C + C_{,}})\, V_{,}$ and a repelled charge $+(C_{,}/\overline{C + C_{,}})\, V_{,}$. Withdraw the latter by earth-contact. Remove the second plate to a distance *d* ; the capacity of the condenser decreases in the ratio t/d ; the charge remains constant ; the potential of the charge on the second plate rises in the ratio d/t. As the capacity of the condenser diminishes, the inducing plate reverts to its original condition ; and when *d* is great enough the whole of the original charge once more becomes "free."

The **Electrophorus** consists of a cake of resin or vulcanite and an insulated metallic plate. The former is slightly charged by being rubbed with a catskin or a dry silk handkerchief : the metallic plate is then laid upon it. The contact between the two can never be perfect at all points ; practically there is an intervening film of air between the resin and the metallic plate, and the latter is charged by induction with an attracted and a repelled charge. The latter may be withdrawn by touching the metallic plate with the finger, or by making metallic communication between the metallic plate and the earth ; the former charge remains, facing and attracted by the original charge on the resin. Work is now done from without in pulling the metallic plate away from the resin ; as the distance between the metal and the resin increases, the electrostatic capacity of the electrophorus, considered as an accumulator, diminishes ; the potential therefore increases both on the metal and over the resin : the knuckle applied to the edge of the insulated metallic plate may now receive a spark. When the metallic plate is next laid on the resin a new charge is induced in it, which may again become a charge of high potential when the plate is removed. Small original charges may thus induce successive charges of high potential. (Holtz, Voss, Wimshurst.)

Thomson's Replenisher.—This instrument, which is used as a means of keeping the Leyden jar connected with the suspended needle of Thomson's electrometer at a constant potential, is sketched in the accompanying diagram (Fig. 213*a*). A, B, two metal half-cylinders, insulated from one another; C, D, two metallic plates insulated from one another and capable of rotation round the axis O; E, F, an insulated spring capable of touching both C and D when they are in the position shown in the figure; G, H, two springs connected with A and B and capable of being pressed upon by C and D as they rotate. Start from the position shown in the figure. B is positively charged by contact with one of the plates of the Leyden jar; C becomes negatively, and D positively, charged. Rotate C to the left, D to the right. Their metallic connection with one another is broken and they remain oppositely charged. As they pass G and H, C's − charge is wholly conveyed to A, and D's + charge to B; and thereafter D and C stand in the former positions of C and D. The + charge of the + plate of the Leyden jar may thus by continuous rotation of CD be continuously increased. If, on the other hand, C be rotated to the right, D to the left, C's negative charge is conveyed to B, and the positive charge of the + plate of the condenser may, by continuous rotation in this sense, be reduced to any desired extent, or even reversed. The potential of the Leyden jar may thus be adjusted to any desired amount, which may be determined by a subsidiary pair of plates, connected with the inner and outer coatings respectively and separated by springs, coming to assume a position at any pre-arranged fixed distance from one another.

Fig. 213*a*.

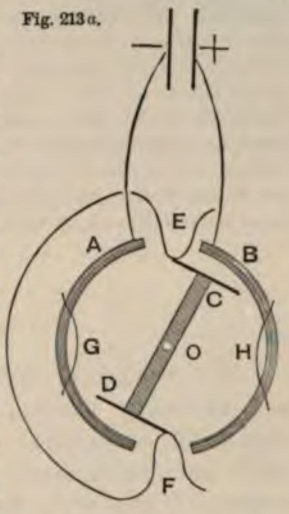
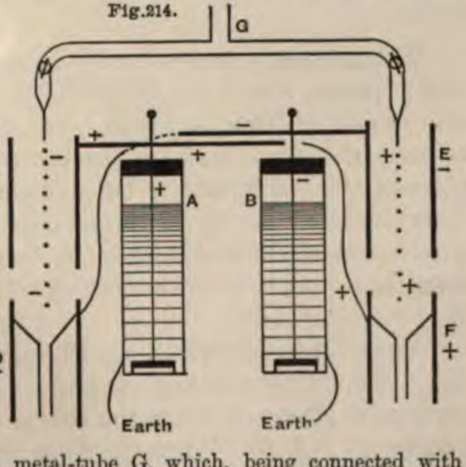


Fig. 214.



Thomson's Water-Gravity Electric Machine.—In Fig. 214, A and B are two Leyden jars, whose inner coatings consist of sulphuric acid and are connected with the metal tubes C, F and D, E respectively. C and D are co-axial: so are E and F. Water falls in drops from the bifurcated metal-tube G, which, being connected with the ordinary water supply, is in communication with the earth, and is therefore at zero Potential. A small initial charge, consisting (say) of positive electricity, is imparted to one of the Leyden jars, say A. Water is made to flow from G in streams so thin as to break up into drops within the tubes C and E. Just before these drops break off from the stream, they are by induction within C charged with negative electricity, while the complementary positive charge is conveyed along G to the earth. When the drops have become separate, they fall down charged negatively. They then fall upon a metallic funnel placed in the tube D, and charge the exterior of that tube negatively : this charge is shared with the Leyden jar B. This Leyden jar, thus negatively charged, by a corresponding inductive action causes the drops which fall through E to become positively charged. When these drops fall upon F they increase the positive charge of the Leyden jar A. Thus the Leyden jars A and B become more and more highly charged, the one with positive, the other with negative electricity on its inner coat. The energy of their electrification is derived from the work done by gravity upon the falling water ; and thus this contrivance, due to Sir William Thomson, is an electrical machine worked by gravity.

ELECTRICAL CURRENTS.

In a conductor, say a wire, along which a current is steadily and uniformly passing, there is no internal accumulation of electricity, no density of internal distribution; there is, on the other hand, an unequally-distributed charge of electricity on the surface of the wire, which results in a potential diminishing within the wire from one end of the wire to the other.

The superficial distribution of electricity over the surface of a conducting aërial wire is not so great as, but otherwise resembles, that over the surface of the metallic core of a submarine cable through which a steady current is flowing.

The **Intensity** of a current—*i.e.*, the Quantity of electricity which passes across any cross-section of the conductor during one second of time — depends, on the one hand, upon the effective difference between the potentials of different parts of the conductor, and, on the other, upon the nature of the conductor—that is to say, upon its size and its substance. A thin wire is a worse conductor—has less Conductivity and offers more **Resistance**—than a thick one: a silver wire conducts better than a copper one.

The relation between E the electromotive difference of potential, I the Intensity of the current, and R the Resistance of a uniform conductor, is, when the flow is uniform, expressed by the equation, $I = E/R$. When there are several sources of difference of potential within the circuit and several successive conductors, each of which offers its own resistance to the onward flow of the current, the law assumes the generalised form that $I = \frac{\Sigma E}{\Sigma R} = \frac{\text{effective sum of all the differences of potential}}{\text{sum of all the successive resistances}}$. This is **Ohm's Law.**

The C.G.S. Electrostatic Unit of Intensity is the intensity of a current in which one C.G.S. electrostatic unit of quantity passes a given section of the conductor during one second. It is the current which passes when the difference of potential $E = 1$ C.G.S. electrostatic unit, and the total resistance is also $R = 1$ C.G.S. electrostatic unit of resistance.

The C.G.S. Electrostatic Unit of Resistance is the resistance offered by a conductor which, when it is interposed between two bodies whose potentials are maintained at a constant difference of one C.G.S. electrostatic unit, allows one C.G.S. Electrostatic unit of Quantity to pass along it, per second.

These units are inconvenient for practical purposes, and electricians use as their practical units certain fractional or integral multiples of these.

The Resistance of a uniformly-cylindrical conductor, such as a wire, depends upon three things: (1) its length l, directly; (2) its cross-section s, inversely; (3) its Specific Conductivity c, inversely. It is therefore equal to $l/sc = R$.

The reciprocal of c, $(1/c) = r$, the Specific Resistance of a substance. The resistance of a conductor of length l and cross-section s is therefore equal to $lr/s = R$.

The specific conductivity, c, of any substance is a constant, special to each substance, and even found to differ from sample to sample of that which is nominally the same substance. It represents numerically the number of electrostatic units of electricity which can pass per second between two bodies kept at a constant potential-difference of one unit, when the conductor interposed between these bodies has a length of 1 cm. and a cross-section of 1 sq. cm. It varies very greatly from one substance to another.

In the following table the first column of figures gives the specific resistances, the next column the specific conductivities, of a certain number of substances; while the third column gives the numbers which denote their relative conductivities when the conductivity of mercury is taken as a standard and called unity. It is very usual to take the conductivity of silver as a standard = 100.

The numbers in the following table have (with the exception of those for the last four substances) been calculated from the data of the authorities named, on the assumption that a rod of mercury 1 metre long and 1 sq. mm. in section has a resistance of 953/900,000000,000000 C.G.S. electrostatic units = ·953 Ohm; or, in other words, that the **Ohm**, the British Association or practical unit of resistance (the resistance of a certain standard

Substances.	Specific Resistances in C.G.S. Electrostatic Units.	Specific Conductivities in C.G.S. Electrostatic Units.	Conductivities; Hg = 1.
Mercury	95300 ÷ v^2	·000010,49318 × v^2*	1 (Matthiessen)
Soft Silver	1451·86 ,,	·000688,772 ,,	65·64 ,,
Soft Copper	1544·57 ,,	·000647,429 ,,	61·70 ,,
Soft Gold	1993·31 ,,	·000502,833 ,,	47·92 ,,
Zinc	5439·50 ,,	·000183,840 ,,	17·52 ,,
Platinum	14752·04 ,,	·000067,7872 ,,	6·46 ,,
Antimony	34157·70 ,,	·000029,2756 ,,	2.79 ,,
Bismuth	127066·30 ,,	·000007,869886 ,,	0·75 ,,
Nitric Acid, 29·7 % (Kohlrausch)	1300,136300 ... ,,	·000000,000769 ,,	0·000073,3
Sulphuric Acid, 45·84 % (Wiedemann)	1173,672500 ... ,,	·000000,000852 ,,	0·000081,2
Do. 11·42 % do.	2171,500500 ... ,,	·000000,000461 ,,	·000043,89
Do. 3·37 % do.	7361,267960 ... ,,	·000000,000135,85 ,,	·000012,95
Do. concent. do.	7494,036000 ... ,,	·000000,000133,44 ,,	·000012,72
$ZnSO_4$ sol. saturated (Kohlrausch)	21084,070000 ... ,,	·000000,000047,43 ,,	·000004,52
$CuSO_4$; sat. sol. (Kohlrausch)	21659,100000 ... ,,	·000000,000046,17 ,,	·000004,40
Pure water (Kohlrausch)	...	...	·000000,000025
Glass (Beetz); the mean of several experiments—			
At about 345° C.	...	·000000,000000,002623 ,,	·000000,00025
At about 190° C.	...	·000000,000000,000058,3 ,,	·000000,000000,55
At ordinary temperatures	Immeasurably great.	Immeasurably small.	Immeasurably small
Guttapercha (Ayrton and Perry)—			
At 83° C.	500,800000,000000,000000 ... ,,		
At 24° C.	83000,000000,000000,000000 ... ,,		
Paraffin (Ayrton and Perry)—			
At 46° C.	34,000000,000000,000000,000000 ... ,,		
At 77°·8 C. (melted)	1,354000,000000,000000,000000 ... ,,		
Vulcanised rubber (Ayrton and Perry)—			
At 67° C.	5,391000,000000,000000,000000 ... ,,		
At 90°·7 C.	1,015000,000000,000000,000000 ... ,,		
Ebonite (Ayrton and Perry)—			
At 36° C.	61,030000,000000,000000,000000 ... ,,		
At 96°·8 C.	9,696000,000000,000000,000000 ... ,,		

* v^2 = 900,000000,000000,000000. (See p. 658.)

wire, which is intended to represent 1000,000000 "electromagnetic" C.G.S. units, or the 900,000,000000th part of a C.G.S. electrostatic unit of resistance), is the resistance of a column of mercury 1 sq. mm. in section and 1·049318 metre in length. If, in the report of the Committee of the Paris Electrical Congress of 1881, appointed to consider this matter, it be declared that any other value of the Ohm (as measured in lengths of mercury-column) is the true one, then the absolute values in the first two numerical columns of the above table will have been found to be erroneous, and will all have to be proportionately corrected; but the relative values in the third column will remain. In the meantime, Lord Rayleigh and Mrs. Sidgwick find that the value of the British Association Standard Coil is 0·98651 times, or by a later measurement, 0·98677 times its intended value in C.G.S. units.; Messrs. Glazebrook, Dodds, and Sargant assign to it the value 0·986439 times the theoretical Ohm, and Messrs. Glazebrook and Sargant a value of 0·98665; Prof. Rowland gives the number 0·9912; the mean value seems to be (Brit. Assoc. Committee, 1883) = 0·9867; while Joule assigned to it the value 0·9873.

Joule's conclusion was arrived at thus: the heat required to raise 1 lb. water from 60° to 61° F., was equal to 24,868 foot-poundals; the heat derived from a known absolute current passing through a resistance of known amount (as compared with the B.A. Standard Coil), gave the value 25,187 foot-poundals to the same quantity of heat, on the assumption that the B.A. Standard Coil represented its intended value; these discrepant values can be reconciled by assuming that the B.A. Coil is really equal to ·9873 Ohm.

From the value B.A. Coil = 0·9867 Ohm, coupled with the value assigned by Matthiessen to the specific resistance of mercury, it would follow that the resistance of a metre of mercury 1 sq. mm. in section is not 0·953 but 0·9403251 Ohm; but the conductivity of mercury is such as still further to modify this value, and to replace the figure 0·953 above not by 0·9403251, but by 0·9413: and accordingly the Ohm is the resistance of a mercury column 1 sq. mm. in section, and 106·24 cm. in length.*

The conductivity of an Ohm-Coil may (Sir Wm. Thomson) be called a Mho.

If the foregoing table be read without the multiplier or divisor, v^2, it then expresses the specific resistances and conductivities in another system—the Electromagnetic system of C.G.S. units, from which the Ohm and the Volt are primarily derived, the Ohm being 10^9 electromagnetic units of resistance, and the Volt 10^8 electromagnetic units of potential-difference. This system depends upon the laws of Magnetism, afterwards to be explained.

The conductivity of metals decreases, that of some bad conductors or insulators increases, with their temperatures: a heated wire or dynamo-electric machine increases the resistance in the circuit of which it forms a part. Very roughly, and with well-marked exceptions in the cases of iron and mercury, the resistance of a metallic conductor is proportional to its absolute temperature.

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* **The Congress-Ohm.**—The above paragraphs have been allowed substantially to remain because the Paris Committee in May 1884 issued a report not declaring the true value of the Ohm, but recommending a practical, but only an approximate, representation of that unit of resistance in the shape of a column of mercury 1 sq. mm. in cross-section and 106 cm. in length at 0° C.; the mean of all the values published before that date being 106·02.

When metals melt their conductivities fall suddenly. Alloys are in general worse conductors than the arithmetical consideration of their percentage composition and the conductivities of their component metals would lead us to expect.

There is a broad resemblance between the conductivities of metals for electricity and for heat: the best conductors of the one are in general the best conductors of the other; and in both cases alloys offer a relatively high resistance. The series are, however, not identical.

Variable Conductivity.—Conductivity varies not only with varying temperature, but also with varying magnetisation, tension, torsion, or pressure. It increases with longitudinal stretching, diminishes with longitudinal compression of a wire, and diminishes in iron, but increases in tin and zinc, when the stress, being transverse, tends to widen the wire (Tomlinson). In powders or porous material, such as metal filings, platinum sponge, charcoal, it increases with the pressure; and if the pressure vary within small limits, the variations of conductivity follow and are proportional to the variations of pressure. This is the principle of the Microphone. In such materials Heat raises the internal pressure and therefore the contact, and this modifies the amount of resistance and the heat produced within the conductor: this last itself affects the conductivity, as in the Tasimeter, which detects changes in temperature by the variation of a current passing through a rod of carbon fixed between metallic supports, and exposed to varying temperatures. Selenium, which in the amorphous form is a non-conductor, but in the crystalline form is a conductor, varies in conductivity with its state of aggregation, its temperature, the length of time during which a current has been passing through it; and crystalline selenium, when acted upon by light (especially the yellow and the red), and to a less extent when acted upon by dark rays, increases in conductivity: in the case of very bright sunlight this increase being sometimes even tenfold. Light of variable intensity produces corresponding and rapidly-responding variations in the conductivity of the crystalline selenium upon which it may fall—a fact utilised in the construction of the Photophone. Metals, unlike selenium, become worse conductors as the temperature rises; but Siemens asserts that at 210° C. selenium changes its character and comes to act like a metal.

Reduced resistance and reduced length of a Conductor.—This may be explained by a few numerical examples. We suppose the unit of resistance to be the Congress-Ohm, as above defined, the resistance of freshly-distilled mercury in a column of 1 sq. mm. section and 1·06 metres in length.

1. What length of soft-copper wire of 1 sq. mm. sectional area will give a resistance equal to one Congress-Ohm? $1{\cdot}06 \times 61{\cdot}70 = 65{\cdot}402$ metres. The figure 61·70 is taken from the table of conductivities above. The Resistance of 65·402 metres of copper is thus equal to that of 1·06 metres of mercury: the Reduced Length of 65·402 metres of copper is 1·06 metres of mercury.

2. What will be the resistance of a column of mercury 100 metres long and 1 sq. cm. in section? It will be equal to that of a column of mercury 1 sq. mm. in section × 1 metre in length multiplied by $l = 100$, and divided by $s = 10^2$. It is therefore 0·9434 Congress-Ohm. Its reduced length is 1 metre of standard mercury-column.

3. What will be the absolute resistance, and what the resistance in Ohms, of 100 metres of platinum wire whose diameter is $\frac{1}{4}$ millimetre? Its sectional area $s = \pi r^2 = \pi \left(\frac{1}{80}\right)^2$ sq. cm. $= \frac{\pi}{6400}$ sq. cm.; its length $l = 100{,}000$ cm.; its specific resistance r is $14752{\cdot}04 \div v^2$; the Resistance of the wire is

$$\frac{lr}{s} = 100{,}000 \times \frac{14752{\cdot}04}{v^2} \times \frac{6400}{\pi} = \frac{3{,}005{,}320{,}000000}{v^2}$$

C.G.S. electrostatic units, or 3005·32 Ohms, such as those of the table, p. 580 (*i.e.*, resistances of 1·049318 metres mercury-column), or $\{3005{\cdot}32 \times (1{\cdot}049318 \div 1{\cdot}06)\}$ Congress-Ohms.

The **intensity** of a steady current is measured by a Galvanometer (pp. 647, 661), round the magnetic needle of which the current is passed: in the Tangent Galvanometer the tangent of the angle of deflection of the needle is proportional to the intensity of the current.

The intensity of a current is equal throughout all parts of a circuit in which there is a steady flow. A magnetic needle is equally deflected when brought into the neighbourhood of any part of the circuit, whether the circuit be locally composed of solid, of liquid, or of heated or rarefied gas.

The practical unit of intensity is the intensity of that current which is produced in a conductor whose total resistance is 1 Ohm (= 1/900,000,000000 C.G.S. electrostatic unit), when there is kept up between its extremities a potential-difference which constantly amounts to one Volt, or 1/300 C.G.S. electrostatic unit.

$$\text{Since } \mathrm{I} = \frac{\mathrm{E}}{\mathrm{R}} = \frac{1 \text{ Volt}}{1 \text{ Ohm}} = \frac{1/300 \quad \text{C.G.S.E.S. unit}}{1/900{,}000{,}000000 \ \text{C.G.S.E.S. unit}} = 3000{,}000000,$$

the practical unit of intensity, the **Ampère**, is equal to 3000,000000 C.G.S. electrostatic units of intensity.

In a current whose intensity is one Ampère, the practical unit of quantity, the **Coulomb**, passes any given section during each second: the Coulomb is thus equal to 3000,000000 C.G.S. electrostatic units of quantity.

Electrical engineers have adopted the Ohm, the Volt, etc., as means of practical measurement. The Ohm and the Volt in electrical workshops are not abstract calculations, but standard wires and standard batteries (or multiples or fractions of these), by comparison with which the resistance or the E.M.D.P., the so-called electromotive force, of any given combination of materials may be relatively measured.

Dimensions of Electrostatic Measure.—Intensity—a quantity passing per second: $[I] = [Q/T] = [M^{\frac{1}{2}}L^{\frac{3}{2}}/T] \div [T] = [M^{\frac{1}{2}}L^{\frac{3}{2}}/T^2]$.

Resistance: $[R] = [E/I] = [M^{\frac{1}{2}}L^{\frac{1}{2}}/T] \div [M^{\frac{1}{2}}L^{\frac{3}{2}}/T^2] = [T/L]$; Conductivity $= [1/R] = [L/T]$, a Velocity.*

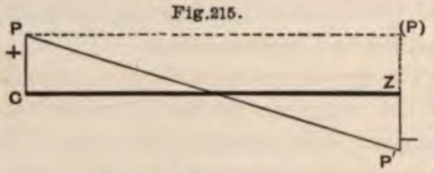
Specific Resistance: $[r] = [R] \times [s/l] = [T/L] \times [L^2/L] = [T]$.

Specific Conductivity: $[c] = 1 \div [r] = [1/T]$.

The above dimensions are based on the assumption that the Quantity of electricity in a current is the same thing as the Quantity of an electrostatic charge: they are therefore called Dimensions in Electrostatic Measure.

Fall of Potential in a Homogeneous Conductor of uniform thickness.—During the maintenance of a steady current one end of a homogeneous conductor is at a higher, the other at a lower potential, and between these points the fall is gradual, so that intermediate points are at intermediate potentials. Fig. 215 shows that if the length of the conductor be represented by CZ, the end C connected with the positive pole of the battery is at a potential which differs by (P)P′ from the potential of the end Z, connected with the negative pole. The Fall is steady, and depends (1) upon the difference of potential between the ends of the conductor, and (2) upon the length of the conductor; it is measured by the Slope of the line PP′, the amount of fall of potential per unit of length. It is not easy with short pieces of wire to observe the different potentials at different parts of the conductor; but when long wires are used, or when the current is made to pass through a column of water, electroscopes may then be attached to different parts of the wire or of the column of water, and will, if the battery be connected at its midpoint with the earth, show that the conductor is near C at a positive potential, that towards the midpoint this diminishes, that the midpoint of the conductor is a point of zero potential (the

Fig. 215.



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* Suppose a sphere of radius r and therefore of capacity $= r$ to be charged with quantity Q; the potential will be $V = Q/r$; and $Q = Vr$. If this sphere be connected with the earth by a wire, whose resistance is R, for a short time t, a current will run through that wire, whose mean intensity is I; the quantity conveyed by that current in time t is It; and this is lost by the sphere, whose charge sinks to Q'. Hence $Q - Q' = It$. If the potential of the sphere is not to sink, the radius must diminish. If the radius shrink to r' in time t, the velocity of its contraction is $(r - r')/t$: and $Q = rV$ as before; and also, $Q' = r'V$, V being unchanged. From these we find that $(r - r')/t = I/V = 1/R = C$, the conductivity of the wire. But $(r - r')/t$ is a Velocity; whence, in electrostatic measure the Conductivity of a wire is a Velocity.

potential of the earth), and that as we approach Z we find the potential increasingly negative.

The same diagram represents for a uniform conductor, such as a wire, the variations in the density of the superficial charge borne by the wire, and in virtue of which the potential falls: the potential within the wire varies as its superficial density, and the equipotential surfaces within the wire are equidistant and approximately plane. In conductors of less simple form the relation between the superficial charge and the internal equipotential surfaces is more complicated than this.

Resistance in a Heterogeneous Conductor.—When a conductor is made up of a succession of conductors which, on account of their differing materials or conditions or thicknesses, present different resistances to the current, it may become necessary to consider each conductor as reduced to an equivalent length of a standard conductor, such as a column of mercury 1 sq. mm. in cross-section. For example: a current passes successively along (1) a metre of mercury 1 sq. mm. in section; (2) 10 metres of mercury 1 sq. cm. in section; (3) 1 mm. of pure water 1 cm. in section; (4) 61·70 metres of soft copper wire 4 sq. mm. in cross-section: what is the total resistance of this combination? We must reduce all to a common term, to reduced lengths of our standard mercury column. Then above (1) is equivalent to a metre of such a column, (2) is equivalent to $\frac{1}{10}$ metre, (3) to 140,845 metres, and (4) to $\frac{1}{4}$ metre of such a mercury column; and the whole resistance is that of 140,846·35 metres of the standard conductor.

In a galvanic circuit we have to consider two sets of resistances: those internal to the cells, the internal resistance, R_i; those in the conducting media, the external resistance, R_e. Then Ohm's Law is $I = E/\overline{R_i + R_e}$.

Let n cells be arranged side by side, copper to copper, zinc to zinc; the E.M.D.P. of the combination is the same as that of one cell, and = E Volts; the internal resistance (the combination being virtually one cell of n-fold surface) is R_i/n Ohms; the external resistance is unaltered. The intensity is therefore

$$I = \left\{E/\overline{(R_i/n) + R_e}\right\} = \left\{nE/\overline{R_i + nR_e}\right\} \text{ Ampères.}$$

If the internal resistance be extremely small in comparison with the external, R_i may vanish from this expression; then $I = \{nE/nR_e\} = E/R_e$ Ampères, and there is found to be little advantage in the use of many cells; but if the external resistance be extremely small, the intensity becomes $\{nE/R_i\}$, and the side-by-side arrangement in Surface* is the best.

If n cells be arranged in file, copper to zinc, the E.M.D.P. is nE Volts;

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* Obsolete synonym—"*in Quantity.*"

the internal resistance is nR_i Ohms; and the external, as before, R_e Ohms. The intensity is now $I = \left\{nE/\overline{nR_i + R_e}\right\}$ Ampères. This arrangement of cells behind one another in Indian file or in Series* is the best when the internal resistance is extremely small in comparison with the external; for then, R_i vanishing, the intensity is $\left\{nE/R_e\right\}$ Ampères; while if the external resistance, on the other hand, be exceedingly small in comparison with the internal, the intensity is $\left\{nE/nR_i\right\} = E/R_i$, which differs but little from $\left\{E/\overline{R_i + R_e}\right\}$, the intensity of the current produced by one cell.

For extremely great external resistances, then, arrange in Series; for extremely small external resistances, arrange in Surface.

When neither the internal resistance nor the external can be considered as vanishingly small the one in comparison with the other, the best arrangement is to unite cells, ab in number, into a series of b each: in each series of b, the b cells are placed side by side, copper to copper, zinc to zinc; then a such series are arranged in file, the copper terminal of each series being connected with the zinc of the next. In this way we virtually make up a large cells, each of b-fold surface, and we arrange these in file.

In each of these virtual large cells the E.M.D.P. is E Volts; the resistance is 1-bth of r Ohms, the resistance of a single cell. Now couple a such large cells; the E.M.D.P. of the combination is aE Volts; the internal resistance of the whole, R_i, is equal to $a \times (r/b)$ Ohms; the intensity of the current produced is

$$I = \frac{aE}{a\,\frac{r}{b} + R_e} = \frac{aE}{\frac{a^2r}{n} + R_e} = \frac{E}{\frac{ar}{n} + \frac{R_e}{a}}$$

Ampères, where $n = ab$. The denominator of the last fraction is the least possible, and the value of the intensity consequently the greatest, when $R_e/r = a/b$. When the intensity is greatest, R_e is thus equal to ar/b, or the external resistance is equal to R_i, the internal. If the external resistance be equal to nr, and still more if it be greater than nr, the problem of the most advantageous arrangement of the cells in rank and file becomes an insoluble one, and the cells must be arranged in series.

Problem.

Sixty Grove cells, in each of which the resistance is ·6 Ohm, are at disposal: a resistance of 10 kilometres of soft copper wire of 4 mm. diameter is to be encountered; what is the best arrangement of the cells? The external resistance, R_e, is that of 1,000,000 cm. of copper wire of cross-section ·125664 sq. cm. and relative conductivity 61·70: this is equal to 12·16841 Congress-Ohms. Now in the equation $R_e = ar/b = a^2r/n$, $R_e = 12{\cdot}16841$, $r = 0{\cdot}6$, $n = 60$; whence $a = 34{\cdot}9$. The nearest feasible number corresponding to this value of a is 30; and the best arrangement is the division of the 60 cells into 30 virtual double-surface cells, arranged in Series.

If the external resistance be that of one kilometre of such wire, a being found equal to 11·03, the best arrangement is 12 sets of cells, each containing 5 cells joined in surface, these sets being joined to one another in Series.

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* Obsolete synonym—"*in Tension.*"

The Fall of Potential in a Heterogeneous Conductor.—If we draw a diagram, setting out on a base-line and using as abscissæ the reduced lengths of the several successive conductors which make up a heterogeneous conductor, and if for a moment we let drop from view the local differences of potential set up by contact of different materials, then the line of potentials slopes uniformly down from one end of the heterogeneous conductor to the other end, and from such a diagram we may find the total fall of potential within each component conductor. Fig. 216 very diagrammatically represents the fall of potential in the composite conductor specified in the preceding large-type paragraph, p. 585. AA′ is the potential at the junction of the slender and the thicker column of mercury, BB′ that at the one surface, CC′ that at the other surface of the water, OO′ and DD′ the terminal potentials.

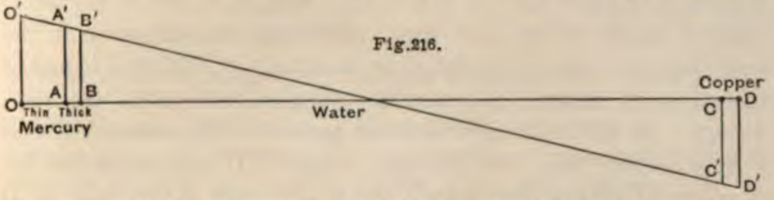


Fig.216.

If now we follow this up with another diagram in which the real lengths of the conductors are supposed to be represented, we find a remarkable appearance presented by it. The potential-line, which indicates the successive falls of potential, is represented by the line O′A′B′C′D′ in Fig. 217. The fall of potential is exceedingly rapid in the bad conductors, for bad conductors keep up a great difference of potential within their substance; and the whole fall of potential is distributed among the component conductors, to each according to its Reduced Resistance.

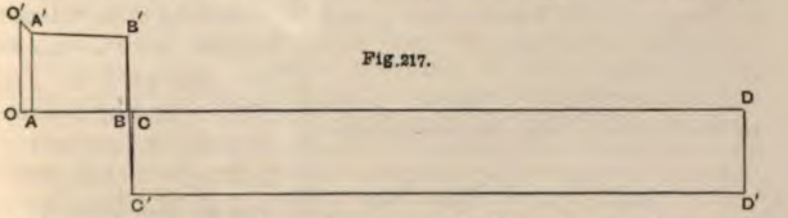


Fig.217.

If there be local differences of potential within the conductor, these must be added to or subtracted from the total fall of potential for which the conductor has to provide. Let AB be a conductor, of which one-half consists of copper wire, the other half of zinc wire, of an equal thickness, and let its extremities be kept at potentials which differ by 2AX. In Fig. 218 AC is the reduced length of the copper wire, and CB ($=\frac{61\cdot70}{17\cdot52}$ AC) the reduced length of the zinc wire. Between the copper and the zinc there is a rise of potential represented by DE, which makes the slope of the line of potentials steeper throughout the conductor. To BX′ add X′F, which is equal to DE: connect X and F by a dotted line. Of this the portion XD represents the fall of potential within the copper: the sudden rise of potential at D brings the line of potentials up to E, whence it is continued parallel to XF, along the course EX′, arriving at the terminal potential X′. From this diagram another may be constructed in which, instead of the reduced lengths AC and CB, the corresponding true lengths may be represented and the corresponding true slope of the potential-line found for each.

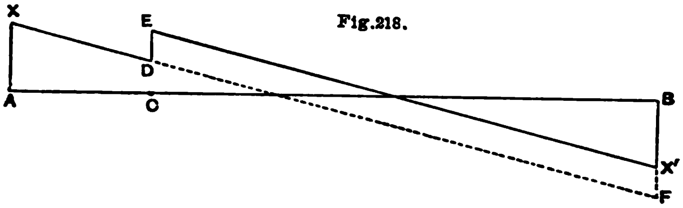


Fig. 218.

If we take as another example of a composite conductor the whole circuit of a single one-fluid galvanic cell, with its connecting wire, we find that the corresponding diagram presents, when the conductors are represented by their reduced lengths, a form such as that indicated in Fig. 219. The zinc is negative, the copper positive, relatively to the fluid; and at the junction of copper and zinc the potential takes a sudden rise, for the zinc is positive at the point of contact with the copper. We might again reduce this diagram to another in which the reduced lengths of the different parts of the circuit would be replaced by their true lengths, and the true slope of the potential-line found for each.

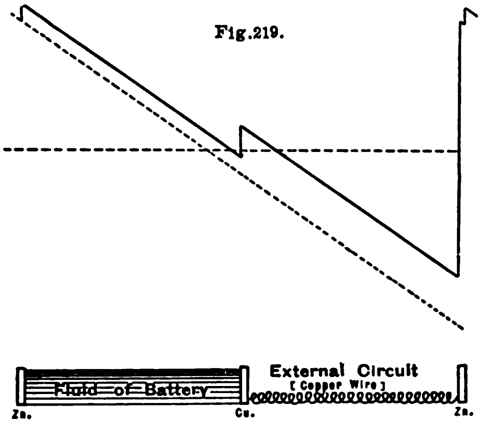


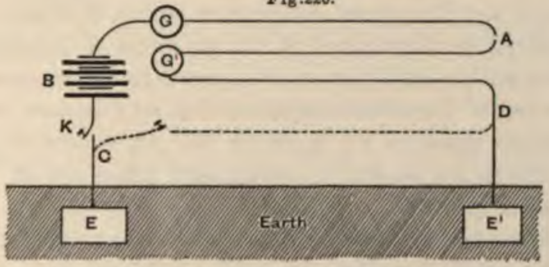
Fig. 219.

From this we find that the actual difference of potentials between the plates of a battery in a *closed circuit* depends upon the relation between the internal and the external resistance.

Flow through large Conductors.—If a conductor be very wide in comparison with the wires leading to and from it, the current widens out, and no part of the conductor is free from equipotential surfaces and lines of flow. If it be practically infinite the resistance offered by it depends on the radius of the wires or plates connecting it with the battery, and on the specific resistance of the conductor itself: not on the distance traversed by the current in the wide conductor.

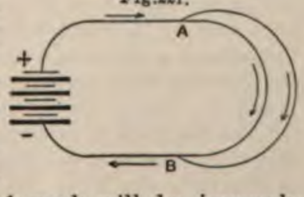
Some hold this to explain the fact that the earth itself can be used in telegraphy in place of a return wire; others hold that the earth in that case does not directly act as a conductor, but as a means of bringing the two plates embedded in it, and connected with the extremities of the single wire employed, to zero potential. In Fig. 220 the battery B is connected with two galvanometers, G and G′, by a long telegraphic wire interrupted at A: at K there is a key by which contact may be made or broken. CD is a wire, the continuity of which may be broken by another key. When C and D are connected through the wire CD, and connection made at K, both galvanometers are deflected. If, however, the connection CD be broken, and connection be suddenly made at K, the galvanometer G is alone deflected: the earth between E and E′ does not simply replace the wire CD between C and D. On the other hand it is beyond doubt that currents do run in the earth's crust. A telephone, part of whose circuit runs in the straight line joining two telegraph stations, will pick up signals from the earth-currents.

Fig. 220.



Derived Currents.—When a current in its course finds the conductor to divide and reunite, it divides into portions which run along the several paths open to it. In Fig. 221 the current arriving at A divides into two moieties; if the two paths be equal in their resistance, these moieties will be equal. If the resistances be not equal, the current passing along each branch will be inversely

Fig. 221.



proportional to the resistance, for the difference of potential between the extremities is the same for every branch, and in each branch the product of the intensity into the resistance is equal to the difference of potential.

The double path acts like a single conductor whose resistance is equal to $1/\{1/r' + 1/r''\}$, where r' and r'' are the resistances of the two branches. The conducting power of the double path (the reciprocal of its resistance) is the sum of the conducting powers of the two branches; these are respectively $1/r'$ and $1/r''$; their sum is $\{1/r' + 1/r''\}$, and the resistance of the double path is the reciprocal of this sum.

Kirchhoff's Laws.—I. Where a steady current branches, the quantity of electricity arriving by the single wire is equal to the quantity leaving the junction by the branches. The algebraical sum of the intensities of the currents passing towards (or passing from) the junction is equal to zero; $\Sigma \mathrm{I} = 0$.

II. In a metallic circuit comprising within it a source of permanent difference of potential E, the products of the intensity of the current within each part of the circuit into the corresponding resistance are, if the elements of current be all taken in cyclical order, together equal to E; $\Sigma (\mathrm{I}r) = \mathrm{E}$. In a metallic circuit in which there is no source of permanent difference of potential, $\mathrm{E} = 0$, and $\Sigma (\mathrm{I}r) = 0$.

This law applies to each several mesh of a wire network as well as to a single metallic loop, and it holds good even when an extraneous current is passed through the loop.

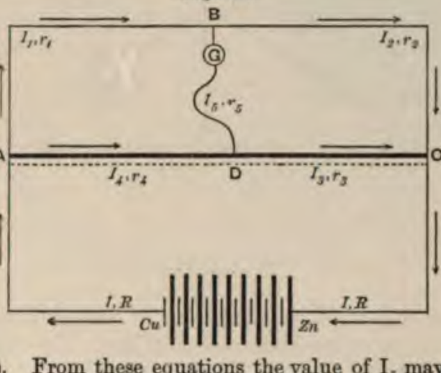
Shunts.—If between A and B (Fig. 221) a single wire run whose resistance is r, a certain current I will pass; if a lateral path or Shunt be made available, the resistance in which is $\frac{1}{99}r$, the current in the shunt is $\mathrm{I}_{,,}$ and the current in the original wire will sink to $\mathrm{I}_{,}$, 1-100th of its former intensity. This result may be found from the equations $\mathrm{I} = \mathrm{I}_{,} + \mathrm{I}_{,,}$ and $\mathrm{I}_{,}r - (\mathrm{I}_{,,} \times \frac{1}{99}r) = 0$. If the original wire contain a galvanometer which would suffer risk of damage if the whole original current were sent through it, the intensity can thus, by the use of shunts, be moderated to any desired degree.

If the shunt have a very high resistance the current running in it is proportionately very small, and the distribution of potential in the circuit, as well as the intensity of the current in the original path, is very little interfered with by the interposition of this new path. If in this new path there be arranged a galvanometer, the indications of this instrument will measure the intensity of the current, and therefore the difference of potential between A and B. This is realised in Sir William Thomson's Voltmeter.

The resistance of two conductors may be compared, by means of a Voltmeter, by observing the relative differences of potentials between pairs of equidistant points in the two conductors, when these conductors are successively, in the same circuit, traversed by one and the same current.

Wheatstone's Bridge.—In Fig. 222 there is represented an arrangement of conductors known by this name. The respective resistances, intensities, and directions of the current are indicated in that figure. Kirchhoff's Laws give us the relations between these. Law I. shows that at A, I (the intensity in the wire CuA) $= I_1 + I_4$ (1); that at B, $I_1 = \pm I_5 + I_2$ (2); and that at C, $I_2 + I_3 = I$ (3). Law II. shows that within the loop CZnCuABC, in which there is a source of difference of potential E, the E.M.D.P. of the battery, $E = IR + I_1 r_1 + I_2 r_2$ (4); while within the loop ABD there is no galvanic cell, $E = 0$ and $I_1 r_1 \pm I_5 r_5 - I_4 r_4 = 0$ (5); and similarly within the loop BCD, $I_2 r_2 - I_3 r_3 \pm I_5 r_5 = 0$ (6). From these equations the value of I_5 may be shown to be equal to zero, when $r_1 : r_2 :: r_4 : r_3$, or as regards resistances, AB : BC :: AD : DC.

Fig.222.



To compare the resistances of two conductors, one is placed between A and B, the other between B and C; then the end of the wire BD is moved along AC until the galvanometer G gives no deflection. At that moment the resistance of the conductor in AB is to the resistance of the conductor in BC as the length of AD is to the length of DC. A scale under the wire AC enables this last ratio to be read off. If one of the conductors compared—say that in AB—be a wire of known resistance, a Standard Resistance-Coil, the resistance of the other may be absolutely measured. If the resistance in AB be exchanged for a tenfold resistance, the value of the resistance in BC will seem to be numerically diminished to a tenth, and thus resistances ten times as great as before can be measured by being placed between B and C. In this way the range of the instrument can be increased.

When the above ratio obtains between the several resistances, the currents will remain unchanged whether BD remain open or closed, or be closed intermittently by a key.

Sometimes the battery is not kept continuously in action, and a key is interposed in BD: the resistances are adjusted until closing both the battery-circuit and the galvanometer-circuit produces no deflection in G; but to avoid complications due to Self-Induction (p. 639) the battery-circuit must be closed first and then the galvanometer-key pressed down. One key is so arranged as to perform these operations successively.

Measurement of the E.M.D.P. of a Galvanic Cell or Battery while in action.—One method out of many may be selected as an illustration. Take that of Wiedemann. Two batteries or cells, the one a standard; connect in series: the joint E.M.D.P. is $E_{,} + E_{,,}$; the total resistance is $r_i' + r_i'' + R_e$; the intensity of the current produced is $I = (E_{,} + E_{,,}) / \overline{r_i' + r_i'' + R_e}$. Now turn one of the batteries round, and connect so that the two now oppose one another; the joint E.M.D.P. is $E_{,} - E_{,,}$: the total resistance is as before: the intensity is $I' = (E_{,} - E_{,,}) / \overline{r_i' + r_i'' + R_e}$. Hence $E_{,} : E_{,,} :: (I - I') : (I + I')$. $E_{,}$, the E.M.D.P. of the standard cell, is known; I and I′, the intensities, can be observed: whence $E_{,,}$ can be calculated.

The Internal Resistance of a Battery may be measured (by Mance's method) by making it one of the four resistances within a Wheatstone's bridge (Fig. 223); one other of the resistances, say AB, being rendered adjustable either by making AB consist of standard resistance-coils or by the use of a Rheostat or Rheochord, by which variable quantities of wire or mercury, or fluids of various kinds equivalent to so many Ohms resistance, may be introduced into AB. A galvanometer is placed in AGC; a key in BD. The adjustable resistance in AB is varied in amount until the deflection of the galvanometer becomes unaffected by making or breaking contact in BD. The relation $R_{AB} : R_{BC} :: AD : DC$ again holds good.

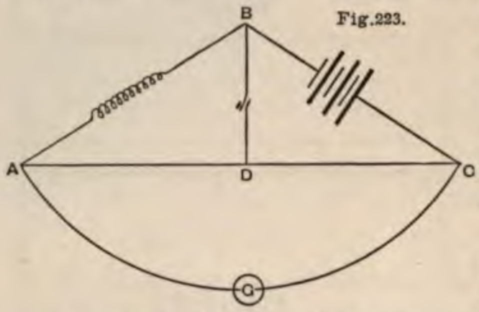


Fig.223.

If we make the galvanometer G and the battery between B and C exchange places, we have (Sir Wm. Thomson) a very easy method of finding the resistance of a galvanometer coil.

The Intensity of a Current is easily measured in Ampères when the number of Ohms resistance and the number of Volts which generate it are known. It may also be inferred from some of the effects of the current.

The Energy of a Current.—In a current of intensity I, a quantity, I, of electricity passes during each second from a place where the potential is $V_{,}$ to a place where it is $V_{\circ}$; but $V_{,} - V_{\circ}$, the fall in its potential, is E, the electromotive difference of potential within the circuit. This fall is constant, for the electromotive difference of potential is kept up: the Energy of the current per second is therefore $I \times (V_{,} - V_{\circ}) = IE$; or, in the course of that period of time during which a quantity Q passes, the Total Energy is equal to QE.

Since by Ohm's law $I = E/R$, we find that the Energy per second, $IE = E^2/R$; and that it is also equal to I^2R, per second.

The energy of a current of one Ampère-intensity under an electromotive potential-difference of one Volt is equal, since Energy per second = EI, to $(\frac{1}{300} \times 3000,000000) = 10,000000$ C.G.S. units or Ergs per second. This Rate of Transference of Energy, an Activity of 10 megergs (one Joule) per second, is sometimes called an Ampère-Volt or a Watt; and it is equal to 1/746 Horse-power nearly, or to 1/735·75 Cheval-vapeur.

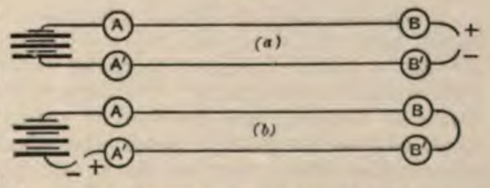
Static Charge of a Conductor.—If an insulated galvanic cell be symmetrically connected with two long wires, also insulated, the two wires become charged each with a uniform static charge, positive or negative, as the case may be. The potential is uniform on each side of the galvanic cell.

Two condensers may be charged to equal and opposite potentials by being metallically connected with the opposite poles of a battery of which the midpoint is connected with the earth.

If the copper of a cell be abruptly connected with the earth, the copper becomes reduced to zero potential, and the zinc side of the cell, together with any wire which may be attached to it, has its negative potential doubled; if, on the other hand, the zinc be suddenly connected with the earth, the positive charge borne by the copper, and by any wire connected with it, is doubled. In the former case there is an instantaneous current from the negative wire through the cell to the earth: in the latter case there is an instantaneous current from the earth to the insulated positive wire.

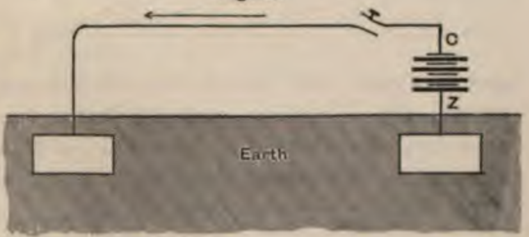
When an open circuit is abruptly closed for an instant, an instantaneous current is produced in the wire; this current is not felt simultaneously over the whole circuit. In the case (*a*) of Fig. 224 the galvanometers B and B′ twitch first when the interrupted circuit is momentarily completed; in case (*b*) of that figure, under similar circumstances, the galvanometers A and A′ twitch first.

Fig. 224.



When a current is actually passing along a conductor, the conductor bears a certain Superficial Charge, varying in density from point to point of its surface: if such a conductor be suddenly isolated by both ends from the circuit, it retains this charge; and not until a wire has had this requisite charge imparted to it can a Steady Current flow along the wire. Between the instant at which the current begins to flow and that at which it becomes steady, there is a period of adjustment, the **Variable Period.**

Fig. 225.



When, as in Fig. 225, a battery of which one pole is connected to earth has its other pole suddenly brought into communication with a long wire whose other extremity is connected with the earth, the time which elapses before the current through the wire becomes steady is found to vary as the square of the length of that wire. This has been proved by experiments on aërial telegraphic lines, and on extremely bad conductors, such as cotton threads, whose conductivity is due almost entirely to the slight film of hygroscopic moisture which covers their surface.

The time spent in acquiring at any point of the wire a certain fixed intensity of current depends (1) inversely upon the effective difference of potential set up by the galvanic cell; (2) directly upon the specific resistance of the conductor; (3) inversely upon its cross-section; (4) directly upon the square of its length; and (5) directly upon its electrostatic capacity.

The time which elapses before a certain proportion of the ultimate intensity is attained varies directly as the specific resistance of the wire, and very roughly as the square of the distance of the point from the cell, and also directly as the capacity and inversely as the cross-section: it is also, in practice, not independent of the effective difference of potential set up by the galvanic cell employed, or of the state of insulation of the wire.

A lightning discharge through a lightning conductor is so brief that the laws of steady flow do not hold good: it is of advantage, in order to diminish the risk of lateral divergence, to render the current more uniform, or, in other words, to retard it; for this purpose the capacity of the conductor should be increased, and therefore its surface; and lightning conductors should be broad flat plates of metal rather than compact rods. The effect of self-induction of the current also aids in bringing about this result; currents running parallel and in the same direction retard one another.

In a uniform wire, OL, between whose extremities a difference of potential is maintained equal to OP (Fig. 226), the ultimate Line of Potentials is PL; the same line indicates the uniformly-varying Density of the Superficial Charge from point to point; and when such a distribution has once been produced over the surface of the conductor, Ohm's law is obeyed; but at various instants during the preliminary variable period, the distribution of potentials over the wire is such as is indicated by the curved lines, 1, 2, 3, etc., sketched in Fig. 226.

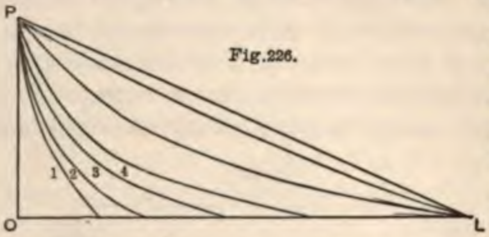


Fig.226.

The momentary and local intensity is always the momentary and local E/R, but during the variable period it varies from point to point and from instant to instant.

When the extremity of a long wire is momentarily charged by contact with a charged conductor or with one pole of a battery, its home end suddenly acquires a high potential, which is immediately thereupon reduced by communication of the charge acquired by the extremity of the wire to the rest of the wire. In Fig. 227 the end O of the conductor OL is suddenly raised to the potential OP. A point such as A is found, as it were, to leap up to a high potential and then to descend. A wave of sudden increase of potential thus travels along the conductor, but falls off progressively, both in abruptness and in height, the farther it travels.

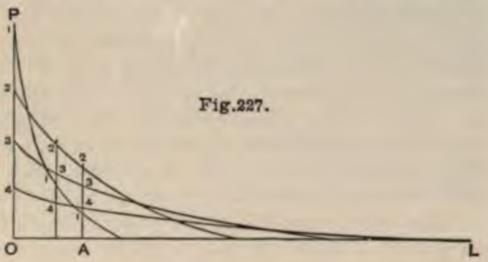
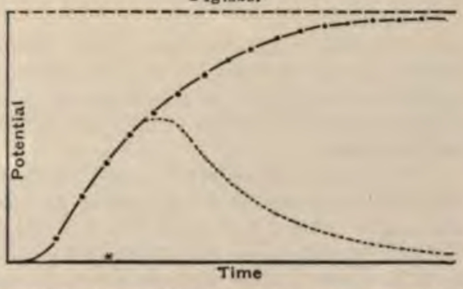


Fig.227.

At the distant end, for a short interval after the circuit has been actually completed, no effect is perceived; the current then begins to become sensible: and it then, if the contact be kept up at the home end, appears to increase in intensity after the manner indicated by the Arrival-Curve represented in Fig. 228. A current, even though it be constantly maintained at the home end, would take an infinite time to acquire its maximum value at the distant end of such a conductor as an Atlantic cable, if that conductor had, when the current commenced to traverse it, been uncharged; it would, however, require only about 108 seconds to attain $\frac{9}{10}$ of its maximum value, and about the fifth part of a second to attain $\frac{1}{100}$ of its maximum value. The apparent velocity of transmission of signals in a given conductor is thus seen to be entirely an affair of the delicacy of the instruments which detect the current on arrival at the distant end, and is perfectly distinct from the velocity of propagation of an electromagnetic disturbance; and it depends on the capacity of the conductor, the transmission being greatly delayed in conductors whose capacity is great, such as submarine cables, appreciably so in long air-lines, inappreciably so in short air-lines.

Fig.228.



When the current suddenly stops after having acquired a steady flow, its cessation at the distant end presents a similar deliberation.

When a wire is momentarily connected with a charged body and then connected with the earth, or "put to earth," the arrival-curve at its distant end is a curve due to the superposition of two arrival-curves; the first of these is the arrival-curve, resembling that of Fig. 228, due to the contact with the charged body; the second is curved in the opposite sense, and is due to the sudden discharge of the conductor. The dotted curve of Fig. 228 is the result of the superposition of two such opposed arrival-curves. This curve indicates that there is an abrupt and brief variation of potential at the end of the wire distant from the galvanic cell.

More effective still than this in producing an abrupt and brief current is the process of following up each positive charge immediately after putting to earth, with a negative one, after which the wire is again put to earth. The disadvantage of this is that the potential, while it abruptly ceases to be positive, sinks at once to a negative condition, as in Fig. 229; for which reason it is customary so to arrange the mechanism at the signalling station that each apparently simple making of contact is in reality a complex operation, in which an odd number of currents of opposite kinds are sent in rapid succession into the wire, the wire being, after each, put to earth; each of these currents being briefer than its predecessor, and correcting it. The arrival-curve for such a combination indicates an abrupt rise of potential, an abrupt fall, and then a slightly-wavy line, which at no point diverges to any material extent from the base-line.

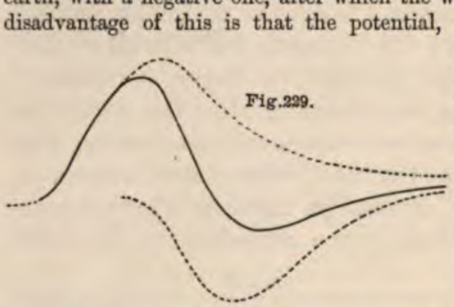
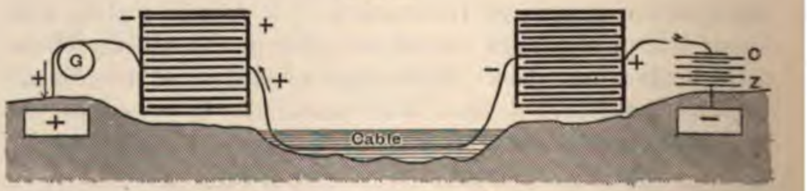


Fig.229.

Even these methods are increased in effect, the arrival-curve being rendered still more abrupt, by the use of Accumulators or Condensers, as they are usually called. Each condenser is composed of a large number of plates of tinfoil separated by waxed paper and paraffin: the alternate plates are in metallic communication with one another. One series of alternate plates in each condenser is in communication with the cable; the other set is in communication with the galvanic battery or with the galvanometer G (Fig. 230).

Fig.230.



Any sudden variation in the potential of the landward plates of the home condenser is immediately followed by an equally-sudden flow of electricity into or from the cableward plates of that condenser: this flow takes place either from or into the cable itself; this disturbance is propagated along the cable; the potential of the cableward plates of the condenser at the receiving station is affected; by induction the distribution of electricity in the landward plates of that condenser is affected, and a current passes through the galvanometer either from the condenser to the earth or in the reverse direction. On connection of the home condenser with the positive pole of the battery employed, a positive current runs through the distant galvanometer G to the earth; and on putting the home condenser to earth a reverse current passes through G, which may be corrected as before.

This cannot be done on land lines on account of defective insulation.

Even if the key be kept permanently pressed down at the transmitting station the current passing through G is but momentary, for both condensers quickly assume a condition of electrostatic equilibrium.

EFFECTS OF A CURRENT.

Production of Heat.—If a circuit be completed and allowed, as it were, to run to waste, no external work being done by it, heat is developed within the cell and in the conducting wire. The Heat produced represents the total Energy of the current, and is equal, like that energy, to I^2R units of energy per second (Joule's Law), or to E^2/R per second, where R is the total resistance of the circuit, and I the intensity of the current actually passing.

Problem.

A uniform copper-wire whose cross-section is 4 sq. mm., and whose length is 106 metres, connects the poles of a cell whose effective difference of potential is one Volt, and whose internal resistance is 4 Ohms. How much heat will be developed during one minute?

E is one Volt $= \frac{1}{300}$ C.G.S. electrostatic units of E.M.D.P. The total resistance, R, is 4 Ohms internal $+ \left(\frac{106}{1{\cdot}06} \times \frac{1}{4} \times \frac{1}{61{\cdot}70}\right)$ Ohms external $= 4{\cdot}405$ Ohms $= \frac{4{\cdot}405}{900000{,}000000}$ C.G.S. electrostatic units. The Heat $=$ Energy $= \frac{E^2}{R}$ per second $= \frac{1}{300^2} \div \frac{4{\cdot}405}{900000{,}000000} = 2{,}270{,}148$ ergs per second $= 136{,}208{,}880$ ergs per minute $=$ about $3{\cdot}3$ *ca* per minute.

When a current is made to pass through a heterogeneous conductor composed of different metals, between which there is developed a constant difference of potential, that energy which is wholly converted into heat when no work is done by the current is divided into two parts. Of these one part obeys Joule's Law, and is equal, per second, to I^2R, the product of the total resistance into the square of the actual intensity: the other, which may be positive or negative, goes to produce **Peltier's effect,** which is the following:—Consider a junction of metals, A and B, such that when this junction is made the hot junction of a thermo-electric circuit a current passes through it from A to B: let a current be made to run *ab externo* through that junction in the same direction, A to B; that junction will under such circumstances be cooled, while if, on the other hand, the current be made to flow from B to A, the junction will be heated.

In Fig. 231 OL is a conductor composed partly of iron, partly of copper; a current is made to flow through the conductor from O to L, between the potentials $V_{,}$ and V_{IV}. At the junction J there is a sudden fall of potential $(V_{,,} - V_{,,,})$. In OJ the intensity $= \frac{\text{Fall of potential}}{\text{Resistance of OJ}} = \frac{V_{,} - V_{,,}}{R_{,}}$; in JL the intensity is $\frac{V_{,,,} - V_{IV}}{R_{,,}}$. In both it is equal: hence

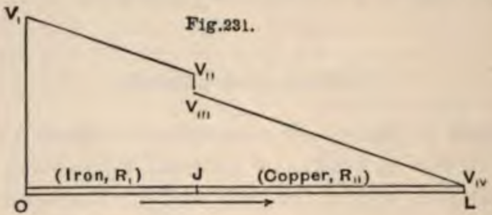


Fig. 231.

$$I = \frac{V_{,} - V_{,,}}{R_{,}} = \frac{V_{,,,} - V_{IV}}{R_{,,}} = \frac{(V_{,} - V_{IV}) - (V_{,,} - V_{,,,})}{R_{,} + R_{,,}} = \frac{E_{OL} - E_{J}}{R},$$

where E_{OL} is the total fall of potential, E_J the fall at J, and R the total resistance. Hence $E_{OL} = RI + E_J$. The Energy of the current is $E_{OL} \times I = RI^2 + E_J I$. The first part of this expression, RI^2, represents heat distributed over the whole conductor; the second part, $E_J I$, represents heat locally developed at J, and proportional to the *fall* of potential there. If the current be made to pass from copper to iron there will be a rise, a negative fall: the heat developed at the junction will be a negative quantity, and the junction will be cooled.*

In a thermo-electric circuit of copper and iron, the current flows from the copper to the iron across the hot junction. At the hot junction the current passes through a rise of potential; the current therefore tends to cool the hot junction. At the cold junction the current passes through a fall of potential; it therefore tends to heat the cool and to cool the hot junction. This is Peltier's Effect.

When a current is passed *ab externo* through iron, copper, iron successively, it again heats the iron-copper and cools the copper-iron junction.

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* Prof. O. L. Lodge points out that there is no such phenomenon at a junction of copper and zinc: whence he concludes that there is at such a junction no real fall of potential, and that the apparent D.P. of copper-zinc is really the sum of a copper-air and a zinc-air contact-difference.

The main current is weakened by the reverse thermo-electric current secondarily produced.

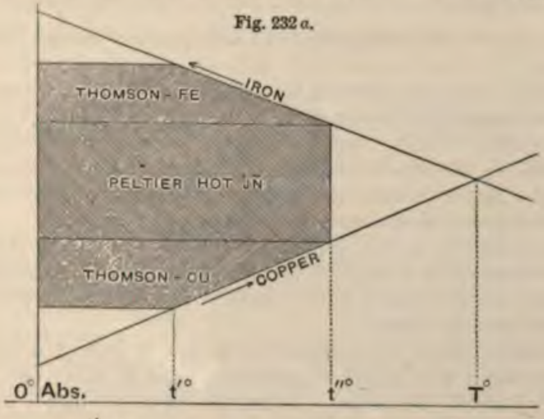
Thomson's Effect.—The same thing may occur even within a single metal. Hot iron is negative (or thermo-electrically positive) to colder iron; a current made to pass within a mass of iron from a hotter region to a colder travels against progressively rising potentials and cools the iron in the cooler region; made to pass from cold to hot iron, it heats the iron in the hotter region. It thus tends to exaggerate the existing differences of temperature. These effects are reversed in copper.

The convection of heat by a current of electricity in unequally heated iron is negative, for it is opposed to that convection of heat which would be brought about by the flow of water through an unequally heated tube. In copper, on the other hand, the electric convection of heat is positive.

In a thermo-electric circuit, therefore, the current, as it travels in the iron from hot to cold, absorbs heat; in the copper, travelling from cold to hot it again absorbs heat.

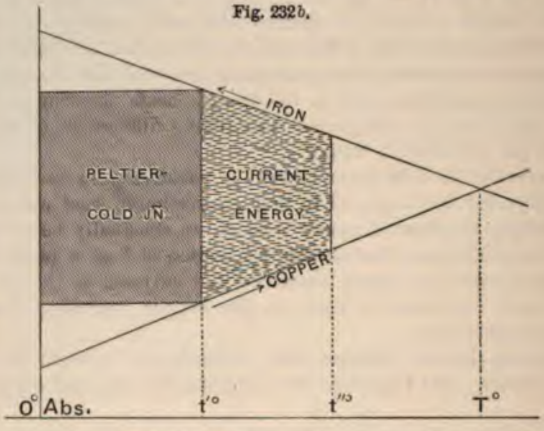
The Thermo-electric Diagram may be made to represent the Thomson and Peltier effects. Let Fig. 232*a* be a diagram for iron and copper between the temperatures $t_{\prime}$ and $t_{\prime\prime}$. The area marked "Peltier-hot junction" represents the amount of energy absorbed at the hot junction when a unit-current passes; the area marked "Thomson-Fe" represents the energy absorbed from the iron when a unit-current passes in it from hot to cold; the area marked "Thomson-Cu" in the same way represents the Thomson effect in the copper, the amount of energy absorbed from the copper when a unit-current passes in it from cold to hot. The whole shaded area thus represents the energy absorbed by the cooling of the hot junction and of the unequally heated iron and copper when the current runs in the direction indicated by the arrows. Plainly, if the hotter junction be heated to T, the neutral point, we shall have two Thomson effects, and, at the hot junction, no Peltier effect.

Fig. 232 *a*.



Now turn to the energy evolved. This takes two forms: (1) Heat liberated at the colder junction (Peltier effect); and (2) the Energy of Electric Current. The latter, when the current-intensity is unity, is equal to the E.M.D.P.; and we have already seen that this E.M.D.P. is represented by the area between two metal-lines and the ordinates corresponding to the two temperatures. Hence the accompanying diagram (Fig. 232*b*) needs little explanation. If the colder junction be at a temperature of T, there will at that junction be no Peltier effect, no absorption of energy by the colder junction.

Fig. 232*b*.



The student may now exercise himself in showing that when the colder junction is at temperature T the effect is the reverse of that obtained when the hotter junction is at T; that when one junction is as far below T as the other is above T, the area representing the Current-Energy vanishes; and that when the hotter junction is at a temperature farther above T than that of the colder is below it, the current is reversed.

In these figures the energy supplied is equal to the energy accounted for. The whole arrangement is a kind of thermic engine, in which Heat is absorbed from a Source, partly restored to a Condenser, and partly converted into the Energy of an Electric Current.

The Thomson effects are themselves reversed in iron at a low red heat, and probably again at a higher temperature, so as to make one if not two new neutral points. The same phenomena occur in nickel at low temperatures.

When a circuit is composed of various conductors which successively offer different resistances to the current, the heat produced is distributed among them, to each according to its resistance.

Numerical Example:—A circuit consisting of one cell whose E.M.D.P. is 1·8 Volts, and whose internal resistance is 0·7313 Ohm, and of an external conductor composed of 6·170 metres of soft copper-wire 4 sq. mm. in cross-section, in which is interpolated a piece of platinum wire $\frac{1}{10}$ mm. in diar. and 4 cm. in length, will have a total resistance amounting to—Battery 0·7313 Ohm, copper wire $\frac{1}{40}$ Ohm, and platinum wire (equivalent to a mercury column $\left(\frac{4}{100} \times \frac{1}{6 \cdot 46}\right)$ metres long and ·007854 sq. mm. in section) ·7437 Ohm; or on the whole 1·500 Ohms, or $\frac{1 \cdot 5}{900000{,}000000}$ C.G.S. electrostatic units. We assume that a steady current can be set up and maintained for a second within such a circuit, and further, that radiation and conduction of heat may be set aside. Of the total heat produced $\frac{0{\cdot}7313}{1{\cdot}500}$ is developed in the battery, $\frac{{\cdot}025}{1{\cdot}500}$ in the copper wire, and $\frac{{\cdot}7437}{1{\cdot}500}$ in the small piece of platinum wire. The total heat produced in a second is

$$\frac{\mathrm{E}^2}{\mathrm{R}} = \left\{ \left(\frac{1{\cdot}8}{300}\right)^2 \div \frac{1{\cdot}5}{900000{,}000000} \right\}$$

C.G.S. units or ergs; this is 21,600000 ergs or (21,600000 ÷ 41,593,010) *ca.* The heat evolved in the battery—·4875 of the whole—would be, if we suppose the battery to contain 1 kilogramme of material of a mean specific heat of 0·8, sufficient to raise its temperature by about ·000313° C. in a second; that evolved in the copper wire (whose weight is about 217 grammes and sp. heat = 0·095) by about ·0004° C.; while that liberated in the platinum wire (whose weight is about 0·0276 grms., and whose sp. heat = 0·0325) would be competent to raise it in a second to the temperature of 290° C.

Production of Light.—When one part of a circuit presents a relatively-great resistance, the greater part of the heat developed within the circuit is concentrated within that part. When the local resistance is due to a thin platinum wire or a thin filament of carbon or of carbonised paper or vegetable fibre, that bad conductor is so far heated as to emit a considerable amount of light. This is illustrated by the various forms of incandescent lamps or electric "glow-lamps."

Those in which the carbon filament is arranged within a vacuum give out, according to the type of lamp, the number in circuit, and the intensity of the current employed, a light equal to that of from 8 to 32 candles each; the fall of potential in each lamp, when not overdriven, is from 36 to 100 Volts, the resistance is from 42ω to 157ω (*i.e.* Ohms) each, and the intensity of the current necessary to keep up the light is from 0·60 Amp. to 1·37 Amp.; a light equal to that of from 90 to 250 candles being produced for each horse-power spent on the dynamo-electric machines which generate the electric current.

If the carbon filament be so constructed as to present the form of a hollow tube, of relatively-great surface and small actual cross-section, the luminous efficiency of the lamp is greatly increased (Bernstein, Cruto).

When too strong a current is driven through such a lamp, the superficial particles of the heated carbon are scattered throughout the vacuum; the carbon is volatilised and condensed on the wall of the lamp; so with platinum and iridium heated above 1700° C.

When a strong electric current is driven through a carbon rod to a thicker piece of carbon, the thin rod becomes heated; when this takes place in air the carbon burns away rapidly; but if the rod rest loosely by one end upon the thicker mass, the contact is always maintained, and the light is fairly steady so long as any carbon remains.

When the interposed resistance is that of a certain thickness of air, the current will not pass unless the interval be so small or the difference of potential on its two sides be so great that a spark can fly across it. When this is the case the current is established across the interval. If the poles be of carbon, their extremities become intensely hot and wear away by oxidation in the air. The intervening air is so good a conductor when intensely heated that, when the arc has once been established, the poles may be separated to a distance greater than the striking distance in cold air; still the resistance of the hot air will not alone explain the resistance offered by the voltaic arc to the transmission of the current. This is due to a kind of thermo-electric effect. The positive pole is hotter (4000° C.) than the negative (3000°–3500° C.); it becomes thermo-electrically negative; this tends to produce a reverse current. A phenomenon similar to this is observed between the various parts of a flame.

The positive pole, being the hotter, wears away about twice as fast as the negative, and becomes hollowed. The problem of electric lighting is to keep the arc in the same place, the carbons being allowed to approach one another as far as, and only as far as, is necessary in order to make up for their wear.

In Jablochkoff's and Jamin's candles the two carbons are rods, parallel to one another and of equal length; the arc passes between their apices. If the current passed in one direction only, one carbon would wear away faster than the other; the carbons would thus cease to be of the same length. The currents used must rapidly alternate in their direction; both carbons are thus equally worn away and the length of the arc is constant; but all such alternating currents make much noise in the lamp.

The usual fall of potential in Jablochkoff's lamps is from 42 to 43 Volts; the intensity of the current producing the light from 8 to 9 Ampères, and the candles per horse-power 300 to 360.

In arc-lamps two carbon points are placed opposite to one another, and it is the part of a special regulatory mechanism to keep the carbons at a constant distance apart. Such regulating mechanisms depend for their action (1) upon variations of intensity of the current traversing the lamp, or (2) upon variations in the differences of potential between the two ends of the arc, or (3) upon departures from a predetermined relation between this difference of potential and the intensity of the current, or (4) upon variations in the amount of heat developed in the arc.

The resistance of the voltaic arc is $1\frac{1}{2}\omega$ to 6ω; the fall of potential is from 32 to 58 Volts; a 12,000-candle lamp consumes about 7 horse-power, a 500-candle lamp from 1 to $1\frac{1}{2}$.

The heat developed in the arc has been utilised by Messrs. Siemens and Huntington, who produced the electric arc within the interior of a crucible, and by its means fused very refractory metals with considerable expedition.

When the electric arc is produced between carbons *in vacuo* a beautiful glow is obtained, the negative pole being surrounded by a blue aureole, and the positive by a stratified pale-blue light. The carbon evaporates, the vessel becomes filled with a blue vapour which darkens to indigo, and this condenses and renders the whole opaque.

If a very little vapour of bisulphide of carbon be introduced into the vacuum, the light becomes unsupportably bright, and of an extremely brilliant green. Its spectrum presents channelled regions in the red, yellow, green, and violet, which look like duplicates of one another, reproduced in different colours (Jamin).

Geissler's Vacuum Tubes.—When a discharge of high E.D.P., as from an electric frictional-machine, or an "induction coil," or a battery of 400 Groves, is sent through a mass of rarefied gas contained within a so-called vacuum tube, that gas glows with a bright light, characteristic, as regards its spectrum, of the gas exposed to this operation. The positive pole is surrounded by a bright glow, the negative by a set of striæ, and in this case the negative pole is the hotter.

If the vacuum be very good and the tube containing the rarefied gas be somewhat narrow at its middle, the glow breaks up into striæ, which flow and flicker if the current which produces them be in the slightest degree variable.

The discharge through a vacuum is shown to be disruptive by the fact that the fall of potential in the vacuum tube is independent of the E.M.D.P. of the circuit; and the striæ seem to indicate that it is pulsatile, not continuous.

The approach of external conductors repels the internal glow and causes its deflection; and the glow is deflected by a magnet in the same way as a wire bearing a current.

In very high vacua the discharges from the two poles of a vacuum tube appear to be independent of one another: each pole discharges itself without, as it were, feeling the condition of the opposite pole of the tube. Even where both are positive they may discharge towards one another into the same space.

Electrification of Radiant Matter.—When the rarefaction of a gas is extreme (one-millionth) its matter becomes radiant. The movement of its molecules may be guided and rendered manifest by electrification. In a Geissler's tube, the matter filling which is radiant, the molecules which come in contact with the negative pole are at once repelled from it in lines at right angles to its surface. Energy is imparted to these molecules by the electrified negative-pole, and where these molecules strike each other or other molecules they produce an internal glow; where they strike glass (or diamond or ruby) they produce light and cause phosphorescence; they also produce heat, so that when they are directed from a concave negative-pole upon a piece of platinum, the energy conveyed by them brings that piece of metal to its melting point; and when they strike a movable body they produce obvious mechanical effect (Crookes).

Two streams of molecules proceeding from a forked negative-pole repel one another like two similarly-electrified gold leaves, and the negatively-electrified particles which constitute such a stream are attracted and deflected from their course by a magnet.

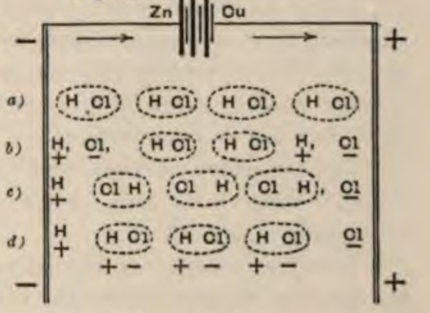
Chemical Effect of Electrification.—When two metal tubes, of which the one is positively, the other negatively charged, are arranged so as to form a double tube with an annular channel, and when gases are passed through this channel, then chemical effects are sometimes produced, as where oxygen subjected to this treatment becomes in part converted into ozone.

Prof. Schuster has shown cause for believing that in highly-attenuated gases there is around the negative electrode a dissociation of molecules into atoms. In mercury-vapour, which is monatomic, the phenomena of glow are the same round both terminals.

Chemical Effects of a Current—Electrolysis.—In most cases if a liquid permit the passage of a current through it, it is decomposed by the conductor; in other words, most liquids which possess conductivity are electrolytes. A few liquids, such as alcohol and ether, though not absolutely non-conductive, are not decomposed by the passage of a current. As a rule, substances which conduct when melted, but insulate when solid and cold, are electrolytes.

Let us take as an example the effect of a current upon a solution of hydrochloric acid in water. In such a solution insert two platinum plates,—the one, the **positive-electrode,** connected by wire with the copper plate of a sufficient battery; the other, the **negative-electrode,** with the zinc or negative plate. Each molecule of hydrochloric acid, HCl, consists of one atom of hydrogen, H, and one of chlorine, Cl; the former is electropositive, the latter negative. A molecule of HCl (the chemical symbol being used as an abbreviation of the name) near the negative-electrode is torn asunder; its electropositive hydrogen is attracted by the negative-electrode and liberated on its surface; the electronegative chlorine, by one means or another, finds its way to the positive-electrode, which by a secondary reaction it attacks and dissolves with the formation of $PtCl_4$. It is possible that the free chlorine-atom may travel directly to the positive-electrode, for when liberated it is in effect a charged particle within a field of force; but it seems more probable that a chlorine-atom is liberated at the surface of the positive-electrode at the same time as a hydrogen-atom is liberated on the surface of the negative-electrode, and that the remaining atoms and the intervening molecules readjust their mutual relations according to a method the mechanism of which is in all probability that indicated in Fig. 233. In that figure (*a*) represents a line of molecules arranged end to end along a line of force; (*b*) the effect of the decomposition of a molecule at each end of the line; (*c*) the mode of readjustment by what may be called a wholesale exchange of partners on the part of all the atoms all along the line, and the formation of a new set of molecules out of the materials of the old,—this being a process which there is chemical evidence for believing to be always going on irregularly, but which is here directed by the condition of the field of force between the electrodes; and (*d*) the reversal of the direction of all the molecules thus formed, every molecule being turned round in the field of force, so that its negative chlorine comes to face the positive-electrode. In course of time all the chlorine will be found to have travelled to the positive-electrode, and all the hydrogen of the hydrochloric acid to the negative-electrode.

Fig. 233.



In such a molecule as that of copper sulphate, $CuSO_4$, the copper plays the part of the hydrogen of the previous example, while the part of the atom of chlorine is taken up by the atom-group or salt-radicle SO_4. The group SO_4 could not undertake a trajectory through a fluid; it would break up into SO_3 and O; whence the necessity for the apparently complicated explanation of the last paragraph for the simpler but analogous case of hydrochloric acid.

The copper liberated at the negative-electrode forms a film upon that electrode; the SO_4 liberated at the positive-electrode reacts secondarily upon the water present; $SO_4 + H_2O = H_2SO_4 + O$; the positive electrode is surrounded by sulphuric acid, and oxygen is liberated on its surface.

If a solution of sulphate of potash be decomposed, the molecule K_2SO_4 is divided into K_2 and SO_4; the SO_4 causes the evolution of oxygen as a secondary product at the positive-electrode; the potassium, by the reaction $K_2 + 2H_2O = H_2 + 2KHO$, causes the evolution of hydrogen at the negative-electrode. Here the water seems to have been decomposed; the apparent decomposition of water is, however, a secondary result of the decomposition of the K_2SO_4. Pure water can only with extreme difficulty be decomposed; it is an extremely bad conductor, and perhaps if perfectly pure it would, like alcohol, not be found to be an electrolyte at all, and possibly not even a conductor. When metallic salts are dissolved in it, so that its conductivity is improved, it becomes an electrolyte—a property not confined to water, but possessed also by such substances as oil and bisulphide of carbon, which in the pure state are non-conductors.

The secondary reactions met with in electrolysis depend upon the time which is allowed for them, and are therefore favoured by currents of small intensity.

If copper chloride be electrolysed between copper electrodes, the one, the negative-electrode, is thickened by a deposit of copper, while the other is worn away, being attacked by the chlorine; and the intervening solution of copper chloride is, if the decomposing current be feeble, maintained in its state of saturation; but if the decomposition be very rapid the solution of the copper electrode does not keep pace with the evolution of chlorine upon it, and the solution becomes weaker in copper and acid in its reaction.

The secondary reactions observed are sometimes very peculiar. When hydrochloric acid is decomposed the chlorine evolved at the positive-electrode attacks the water present and liberates oxygen, which in its turn attacks some of the hydrochloric acid formed and produces chloric and perchloric acids. When Na_2CO_3 is decomposed Na appears at the negative pole (producing a secondary evolution of hydrogen) and $CNaO_3$ at the positive pole; this last group reacts upon water and forms oxygen and $NaHCO_3$; $2CNaO_3 + H_2O = O + 2HNaCO_3$. When $NaHCO_3$ is decomposed it produces Na and CHO_3, and then $2CHO_3 = 2CO_2 + H_2O + O$. When formic acid (H.COOH) is electrolysed it breaks up into H and COOH; then $2COOH + H_2O = 2H.COOH + O$, or formic acid and oxygen; but the oxygen reacts upon some of the formic acid present, and then $H.COOH + O = H_2O + CO_2$. When fused caustic-potash is electrolysed, K appears at the negative, HO at the positive ; this coalesces into H_2O_2, and breaks up into H_2O and O ; but if the action be slow the K acts upon some of the water, and hydrogen is evolved. The nascent hydrogen evolved at the negative pole will attack aldehydes, forming alcohols, and thus certain ill-tasted rough alcohols may be greatly improved.

Faraday's Laws of Electrolysis.—First Law.—The quantity of an electrolyte decomposed in a given time varies directly as the intensity of the current.

A current whose intensity is n Ampères will decompose 0·000,0935n grammes of water per second.

A Voltameter is an instrument in which a current is made to pass through acidulated water between platinum electrodes ; the acidulated water is in part decomposed, and the hydrogen and oxygen resulting from the secondary decomposition are collected in a graduated tube ; the amount of the mixed gases can be read off, and the weight of the water decomposed readily ascertained ; a simple proportion gives the intensity of the current actually passing through the voltameter. Instead of measuring the quantity of gas evolved, we may weigh the quantity of silver deposited from a solution of pure silver nitrate ; a current of one Ampère intensity deposits 0·001,118 gramme of silver per second (Rayleigh) or 4·025 grammes per hour ; this corresponds to 0·000,010,352 gramme of H per second. Kohlrausch finds 0·001,1183 gramme Ag or 0·000,010,355 gramme H per second ; Mascart, 0·001,124 gramme Ag or 0·000,010,415 H ; Gray, 0·000,010,478 gramme H ; the mean of these is 0·000,0104 gramme H or ·000,0935 gramme H_2O per second.

Instead of sending the whole current through the voltameter, we may send a known fractional part of it by arranging the voltameter in a shunt.

Faraday's Second Law.—The Law of Electrochemical Equivalence may be divided into the following propositions, of which the fourth may be regarded as a paraphrase of the law itself:—

1. The gramme-equivalent of a metal is that quantity which will chemically replace one gramme of hydrogen. For example: in comparing HCl ($H = 1$, $Cl = 35·5$) with AgCl ($Ag = 108$, $Cl = 35·5$), we find Ag ($= 108$) to be equivalent to H ($= 1$); the gramme-equivalent of silver is 108 grammes. In comparing $CuSO_4$ with H_2SO_4 we find Cu ($= 63·46$) equivalent to H_2 ($= 2$); the gramme-equivalent of copper is 31·73 grammes.

2. The gramme-equivalent of a salt-radicle or halogen is that quantity which will combine with one gramme of hydrogen. In HCl, 35·5 grammes of chlorine unite with 1 gramme of hydrogen; the gramme-equivalent of chlorine is 35·5 grammes. In HNO_3 ($H = 1$, $NO_3 = 62$), 62 grammes of NO_3 unite with 1 gramme of H; the gramme-equivalent of NO_3 is 62. In H_2SO_4 ($H_2 = 2$, $SO_4 = 96$) the gramme-equivalent of SO_4 is 48 grammes. In H_3PO_4 ($H_3 = 3$, $PO_4 = 95$) the gramme-equivalent of PO_4 is $31\frac{2}{3}$.

3. The gramme-equivalent of a salt or acid is that quantity which contains 1 gramme-equivalent of the halogen or salt-radicle.

4. When a current whose intensity, after the current has become steady, is equal to n Ampères passes through a solution of a salt, 0·000,0104n gramme-equivalents of the salt are decomposed during each second; the same number of gramme-equivalents of the salt-radicle or halogen being liberated at the positive pole, and a corresponding quantity of the metal at the negative.

A current is sent simultaneously through a solution of cupric chloride and a solution of cuprous chloride, and continues to pass through both solutions for five minutes; its intensity is 18 Ampères; compare the amounts of copper deposited on the negative-electrodes in the two solutions. In each the amount of the halogen—the chlorine—liberated is (·0000104 × 18 Amp. × 300 sec.) gramme-equivalents, or (·0000104 × 18 × 300 × 35·5) grammes. In $CuCl_2$ every 71 parts of chlorine are combined with 63·46 of copper; the copper deposited from the $CuCl_2$ is therefore

$$\left\{(\cdot 0000104 \times 18 \times 300 \times 35\cdot 5) \times \frac{63\cdot 46}{71}\right\} \text{grammes.}$$

In Cu_2Cl_2 every 71 parts of chlorine are combined with 126·92 of copper; the copper deposited from the Cu_2Cl_2 solution is therefore

$$\left\{(\cdot 0000104 \times 18 \times 300 \times 35\cdot 5) \times \frac{126\cdot 92}{71}\right\} \text{grammes,}$$ double the quantity deposited by the same current from a solution of cupric chloride.

Each Coulomb will thus liberate 0·000,0104 gramme-equivalents; each C.G.S. electrostatic unit will liberate one 3000,000000th part of this quantity, *i.e.*, 0·000000,000000,003466 gramme-equivalents. This last quantity is otherwise known as the electrostatic **Electrochemical Equivalent** of the salt-radicle or halogen liberated or salt decomposed.

The Energy of a current passing a quantity Q down a potential-fall E is equal to EQ; when Q = 1 this energy is, numerically, E ergs; but it is also equal to the work done by the unit quantity in decomposing one electrochemical equivalent of a salt. Hence the chemical energy liberated by the production of one electrochemical equivalent of a salt may be measured in terms of a potential-fall or an electromotive difference of potential.

Conversely, the E.M.D.P. in a galvanic cell may be measured in terms of the chemical energy set free in it. Let us enquire what is the value of the E.M.D.P. of a Daniell's cell. Here we have a chemical action going on which liberates energy: this energy, if not converted into the energy of a current of electricity, may wholly appear as heat; this heat has been measured in various ways, and the mean result of several observations is that (between extreme values 714 and 805) the amount of heat liberated when one gramme of zinc is dissolved amounts to 760 *ca* or 31,610,687,600 ergs. The gramme-equivalent of zinc is 32·645 grammes (Marignac); the electrostatic electrochemical equivalent of zinc is (0·000000,000000,003466 × 32·645) = 0·000000,000000,1132 grammes. The amount of energy liberated on the solution of one electrochemical equivalent of zinc is therefore

(0·000000,000000,1132 × 31,610,687,000) ergs or 0·00358 ergs. This being equal to the amount of energy liberated during the solution of one electrochemical equivalent of zinc in a Daniell's cell and the production of a current of one electrostatic unit-intensity for one second, is necessarily equal to the energy which would be spent by a current of the same intensity, and enduring for the same time, in reversing the whole chemical process and in decomposing the one electrochemical equivalent of salt produced. This we have seen to be numerically equal to E, the difference of potential under which the unit current passes. The E.M.D.P. of a Daniell's cell is therefore $E = 0{\cdot}00358$ electrostatic units of difference of potential. This is equal to 1·074 Volts; a theoretical result which does not depart widely from the experimental values, which range between 1 Volt and 1·124 Volt. This mode of computation is due to Sir William Thomson.

In decomposing an electrolyte, Work is done by the current; its Energy is spent in producing Chemical Decomposition. The energy absorbed in the decomposition of say 9 grammes of water is equal to the heat liberated by chemical action and change of physical state when 1 gramme of hydrogen and 8 of oxygen are exploded together and condensed into water.

Work done on decomposition, if such work be other than chemical, causes divergences from Faraday's second law. Such work is done when the products of decomposition of a liquid become gaseous, or when the metal of an electrolyte is lifted up towards the negative electrode; while the energy of the current is increased when these circumstances are reversed.

The relation of the energy liberated by the chemical action of the battery to the heat produced in the battery and the energy spent in doing electrolytic work is represented by the equation—

Battery-energy = Heat evolved in battery + Electrolytic work done.

The last term cannot be greater than the first; if it were, the heat developed in the battery would be a negative quantity, and the battery would go on indefinitely cooling itself as well as all surrounding objects—an impossibility.

In a Daniell cell the replacement of one grm.-equivt. of copper by one grm.-equivt. of zinc is attended with the evolution of 24,200 *ca* of heat; in a Grove the energy evolved is 47,000 *ca* for every grm.-equivt. of Zn dissolved. When 1 grm. H and 8 grm. O unite to form one grm.-equivt. of water, 34,462 *ca* of heat are evolved. A single Grove cell can therefore decompose water; a cell with a potential-difference equal to $\frac{32462}{24200}$ times that of a Daniell would just be able to do so; a single Daniell cannot. Two such cells are, however, able to effect decomposition; the energy supplied by two cells arranged in tension is double that supplied by one, even though the resistances be so adjusted that the current produced is of the same intensity: for Energy per second = EI, and if E be doubled, while I remains the same, the energy is doubled.

A single cell will, however, decompose water if the positive-electrode be of a substance such as copper, which will combine with oxygen, and thereby liberate energy sufficient to make up the deficiency in that supplied by the cell.

The process of electrolysis is turned to practical account in the arts of electroplating, etc.; the article to be covered with a metallic film is made the negative-electrode in a suitable solution of the metal. The positive-electrode is often itself made of the metal to be deposited from the solution; as the metal is deposited from the solution upon the negative-electrode, the positive-electrode is attacked and its substance dissolved in the solution, which is thus kept saturated if the action be not too rapid.

When a mixed solution of acetates of lead and copper (Nobili), or a solution of litharge in caustic potash (Becquerel), is electrolysed between two electrodes, of which the negative is a sharp-pointed platinum wire or steel needle, while the positive is a large plate of silver, german-silver, silvered copper (spangle metal), or even thin sheet-iron, the current being one of relatively-great E.D.P. (15 to 20 very small Bunsen cells mounted in file), there is formed on the positive-electrode a series of rings concentric with the point of the negative-electrode,—**Nobili's rings**, produced by deposition of PbO_2. If the positive-electrode be complex and consist of two or more points, or if currents be made to run through some of these points towards the plate, while in others the direction is reversed, the rings are modified into representations, in iridescent hues, of complete systems of equipotential surfaces (Guébhard).

Polarisation of Electrodes.—When platinum electrodes have been used in the decomposition of water, the negative one is found to contain a certain quantity of hydrogen not only adherent to its surface, but also occluded within its substance; while the positive one carries similarly a certain quantity of oxygen. These oxygen and hydrogen films tend to produce a reverse current, which weakens the main current. When the main current ceases the reverse current continues for some time and dies away gradually.

The occluded hydrogen is very gradually reduced by the oxygen brought to it by the reverse current, and a corresponding quantity of hydrogen is liberated on the positive pole, where it is occluded by the electrode or dissolved by the water. Thus, though a current passes, no products visibly appear.

In Grove's gas-battery a number of such electrodes are arranged so as to give a current, or—which amounts to the same thing—a circuit is arranged in which a current passes through: (1) water; (2) a plate of platinum standing partly in water, partly in hydrogen gas; (3) conducting wire; (4) a plate of platinum standing partly in an atmosphere of oxygen, partly in the water.

If electrodes of palladium be used in the decomposition of water, the negative one absorbs as much as 936 vols. of hydrogen, with which it forms an alloy greater in size than the original electrode.

The electrodes in such a case are said to be **polarised**, and their polarisation within a given substance is, if the resistance of that substance be not so great as to prevent a perceptible current from passing, the best test as to whether that substance is really an electrolyte. Warm glass is thus found to be actually an electrolyte.

The capacity of electrodes so polarised is very great. Two square inches of platinum electrode immersed in dilute sulphuric acid have (Varley) when the E.M.D.P. is one-fiftieth that of a Daniell's cell, a capacity equal to that of an electrostatic condenser whose plates have an area of 80,000000 square inches separated by $\frac{1}{8}$ inch of air ; *i.e.*, a capacity of 175 microfarads : while as the E.M.D.P. increases the capacity increases also.

Electrodes of amalgamated zinc are not at all polarised when used to transmit a current through a perfectly neutral solution of $ZnSO_4$ (Beetz).

Secondary Cells and Batteries.—When acidulated water is decomposed between electrodes of lead, the negative electrode remains bright, but the positive becomes covered with a film of PbO_2. When the current ceases, if the positive and negative electrodes be connected by a conducting wire, a reverse current—a polarisation-current, or Secondary Current—passes ; the film of PbO_2 is negative, the lead positive.

The reverse action appears to be (Gladstone and Tribe) in the main the following :—

$$\underset{-}{\mathrm{PbO_2}} \mid \mathrm{H_2SO_4} \mid \mathrm{H_2SO_4} \mid \underset{+}{\mathrm{Pb}} = \mathrm{PbO} \mid \mathrm{H_2O} \mid \mathrm{H_2SO_4} \mid \mathrm{PbSO_4}$$

which becomes

$$\mathrm{PbSO_4} \mid \mathrm{H_2O} \mid \mathrm{H_2O} \mid \mathrm{PbSO_4}.$$

Sulphate of lead is formed on both plates, while PbO_2 is formed on the positive plate by secondary reactions.

On passing in a charging current, the reaction is, for thin layers,

$$\mathrm{PbSO_4, H_2O, O} ; \mathrm{H_2, SO_4Pb} = \mathrm{PbO_2, SO_4H_2} ; \mathrm{H_2SO_4, Pb}.$$

An arrangement of this kind, into which a current of electricity can be passed and a reverse or secondary current obtained at will, is a Secondary Cell ; and secondary cells may be grouped into Secondary Batteries. Planté's original form of secondary cell consisted of two large lead electrodes, separated by a sheet of felt, rolled up into a spiral and immersed in 10 % dilute sulphuric acid. Faure improved this by covering both electrodes with a layer of red-lead of about 10 kilogs. per sq. metre, held on by layers of felt and parchment between the opposed plates. When a current has been passed into a so-called Faure-accumulator for some time, the red-lead on the surface of the negative electrode is converted into spongy lead, while that on the positive electrode is oxidised to PbO_2. This improvement greatly abridges the tedious process formerly necessary for charging Planté's accumulators.

In the Faure-Sellon-Volckmar accumulators there is no felt; the plates of lead are pierced or cast with holes, into which there is compressed a quantity of red-lead, of reduced lead, or of a salt of lead. One of these, weighing 140 kgr., and composed of 43 plates, gave (Hospitalier) a current of 120 Amp. for 6 hours.

The efficiency of these batteries is in part due to the fact that their porous metal or oxide is in close contact with the lead plate, and is, on account of its porosity, able to retain large quantities of the oxygen or hydrogen which is evolved when an external current is passed through the accumulator.

When the secondary current has passed for some time, both films become mainly converted into similar compounds of lead, and the apparatus is ready for a renewed charge.

When a secondary battery is charged by two or three Grove cells and disconnected from them, it will be rapidly discharged if connection be established between its poles by means of a thick wire. From fifty to sixty per cent, or, in the newer forms, a still greater proportion of the energy actually sunk in it can be recovered in the form of the energy of the secondary current. When the cells of a secondary battery, charged side by side, are disconnected from the source, and then connected in series and discharged, the electric current produced is one of "high tension" or great fall of potential. The E.D.P. of a single cell is about 2·25 Volts, and hence a hundred such cells, first arranged in surface and charged by prolonged connection with a few cells, can be made, if arranged in series, to pass for a short time a current of E.D.P. = 225 Volts; such a current produces a high temperature, together with vibration and crumpling of the conducting wire. The internal resistance of these cells is very small, being only 0·006 Ohm in a single cell whose surface is about 300 sq. cm.

A secondary cell of about 18 lbs. weight, the plates in which are each about 2 sq. feet in area, will, when charged by three Leclanchés, keep at a white heat during 5 to 10 minutes a platinum wire of $\frac{1}{40}$ inch. diar. and 8 inches long. A pile of this weight kept constantly connected with three or four Leclanché elements (which require little attention beyond that of keeping them moist) is a very convenient means of heating such a thing as a galvano-cautery wire, which must be raised to a high temperature for a short time.

Electrical Storage of Energy.—When Energy is stored up in bent steel springs, about 3924 megergs per kilogramme can be stored up—*i.e.*, 40 kilogrs. can be lifted through 1 metre by the elasticity of a spring weighing one kilogramme.

When it is stored up in compressed air, 1 kilogr. of air compressed to one-sixth contains 2,250,000 megergs, of which about 450,000 can be recovered in the form of work.

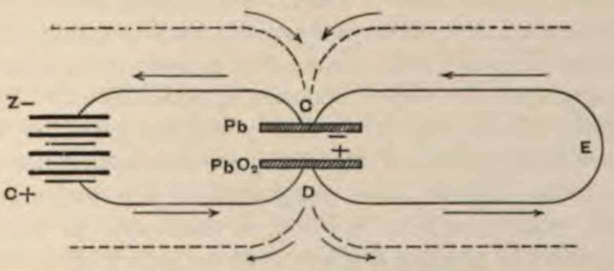
When it is stored in secondary batteries, about 500,000 megergs per kilo. of secondary battery are stored up, of which from 250,000 to 330,000 may be recovered if the batteries be used within a day or two after charging.

The great fault of these accumulators in their present form is their want of durability.

Equalisation of a Current.—A current passed through one plate of an Electrostatic Accumulator will be apparently absorbed when the current is increased, and will be given out equably when the current falls off; such an accumulator is therefore competent to play the part taken by a flywheel in the mechanical transmission of power.

Similarly, Secondary Cells may be made to serve as regulators of a current if they be fitted up in the course of the conductor in the manner indicated in Fig. 234. The direction of the secondary current is indicated by the dotted lines and arrows connected with the secondary cell; it opposes the main current in CuDCZn, but aids it in DEC.

Fig. 234.



THE DYNAMICAL PROPERTIES OF A CURRENT.

The phenomena of a steady electric-current are not confined to the conducting wire, for the space surrounding the wire bearing a current is found to be in a peculiar condition—a condition which can be explained as due to displacement of the ether or other dielectric or medium filling that space, and one which it seems impossible physically to account for on any other satisfactory basis.

The condition of the space immediately surrounding a current may be explored by means of iron filings. If a conducting wire be passed vertically through a hole in a piece of card or of silver-paper adjusted to a horizontal position, and if iron filings be then sprinkled upon the card, and if the card be gently tapped downwards so that the filings may leap into positions spontaneously assumed by them, they will be found to range themselves in concentric circles round the current, while each filing becomes, for the time being, a little magnet.

The space round the current is therefore an **Electromagnetic Field of Force,** permeated by concentric circular Lines of Force and by Equipotential Surfaces, which are at right angles to these. The equipotential surfaces all have the line of the current for their common edge or boundary. If the current be straight the equipotential surfaces are planes; and if they were visible, and if the current could be looked at end-on, so as to appear a mere point, these surfaces would seem to radiate from it like equidistant radii from a centre.

The Lines of Force mark the direction in which an ordinary magnet, such as a small compass-needle, when placed within the field, tends to place itself. The one end of the magnet is driven in one direction, the other end equally in the opposite direction along these lines of force: the magnet is acted upon by a couple, which acts upon the two extremities or poles like the hands on the handles of a copying press—one pole being pushed or repelled, the other being pulled or attracted, until the magnet lies along a line of force. The effective components of the couple gradually diminish as this position is being assumed, and the couple ultimately ceases to produce farther rotation; and further, since one pole is attracted as much as the other is repelled, the magnet as a whole undergoes in such a field no movement of translation.

The direction in which a current tends to throw the positive or north-seeking pole of a magnet placed in its neighbourhood is shown by Fig. 235. This direction is called the **Positive Direction** of the lines of force. The current in the figure passes vertically upwards; the Positive pole is thrown to the Left Hand of the Current. This expression, left hand of the current, is obtained by supposing the current to be replaced by a person whose head is at B and feet at A, and who turns so as constantly to keep the magnet-pole in full view. The relation between the direction of the current and the positive direction of the lines of force is always the same as that between the propulsion of the point and the twist of the hand in the ordinary use of a European corkscrew.

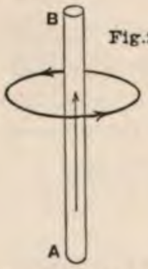


Fig. 235.

Conversely, action and reaction being opposite, stationary positive magnet-poles tend to throw movable upward currents to their left.

For negative magnetic-poles the directions given are reversed.

Further, Linear Currents act upon other Linear Currents; but they do not throw them to the right or left; they attract or repel them directly.

In Fig. 236 AB is a current; round it there are a number of lines of force. Let another current, parallel and in the same direction, be brought to CD: then, between AB and CD the lines of force of the two currents are opposed, but beyond CD they concur in their direction. The result is that the medium beyond CD is in a state of greater constraint than that between AB and CD; so is that to the left side of AB; and the two conductors are impelled towards one another. If the current in AB be opposed to that in CD, the directions of the lines of force coincide between AB and CD, but are opposed beyond AB or CD, and the stress is such as to drive AB and CD asunder. If the currents in AB and CD be at right angles, approaching one another (Fig. 237), the current in AB has lines of force whose direction is upwards on the left side of AB, downward on its right. The current CD has lines which would depress a positive pole placed in the upper, and raise a positive pole placed in the lower part of the figure. The concurrence of lines of force is in the upper part of the diagram (Fig. 237): if the current AB be fixed, the conductor CD is repelled in a direction contrary to that of the course of the current in AB. If the current in CD pass from D towards C, the conductor CD tends to move in the same direction as the current in AB.

Fig.236.

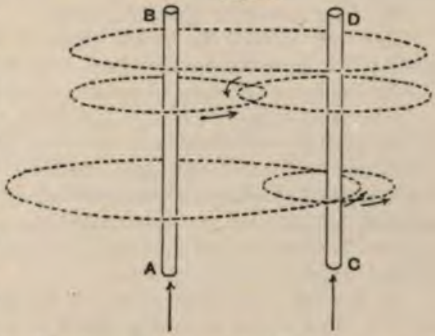
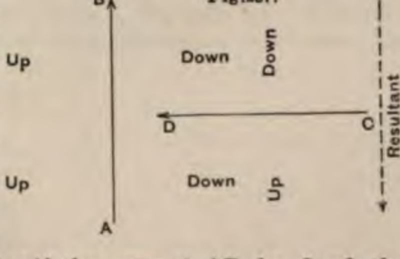
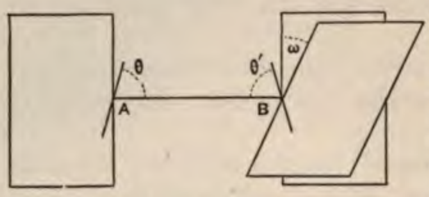


Fig.237.



These results were first summarised in **Ampère's formula**, by which the attraction between very small elements of two linear circuits was stated to vary as $I.I'.ll'\ (2 \sin\theta \sin\theta'.\cos\omega - \cos\theta.\cos\theta')/d^2$, where I and I′ are the intensities of the currents passing in the two wires; l and l' the length of the elements; d the distance between their mid-points; and θ, θ', ω are the angles which determine their relative direction as follows:—Each element makes an angle θ or θ' with the line AB; but further, these elements are situated in planes which make an angle ω with one another.

Fig.238.



We may take some particular cases.

Let both currents lie in the plane of the paper; the angle $\omega = 0$; $\cos\omega = 1$; $F \propto I.I'.ll'.\ (2 \sin\theta \sin\theta' - \cos\theta' \cos\theta')/d^2$.

1. Let both currents be parallel to AB; $\theta = 0$, $\theta' = 0$; $\sin\theta = \sin\theta' = 0$; $\cos\theta = \cos\theta' = 1$; $F \propto -I.I'.ll'/d^2$. Two elements of current, end-on to one another, and running in the same direction, repel one another.

2. Let both be at right angles to AB; $\theta = \theta' = 90°$; $\cos\theta = 0$; $\sin\theta = 1$; $F \propto 2I.I'.ll'/d^2$. Two elements of current, parallel and abreast of one another, running in the same direction, attract one another. If they be opposed in direction, the product I I′ is negative; then F is negative, and the mutual action is one of repulsion.

3. Let one be at right angles to AB, the other parallel to it; $\theta = 90°$; $\theta' = 0$. $\cos\theta = 0 = \sin\theta'$; $\cos\theta' = 1 = \sin\theta$. $F = 0$. There is no appreciable action between two extremely small elements one of which points end-on and at right angles to the centre of the other.

4. Let one be at right angles to AB, the other at any other angle, say 45°. $\theta = 90°$, $\theta' = 45°$. $F \propto \sqrt{2}.I.I'.ll'/d^2$. Two currents diverging from a point attract one another; if one of the currents be reversed, they repel one another; if both be reversed, so that both converge upon an angle, they attract one another.

5. Two elements, B and A, parallel in their direction; the resultant force is one of attraction between B and A. A similar element at A′ attracts B towards A′. Of these two attractions the resultant is towards O. Pairs of such elements symmetrically ranged to an infinite distance on either side of O make up an infinite conductor whose attraction for B is on the aggregate $F \propto (I.I'/OB) \times$ length of element B. The attraction of an infinite current for an element of current running parallel with it is directed along a line at right angles to both currents, and is inversely proportional to the distance between them.

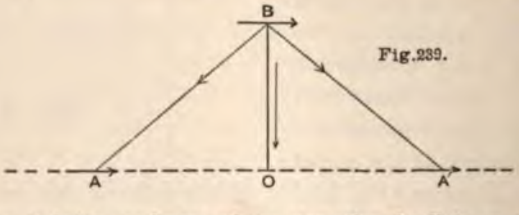
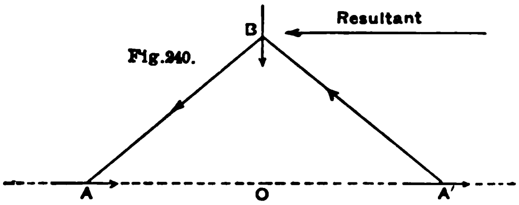


Fig.239.

6. The element B passes towards O ; A and A′ are equal elements, symmetrically arranged, of an infinite current along AA′ : A attracts B : A′ equally repels B ; the resultant is in a line parallel to AA′, and tends to drive the current in the direction A′A.

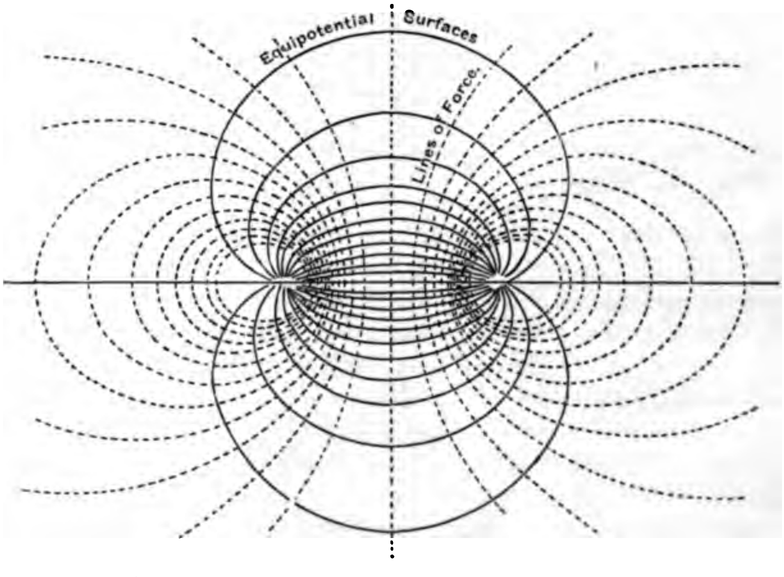
Fig. 240.



The formulæ of Grassmann, Clausius, Helmholtz, and others, we here pass over.

If a wire bearing a straight current be bent into a closed circuit or loop, its equipotential surfaces are modified into a series of bowl-shaped surfaces which still have the wire, the contour of the circuit, for their common edge or boundary. A circular current would have equipotential surfaces whose general form, looked at in section, is indicated by the undotted curves of Fig. 241, in which the lines of force are shown at right angles to the equipotential surfaces. To one side of the circuit the potential is positive, to the other it is negative. The **positive side of the circuit** is such that an observer standing girdled by the current, with his

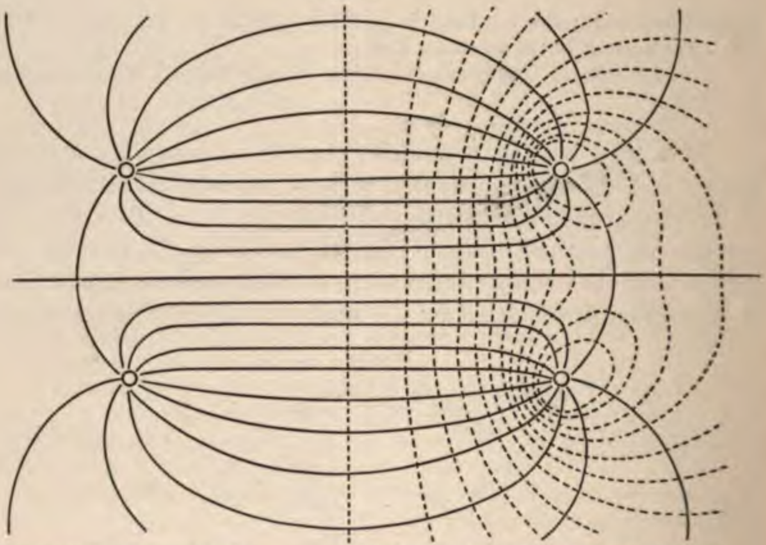
Fig. 241.



head in the positive region, would see the current pass round him from his right towards his left hand.

If another circular circuit be brought near, and if the direction of the current within it be the same as that within the former, the lines of force of the two circuits coalesce, and the two circuits attract one another. The resultant system of surfaces and lines takes the form indicated in Fig. 242. The field of force between the two circuits is approximately uniform. The lines of force are all closed curves, but some of them, those which pass up the centre of the region between the circuits, take a relatively-wide sweep into space, and seem to radiate from or converge upon the external face of either circuit.

Fig. 242.

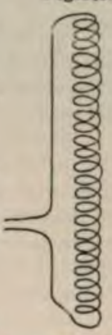

Equipotential Surfaces only.
Equipotential Surfaces & Lines of Force.

A large number of such circular circuits arranged so as to have a common axis, and thus to form, as it were, the outline of a cylinder, would form a so-called **Solenoidal** system. Such a system would have lines of force radiating from each extremity, taking a more or less ample sweep into space, returning into the opposite extremity and passing up the axial region of the cylinder; each line of force being thus a closed curve. The external electromagnetic field of such a solenoid system would be identical with that produced by a system consisting of an attracting disc at the one, and a repelling disc at the other extremity of the solenoid;

and such a solenoid would by one of its extremities attract the north and repel the south pole of a compass-needle; while by the other it would attract the north and repel the south pole. Such a solenoid would, so far as its external action is concerned, act like a bar-magnet, and Ampère's theory of Magnetism is, that magnets and solenoid systems of currents are fundamentally identical.

A solenoid may be roughly realised by winding a wire into a narrow spiral and bringing the two extremities back to the same point. The error introduced in each turn of the spiral by its departure from a perfect ring-form is roughly compensated by the return of the wire (Fig. 243).

Fig. 243.



This identity of action of Magnets and of Solenoidal Current-systems being premised, we now proceed to give a rapid summary of the main phenomena of Magnetism.

MAGNETISM.

Some bodies—a piece of loadstone, a compass-needle, a wire spiral through which a current is passing—tend, when suspended by their centre of gravity, to lay themselves in a definite direction, and to place a definite line within them, their Magnetic Axis, in a definite direction, which, roughly speaking, lies north and south. The same bodies have the power of attracting iron. Such bodies are called Magnets.

Curiously enough, this directive power is, according to Gore, shared by crystals of Cyanite, an anhydrous monosilicate of alumina.

Magnets may be divided into Permanent (loadstone, hard steel magnets) or Temporary (a solenoid current, or an electromagnet, a bar of soft iron whose magnetic properties are induced by the presence of an electric current circulating round it, but which endure, in soft iron, no longer than the persistence of that current); or again, into Natural (loadstone) and Artificial.

The constituent particles of a magnet are themselves magnets. A permanent magnet may be cut into an indefinite number of fragments, each of which will be a little magnet, the original magnetic axis in which will continue to point to the magnetic north and south. When a steel bar is converted into a permanent, or a soft-iron bar into a temporary magnet, some operation must be effected, not upon the mass as a whole, but upon its constituent molecules. The magnetic axis of a bar-magnet or compass-needle coincides more or less closely, but hardly ever with perfect accuracy, with its geometrical axis of figure. The extremities of the magnetic axis are the **Poles** of the magnet.

One mode of expressing the mechanical action of magnets is to feign a distribution of imaginary magnetic matter at the Poles; positive at the one pole, equal and negative at the other; the attractions and repulsions observed are exercised mainly to and from these poles.

Another method is to feign a distribution of magnetic matter partly over the surface, partly within the substance of the magnet (Poisson) or over the surface only (Gauss and Green).

Positive and negative magnetic distribution may be feigned to be either heapings-up of positive matter towards or at positive poles, and of negative matter towards or at negative; or else to be distribution in excess and defect respectively (or inversely) of one and the same all-pervading magnetic fluid.

These modes of representation are convenient for calculation and exposition merely.

A long thin bar so magnetised that all its molecules would, considered as magnets, be absolutely equal, would have its poles at its ends. Such a theoretical bar-magnet is called a **Solenoidal Magnet.** In practice the action of bar-magnets is the same as that of a theoretical Solenoid whose poles are at a somewhat less distance from one another than the extremities of the bar: for which reason the Poles of a bar-magnet are often said to be within its substance, at a short distance from its ends.

The North-seeking pole of a magnet is its Positive pole; the other, its south pole, is its negative pole.

In different magnets, unlike poles attract one another; like poles repel one another.

The force between two poles varies inversely as the square of the distance between them. It also varies directly with the **strength** of each pole, or the quantity m of magnetic matter supposed to be concentrated at each. It is therefore $F = mm'/d^2$, and is repulsive when the poles are similar, both positive or both negative.

Two poles are said to be of Equal Strength when they can replace one another in their action upon external magnetic poles.

Two poles are said to be each of **Unit Strength** if they be equal and have between them a repulsion-force equal to one dyne when their mutual distance is one centimetre through air.

Clausius and Siemens suggest that such a pole should be called a Weber.

A unit pole placed at 1 cm. from a similar pole of strength m units (or Webers) will be repelled with a force of m dynes.

The opposite poles of a magnet are of strengths opposite but numerically equal; these are $+ m$ and $- m$; their sum is zero; the sum of the magnetisms of every magnet is always zero.

It is not possible to isolate a single magnetic pole or to produce any numerical difference of strength between the two poles of a magnet.

Round a magnet there is a **Magnetic Field of Force** permeated by Magnetic Lines of Force and Magnetic Equipotential Surfaces. An isolated positive-pole (if such a thing were possible) placed in the neighbourhood of the positive extremity of a bar-magnet would be repelled and would travel to the negative extremity, not by the shortest path, but by following the wide sweep of any line of force on which it might happen to lie.

The Direction of a magnetic Line of Force is the direction in which a positive pole is driven, or, conversely, a negative pole pulled upon.

A magnet within a magnetic field is acted upon by a Couple: its positive pole is driven, its negative pole drawn, in the direction of the Lines of Force passing through them: the axis of the magnet tends to coincide as nearly as possible with a line of force passing through its centre. If this line of force be curved, the axis of the magnet is set tangentially to it.

The Condition of a Magnetic Field at a point is determined (1) by the Direction of the Line of Force passing through the point, and (2) by the local Intensity or **Strength of the field**—*i.e.*, by the amount of force with which a unit-pole there situated is repelled or attracted.

The unit intensity of field would be that produced by a Weber at 1 cm. distance. Sir William Thomson suggests that this unit of intensity should be called a Gauss.

When the local intensity is F, a magnet whose length is l, and whose poles have the respective values m and $-m$, is acted on by a couple: the force acting on the positive pole is mF, and its moment is $(m\mathrm{F} \times \frac{1}{2}l)$; the moment of that acting on the negative pole is $(-m \times -\mathrm{F} \times \frac{1}{2}l) = \frac{1}{2}m\mathrm{F}l$: the moment of the couple is thus $m\mathrm{F}l$. At a spot where the intensity $\mathrm{F} = 1$—that is, within a unit magnetic field—this moment is equal to ml, the numerical Strength of either pole multiplied by the Length of the

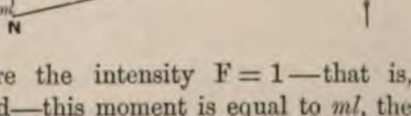


Fig. 244.

magnet. This moment is called the **Magnetic Moment** of the magnet.

If a magnet of length l be broken into n fragments, each of length l/n, the magnetic moment of each fragment is $m \times l/n$; the sum of the magnetic moments of all the n fragments is $n \times m.l/n = ml$, the magnetic moment of the original magnet, and each fragment possesses poles of strength m, equal to those of the original magnet.

When two equal magnets are arranged thus—

N———S, N———S,

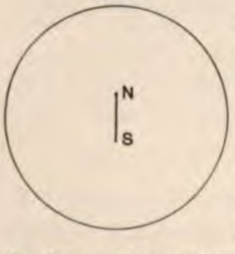
the extreme poles are effective, the intermediate ones mask one another; when work is done upon them in separating them, the original condition is restored and all the poles are again manifest. When n such magnets are connected in this way, all the poles except the extremes mask one another. A uniform bar-magnet l cm. long and a sq. cm. in cross-section, and therefore having a volume of la cub. cm., may be considered as a collection of la magnets, each 1 cm. in length and 1 sq. cm. in section, and therefore each of unit volume. The magnetic moment of the whole is equal to that of la such magnets: the magnetic moment of each of these unit volume magnets is that of the entire magnet, ml, divided by la, the volume, and would therefore be equal to m/a. This is the so-called **Intensity of Magnetisation** of the bar-magnet.

If the intensity of magnetisation of a bar-magnet were equal throughout, its poles would be situated exactly at its extremities. We generally find, however, that magnets present abnormalities in this respect, and that they may even have secondary poles, produced by local inequalities in the intensity and by consequent deficient compensation of the internal poles.

Such complex distributions can, within a bar, be generally represented by the superposition of a number of solenoids of different lengths.

A steel sphere will be magnetised uniformly if it be placed for some time within a uniform magnetic field. It then has a moment equal to that of a small axial magnet NS, Fig. 245; and it tends to lay its axis NS along that line of force which passes through its centre.

Fig. 245.



The Earth considered as a magnet is not uniformly magnetised; its intensity of magnetisation is not equal throughout; it does not act upon bar-magnets placed near its surface exactly as a distant bar-magnet would do, for the law of its action is not even approximately expressible by any formula less complicated than one which contains at least twenty-four coefficients (Gauss).

To find the Magnetic Moment of a Magnet.—We must combine the magnetic moment M of the magnet with H, the horizontal component of the intensity of the earth's magnetic force at the place of observation. By one process we can find the value of MH; by another we can find that of M/H; from these data we can find not only M, the magnetic moment (for $\mathrm{MH} \times \mathrm{M/H} = \mathrm{M}^2$), but also H, for $\mathrm{MH} \div (\mathrm{M/H}) = \mathrm{H}^2$.

1. To find MH ("Method of Vibrations"):—Suspend a very long magnet by one thread attached to its centre; load it so that it may swing horizontally round a vertical axis: observe the time of its oscillation under the earth's magnetic attraction of one pole and repulsion of the other. The time of a *complete* oscillation is $\mathrm{T} = \pi\sqrt{\mathrm{I/HM}}$, where I is the moment of inertia (page 150). In the simple-pendulum formula (page 197) $\mathrm{T} = 2\pi\sqrt{\mathrm{I}/wl}$. Here w, the weight of the pendulum, is replaced by Hm, the attraction of the one, and $-\mathrm{H}m$, the repulsion of the other pole. Hence $\mathrm{T} = 2\pi\sqrt{\mathrm{I}/2\mathrm{H}m.l}$; but $2lm = \mathrm{M}$, the magnetic moment of the magnet of length $2l$, which swings suspended on its midpoint like a couple of simultaneously oscillating pendulums each of length l. Whence T, the period of a complete or double oscillation, $= 2\pi\sqrt{\mathrm{I/HM}}$. T can be found; I can be ascertained for any given needle; whence (HM) may be calculated.

This operation is difficult; for M, the magnetic moment, and therefore the rate of oscillation, varies with every slight vibration or change of state or of the temperature of the suspended magnet.

If we take the torsion of the suspending thread into account we find that a restitution-pressure has been developed, proportional to the displacing force Hm and also to the arm of the lever, l, the distance of the pole from the thread; it is therefore proportional to $\mathrm{H}m.l$ and consequently to HM; call it p.HM. The force tending to restore the needle to its mean position is not HM but $\mathrm{HM} + p.\mathrm{HM}$; and the divisor of the value of T is not $\sqrt{\mathrm{HM}}$ but $\sqrt{\mathrm{HM} + p\mathrm{HM}}$.

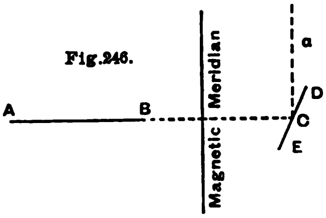
To find the value of I, the Moment of Inertia, we must attach to the oscillating needle a mass whose moment of inertia $\mathrm{I}_{,}$ is known from geometrical considerations. Let this be, for instance, a ring of rectangular cross-section whose mass is m and whose radii are $r_{,}$ and $r_{,,}$; the moment of inertia is (No. 6, p. 151) $\frac{1}{2}m\,(r_{,,}^2 + r_{,}^2)$, or half the weight × $(r_{,,}^2 + r_{,}^2)/g$. When this ring is fixed to the needle in such a way that the horizontal oscillations of the needle cause the heavy mass to rotate horizontally round its own centre, the time of a complete oscillation is increased to $\mathrm{T}_{,}$, which is equal to $2\pi\sqrt{(\mathrm{I} + \mathrm{I}_{,})/\mathrm{HM}}$. These data are sufficient to give the value of I, the moment of inertia of the needle.

If the needle be a straight wire of length $2l$, $\mathrm{I} = \mathbf{m}l^2/3$; whence $\mathrm{HM} = 4\pi^2.\mathbf{m}l^2/3\mathrm{T}^2$, where **m** is the mass of the needle in grammes (Equation **a**).

2. To find M/H.—We may use the same needle to produce a deflection in a compass-needle free to swing round its midpoint; by observing the deflection from the Magnetic Meridian (p. 626) when the compass-needle has come to rest under the influence of the two couples, we find the ratio of these couples and thus learn the value of M/H. These are two main methods; the End-on deflection-method and the Broadside deflection-method.

In the former, the **End-on**, Fig. 246, DE is the deflected magnet or compass-needle; AB the magnet whose moment we are investigating; a the deflection; d the distance between C and the midpoint of AB, a distance very considerable as compared with the dimensions of DE; $2l$ the length of AB; then

Fig. 246.

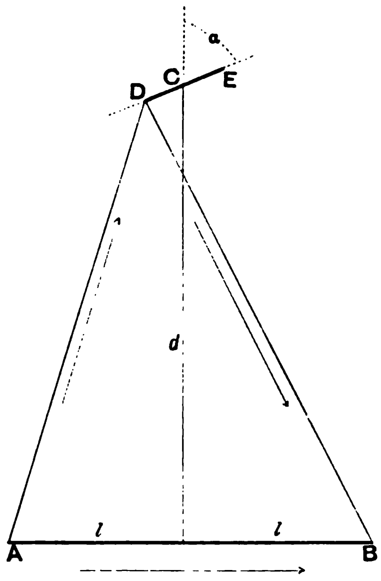


$$\tan a = \frac{M}{H} \cdot \frac{2d}{(d^2 - l^2)^2}, \text{(b)}:^*$$

a formula which, when the length $2l$ of the magnet AB is small in comparison with the distance d, becomes $\tan a = 2M/Hd^3$.

In the latter, the **Broadside** method, Fig. 246*a*, AB is fixed so that its midpoint is in a line with the magnetic meridian passing through C, and d being, as before, the distance between the centres of the magnets, the deflection a is such that

Fig. 246 *a*.



$$\tan a = \frac{M}{H} \cdot \frac{1}{(d^2 + l^2)^{\frac{3}{2}}}, \text{(c)}:\dagger$$

which, if l be relatively insignificant, becomes $\tan a = M/Hd^3$; the twisting couple in this case is therefore half that due to AB when the end-on method is applied.

By blending equations (**a**) and (**b**) we find that the data of the end-on measurement give

$$M = (\pi l/T)\,(d^2 - l^2)\,\sqrt{2 \tan a \,.\, \mathrm{m}/3d},$$

and

$$H = (2\pi l/T(d^2 - l^2))\,\sqrt{2\mathrm{m}d/3 \tan a},$$

expressions which involve only measurable terms, and which give numerical values for H and M in proper C.G.S. units. Similarly by blending equations (**a**) and (**c**) we may interpret the data of the broadside method.

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* DE is supposed so small that all forces acting on it act along the line BC, and that deflections do not modify the forces upon it. The distance BC is $(d - l)$; the strength of B is $M/2l$; the strength of D is m; the force between B and D is $mM/2l\,(d-l)^2$. Similarly the force between A and D is opposite in sign, and equal to $mM/2l\,(d+l)^2$. The force upon D is thus

$$(mM/2l)\left\{\frac{1}{(d-l)^2} - \frac{1}{(d+l)^2}\right\} = mM \,.\, 2d/(d^2 - l^2)^2.$$

An equal force acts upon E. The couple acting on DE is thus $DE \times mM \,.\, 2d/(d^2 - l^2)^2 = M_,M \,.\, 2d/(d^2 - l^2)^2$ where $M_,$ is the magnetic moment of DE. When DE is deflected through an angle a this couple becomes $2MM_, d \cos a \,.\, /(d^2 - l^2)^2$. When this couple is in equilibrium with the terrestrial horizontal couple $(H \,.\, mDE \,.\, \sin a)$ or $(H \,.\, M_, \sin a)$,

$$2MM_, \,.\, d \cos a \,.\, /(d^2 - l^2)^2 = H \,.\, M_, \,.\, \sin a, \text{ whence}$$

$$\tan a = \frac{M}{H} \cdot \frac{2d}{(d^2 - l^2)^2}.$$

† It is supposed that all parts of DE are appreciably at the same distances from

If we have any doubt as to the true value of $2l$, the distance between the "poles" of AB, we can find it by repeating at a different distance d the observations which lead to, say, equation (b). We now have a different a, a different d, but still the same l. From two such equations we can obtain the numerical value of l.

Magnetic Potential.—A magnetic pole, if isolated, would be surrounded by concentrically-spherical equipotential surfaces traversed by radial lines of force. But a magnet has two poles of opposite kind, and the field of force surrounding it presents a general character which may be approximately represented by Fig. 241, if the lines marked Equipotential Surfaces in that figure be held to represent Lines of Force and *vice versâ.*

The potential at any point due to the positive pole m at distance r is m/r; that due to negative pole, $-m$, at distance r' is $-m/r'$; due to both together the potential is $\frac{m}{r} - \frac{m}{r'}$ or $m\left(\frac{1}{r} - \frac{1}{r'}\right)$, which has the same value for every point on one and the same equipotential surface.

Magnetic Shell.—We may arrange a number of bars side by side, so that their similar poles all point in the same direction; a metal sheet is thus built up, of which the one face is negatively, the other positively magnetised. Such a sheet is called a Magnetic Shell. The Magnetic Moment of such a shell is the sum of the magnetic moments of all its portions. Let it be supposed to be first a continuous shell, and then to be divided into portions each 1 sq. cm. in area. Each such portion will have a magnetic moment, M. The magnetic moment of each such unit-area portion, if this be invariable over the whole shell, is called the **Strength of the Shell**; it is equal to the magnetic quantity per unit of area × the thickness of the shell.

The Potential of a Magnetic Shell upon a Unit Positive-Pole placed at any point facing the Positive Aspect of the Shell will be the product of the Strength of the Shell into the Apparent Surface of the shell, as seen from the unit-mass—this apparent surface being measured by the projection of the shell upon an ideal sphere whose centre is occupied by the unit-mass, and whose

---

AB as the central point C is; *i.e.*, at distance d from the midpoint of AB, at distance $\sqrt{d^2+l^2}$ from either A or B. The pole D is attracted by the one end of AB and repelled by the other; in each case with a force $m \,.\, \mathrm{M}/\overline{2l \,.\, \mathrm{AD}^2}$, or $m\mathrm{M}/\overline{2l \,.\, (d^2+l^2)}$. The resultant force on D is parallel to l and is therefore equal to the whole force acting $\times\, l/\sqrt{d^2+l^2}$; *i.e.*, it is equal to $m\mathrm{M}/\overline{(d^2+l^2)^{\frac{3}{2}}}$. The resultant on E is equal and opposed in direction. The couple on DE is therefore $m\mathrm{M}/\overline{(d^2+l^2)^{\frac{3}{2}}} \times \mathrm{DE} \times \cos\alpha$ $= \mathrm{M}_, \,.\, \mathrm{M} \,.\, \cos\alpha \,./\overline{(d^2+l^2)^{\frac{3}{2}}}$. This is equal to the terrestrial couple $\mathrm{H} \,.\, \mathrm{M}_, \,.\, \sin\alpha$; whence $\tan\alpha = \mathrm{M}/\overline{\mathrm{H}(d^2+l^2)^{\frac{3}{2}}}$.

radius is 1 cm., or, in other words, by the value of the Solid Angle subtended by the Shell at the point.

If the shell be fixed, the positive pole will tend to move away to regions of less potential, and thus to travel round to the negative side of the shell. If the unit pole be fixed, the shell will tend to move in such a way as to diminish its apparent area, and even to present what is equivalent to a negative area, namely, its negative side, to the positive pole; it will therefore tend to rotate.

In the immediate neighbourhood of a magnetic shell the angle subtended by it is 2π; the potential near the positive surface is therefore $2\pi S$, where S is the strength of the shell; near the negative surface it is $-2\pi S$; hence, when a unit positive-pole moves from the + to the − surface, $4\pi S$ units of work are done by it.

The Equipotential Surfaces in the neighbourhood of a magnetic shell are such that from every point on any one of them the area of the shell will for that surface appear invariable. But equipotential surfaces as determined by this criterion are identical in form with those bowl-shaped equipotential surfaces which surround a closed circuit bearing a current of electricity (Fig. 241); provided that the contour of the shell and that of the circuit be the same. A Magnetic Shell and a Closed Current of electricity may therefore have in their vicinity an identical Magnetic or Electromagnetic Field.

Terrestrial Magnetism.—The neighbourhood of the surface of the Earth is a great Magnetic Field, nearly uniform within such small spaces as the interiors of rooms. The lines of force point in the northern hemisphere downwards and northwards: in the southern hemisphere upwards and northwards. A compass-needle thus tends to place itself, in the northern hemisphere, so that its magnetic axis points downwards and to the Magnetic North, which is inclined to the west of the true or Geographical North by a so-called Declination of 18° 5·68′ (at Greenwich, 1885; 21° 7′ at Edinburgh, 22° 8′ in Dublin). This declination towards the west is at present decreasing at Greenwich by 7·38′ per annum. If the needle cannot move except round a vertical axis, its axis cannot point downwards or upwards: it therefore tends to point to the Magnetic North, lying, as it does so, with its magnetic axis in a line situated in the same vertical plane with the true line of force. This plane is the Magnetic Meridian; and the magnetic axis of a given magnet may be found if the magnetic meridian passing through the place of observation be known.

To find the magnetic meridian a single observation is not sufficient. The axis of figure of a needle may not coincide with its magnetic axis, and the needle (which is provided with an agate cup on each of its flat faces) is therefore observed when it lies poised on one side, and again when it lies on the other. The mean of the two positions gives the position of the magnetic axis of the needle, and therefore indicates the magnetic meridian. The downward or upward direction of the lines of force, their departure from the horizontal line, is the Inclination or Dip of the needle. This is downwards in the N. Hemisphere, upwards in the Southern. A needle suspended on a horizontal axle will, by the mean of two readings, give the downward inclination of its magnetic axis. Friction prevents the attainment of very great accuracy in this measurement, though this can be greatly diminished by slinging the axle of the needle upon silk threads. The inclination is at Greenwich, 1885, 67° 2′, diminishing by 2·04′ per annum: at Edinburgh, 70° 30′; in Dublin, 69° 30′.

If the inclination be found, and the horizontal component of the attractive force of the earth's magnetism, acting upon a unit pole, be known, we have the data required for determining the whole intensity of the earth's magnetic field in the direction of the lines of force at any point. The horizontal component at Greenwich in 1885 is 0·18165 dynes, increasing by ·00027 per annum. The vertical component there is 0·43696 dynes, decreasing by ·00008 per annum.

The line along which the lines of force are horizontal, and at which the Inclination or Dip is equal to zero, is the Magnetic Equator, which does not coincide with the geographical equator, and is not a great circle of the earth. The lines, roughly parallel to the magnetic equator, along which the Dip is equal, are the Magnetic Parallels: these are lines along which equipotential surfaces cut the surface of the earth. The intensity of the earth's magnetic force may be indicated by the distance between these parallels, just as those maps, which give contour-lines indicating equal levels, may show by the crowding together or separation of these lines the tendency of water to rapid or to slow flow over the face of a country. The magnetic parallels are not great circles of the earth; they are not even parallel to one another; in circumpolar regions they are irregularly elliptical, and the needle points to their centres of curvature. A Magnetic Pole is a spot where the equipotential surfaces of the magnetic field graze the earth's surface; the needle there stands vertical, the dip being 90°. There are two true poles, one arctic (negative), the other antarctic (positive), together with other points towards which surrounding magnetic needles seem to converge, but which are only the centres of curvature of the irregularly-shaped magnetic parallels. The line joining the Magnetic Poles does not coincide with anything which may be termed the Magnetic Axis of the earth.

The terrestrial magnetic field undergoes remarkable Variations. The direction of the lines of force, and therefore the dip, the declination and the position of the magnetic north, as also the intensity, undergo secular changes; and there are other changes, some of which depend, like the period of sunspots, upon a cyclical period of about eleven years, others upon the rotation of the sun, upon the position of the moon, upon the time of the year and the hour of the day; while other disturbances, productive of electrical currents in the crust of the earth, so powerful and so irregular as sometimes to render telegraphic signalling perfectly unintelligible—disturbances known as Magnetic Storms—are observed to occur with special frequency in sympathy with outbreaks of sunspots and of solar storms and appearances of the Aurora Borealis. The nature of the undoubted connection between the sun and the magnetism of the earth is in the highest degree obscure; it is clear, however, that the sun and moon cannot exercise any important direct effect as magnets, although when one side of the sun is turned towards us the terrestrial magnetic intensity is greater than when that side is turned away.

Magnetic Induction.—Soft iron filings are attracted by a magnet, and themselves become temporary magnets. This they do even though they be not in contact with a magnet, but merely exposed to such forces as can act upon them within a magnetic field. Soft iron completely loses its magnetic properties when removed from the neighbourhood of a magnet; but a steel or hard iron bar, which is with greater difficulty induced to become a magnet, will not, when removed from the field, entirely lose its magnetic state, but preserves a certain Residual Magnetisation. The property of steel or hard iron, in virtue of which it slowly takes up and slowly parts with a magnetic condition, is traditionally named its Coercitive Force. Any vibration or jar which facilitates relative movement of particles of the iron will enable its molecules to yield to the inducing forces, and will facilitate the magnetisation of the iron: and after its removal from the field, such a jar will facilitate its loss of magnetic condition.

A poker suspended near the earth's surface and repeatedly struck will become feebly magnetic; so does an iron ship which is exposed to much hammering during construction; and all working machinery is magnetic.

The effects of the inducing forces within a magnetic field differ from those within an electric field of force in the following respects:—(1) The action is one which affects the state of each molecule; (2) There is no repulsion of a mass of iron or steel which comes in contact with a magnet; and (3) The power of taking up a magnetic condition in any marked degree is limited to a very small number of bodies, though to a slight extent it is possessed by all.

The strength of the poles of an induced magnet depends on the nature of the magnetic field, and therefore on the strength, the distance, the direction, the form, of the inducing magnet; and also upon the nature of the body acted upon, its form, its direction, and its size.

In some substances the magnetisation induced is such that the north pole of the induced magnet lies as far as possible along the lines of force,—as far as possible away from the north pole of the inducing magnet. Such substances—iron, nickel, cobalt, manganese, chromium, oxygen, etc.—are **Paramagnetic** or **Ferromagnetic.**

In other substances the direction of the induced magnetisation is the reverse of this. Such substances—bismuth, antimony, silver, copper, hydrogen, nitrogen, etc.—are **Diamagnetic.**

Intermediate between these are such substances as air, which do not become magnetic, or but very slightly so.

In still other substances the induced magnetisation is not parallel to the lines of force, but along certain **Lines of Induction** within the body, whose direction depends upon the molecular agglomeration or the crystalline constitution of the body.

The Lines of Induction within an induced magnet must therefore be distinguished from the Lines of Force, with which they do not in all cases coincide; within a magnetic field, on the other hand, they are coincident in all respects.

The lines of induction within a magnet are directed from the positive towards the negative pole; a magnet thus tends to repel its own magnetism and to weaken itself by self-induction. The lines of force within an induced magnet are, on the one hand, directed towards the positive pole: for this is the direction in which the induced positive magnetisation is impelled.

The intensity of magnetisation produced in a particle by induction depends upon F, the intensity of the surrounding magnetic field; but not upon this only. It also depends upon the reaction of other particles which have, by induction within the same field, become magnetic. The intensity of magnetisation, I, induced in a particle depends, therefore, on the aggregate force acting upon it, and varies, within certain limits, directly as that aggregate force. It is therefore equal to that aggregate force multiplied by a constant. This constant, peculiar to each substance, is κ, the so-called **Coefficient of Induced Magnetisation** for the substance acted upon, or its Magnetic Susceptibility. In a limiting case, that of an extremely-long cylindrical bar, the force due to the magnet itself—that is, to its distant poles—may be neglected. Then the force due to the field acts alone within the magnet, and $I = \kappa F$. If the field be a field of unit intensity, $F = 1$, $I = \kappa$, and the Coefficient of Induced Magnetisation is equal to the intensity of the magnetisation assumed by an exceedingly long and exceedingly thin bar placed within such a Unit Field. This may be taken as the definition of that coefficient.

If in an induced magnet, whatever be the direction of its magnetic axes, we ideally excavate a disc-shaped cavity at right angles to the lines of induction, that cavity being supposed to be extremely thin as compared with its breadth, the force within the cavity due to the action of the surrounding magnetised particles will be $4\pi I$:* that due to the magnetic field will be F; together, the force within the cavity will be $F + 4\pi I$. This force is called the Magnetic Induction within the magnet. This magnetic induction varies as the strength of the field, and is equal to that strength multiplied by a coefficient, μ. This coefficient is called the **Coefficient of Magnetic Induction**, or the Magnetic Permeability of the substance. It will be seen that, since $I = \kappa F$, $F + 4\pi I = F\,(1 + 4\pi\kappa)$, and $\mu = (1 + 4\pi\kappa)$.

For soft iron $\kappa = 32$, and $\mu = 403{\cdot}125$; for air κ is approximately $= 0$, and $\mu = 1$; for bismuth $\kappa = -{\cdot}000{,}002{,}5$, and $\mu = 0{\cdot}999{,}996{,}858{,}4$. Bismuth is the most strongly diamagnetic substance known, and has the least known permeability.

If we consider the air in the neighbourhood of a magnet, we find a certain number of lines of force or of induction passing through a given bulk of it. If we replace the given bulk of air by an equal bulk of iron, we find the lines of induction gathered together into it from the surrounding magnetic field. The induction within the iron is greater than that previously within the undisturbed magnetic field. The equipotential surfaces are also farther apart within the iron, so that iron may be said to transmit inductive effect better than air or a vacuum.

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* See Proposition (9), page 177; there the force $= 2\pi\sigma$; here we have the force at a point between two such faces, whose breadth is practically infinite as compared with the distance between them: one attracts, the other repels: these effects concur.

A diamagnetic substance has the reverse property; fewer lines of induction pass through it.

The result of this distortion of the lines in the magnetic field is to set up stresses, which tend to cause an iron bar to assume a position parallel with the lines of force; while a bar of bismuth tends, in a non-uniform field, to move into a position of least negative magnetisation, and to lie across the lines of induction, at right angles to them if possible. Substances which in the form of bars take up this cross-position in a non-uniform field—Diamagnets—comprise the great majority of the substances found in nature.

In a uniform field, a magnetically **isotropic** substance (*i.e.*, one in which all directions are magnetically similar), if its form be **spherical,** becomes simply magnetised and remains at rest. If its form be ellipsoidal or otherwise **elongated,** it tends to rotate until its greatest length lies parallel to the lines of force, and this whether it be paramagnetic or diamagnetic.

If the substance be **aelotropic** (*i.e.*, having different susceptibilities, κ, in different directions) and **spherical,** the sphere tends to rotate until it brings its direction of greatest susceptibility parallel to the lines of force. If it be **elongated** we have two cases: (1) in the case of very small susceptibility the form has no effect, and the axis of greatest susceptibility comes to lie along the lines of force; (2) in the case of great susceptibility the longest axis lies parallel to the lines of force.

In a non-uniform field, an **isotropic sphere** tends to move along the Lines of Slope (p. 187) into regions of greater force if the substance be paramagnetic, into regions of less force if it be diamagnetic. An **elongated** body if it be paramagnetic lies along the lines of force, a position in which it lies as far as possible in the strongest part of the field; if it be diamagnetic it rotates so as to lie across the lines of force, a position in which it lies on the whole in the weakest possible part of the field.

If the substance be **aelotropic,** the force tending to produce motion along the Lines of Slope is greatest when the axis of greatest paramagnetic susceptibility is parallel, or that of least diamagnetic is at right angles to the lines of force: and in the case of crystals, the susceptibility being small, there will be rotation into this position whatever the form of the crystal.

A **diamagnetic** substance has therefore two obvious characteristics: (1) a bar of it places itself across the lines of force in a non-uniform field, and (2) a sphere of it in a non-uniform field, such as the neighbourhood of either pole of a magnet or of an electric current, is repelled into regions of weaker force.

A **paramagnetic** substance in bar-form places itself along the lines of force, and a sphere of it is attracted by either pole of a magnet.

With these we may compare a sphere of an **already magnetised** substance, which is attracted by the one pole of a magnet and may be repelled by the other, if the disturbance of its magnetic condition due to induction do not overpower the already existing magnetic distribution.

Limit of Magnetisation.—The intensity of induced magnetism increases with the intensity of the magnetic field, and varies with that intensity; but, in the case of iron, within certain limits only. As the intensity of the magnetic field rises, the induced magnetism of soft iron verges towards a limit. This would appear to favour that theory of induction which regards the magnetisation of induced iron not as created, but as directed by the forces within the field. According to this theory (Weber's), the molecules of iron are already little magnets, but their directions are promiscuously discrepant. When they come into a magnetic field they are directed so as to lie with their axes parallel to lines of force, and the whole mass of iron thereupon becomes obviously magnetic. In diamagnets there is some reason for believing that the particles are magnetised *de novo* in the magnetic field; but the particle-magnets thus produced are feeble, and their strength does not tend to a limit.

The researches of Professor Hughes (*Trans. Roy. Soc.*, London, 1881) have brought to light many curious facts which tend to show that a bar of iron when magnetised, or even when currents pass through it, suffers rearrangement of its molecules, or even an alteration in their shape. According to him, the neutral state is characterised by a perfectly-symmetrical arrangement in which all the little magnets satisfy their mutual attractions. When subjected to induction the particles rotate into their directed position with comparative ease if the induced iron be struck or vibrated or twisted; and the so-called coercitive force is to be attributed to molecular rigidity.

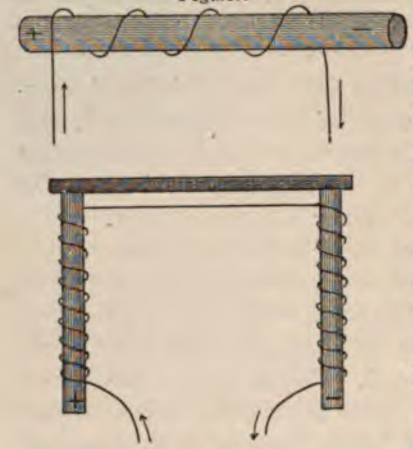
When an iron bar is magnetised there seems to be an actual twist set up in it.

Magnets are usually made by exposing steel for some time to the influence of an existing magnetic field; as by rubbing bent· or straight bars from centre to ends with the opposite poles of bar-magnets, or by leaving them in contact by their extremities with the opposite poles of a strong horse-shoe magnet or electromagnet.

A ring of steel wire may be magnetised by slowly rotating it near one pole of a steel magnet, while slowly bringing it up to and then away from the inducing magnet so as to prevent the formation of secondary poles.

Magnets can also be produced by an electric current. A simple bar-magnet, as we have seen, tends to lie across a current, its positive pole to the left, its negative to the right of the current. If a bar of soft iron be placed along the lines of force within an Electromagnetic Field, it becomes a temporary magnet; there is a kind of separation of magnetisms; the left-hand end of the bar becomes magnetically positive, the right-hand end negative. If the current be wound round the bar, so that every part of the current exerts a similar action upon the bar, the bar becomes strongly magnetic, and is then called an **Electromagnet.**

Fig.247.



If it be of soft iron it loses this property the instant the current ceases; but if it be of steel, and if the current be powerful and continued for some time, it becomes a permanent magnet. Elias made steel magnets by passing steel bars through the axis of a short spiral of about twenty turns, bearing a powerful current; sometimes even one pass was sufficient to saturate the steel.

Heat and extreme cold alike enfeeble magnets of iron and of nickel; cobalt, iron, and steel become more susceptible to magnetic induction when they are slightly warmed. A nickel magnet loses its magnetic power at 635° C.; a steel magnet at a slightly higher temperature; cobalt at the temperature of melting copper.

When an iron or cobalt bar is magnetised it becomes longer and somewhat more slender, but does not appreciably alter in volume; it also emits a slight sound, a "magnetic tick." A nickel or a steel bar shortens and thickens.

Magnetisation induced or residual in a wire is diminished on stretching, provided that the magnetisation corresponds to an inducing force above a certain critical value known as Villari's Critical Value; this being (Sir Wm. Thomson) about 24 times the terrestrial intensity. Below that critical value tension increases the magnetisation of a magnetised wire. The effects of transverse expansive stress are opposed to those of longitudinal stretching.

Energy is absorbed during magnetisation, and if an electromagnet be made and unmade in frequent quick succession, it becomes hot; the energy is derived from that of the intermittent inducing current.

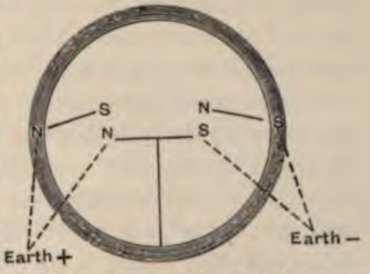
If the induced magnetism of iron be due to the directive action of the magnetic field, the residual magnetism of steel may perhaps be due to a sort of imperfect elasticity of the medium surrounding the particles; the particles are wrenched into definite directions, and retain these as a permanent set, or very slowly reassume their discrepancy of direction (Maxwell).

If we dissolve away the outer skin of a steel magnet by means of acid, we find (Jamin) that the remainder has a very small intensity of magnetisation. Perhaps the outer shell is the hardest part of the magnet and has the greatest amount of the so-called coercitive force, the least amount of elasticity of the medium.

Astatic Arrangements.—A needle tends to point to the magnetic north; but it is often desirable to mask the action of the earth's magnetism, in order to increase the ratio of the moment of any deflecting magnet to the effective terrestrial intensity, and thereby to increase the sensitiveness of galvanometers. This may roughly be effected by bringing another magnet of opposite effect into the neighbourhood, so as nearly to neutralise the earth's directive force; or again by coupling together on the same suspending thread two equal magnets with their poles opposed. In the latter case the earth tends to direct the two magnets in opposite senses, and if the two magnets were equal and their axes parallel, the joint system would be practically unaffected by the earth's directive action.

It is better to enclose a single needle in a shell of very soft iron, as in Sir William Thomson's marine galvanometer. This shell, within the earth's magnetic field, becomes magnetic. The needle is now under the inducing action of two magnets, the earth and the induced shell. The actions of these are opposed, and if the shell be thick enough are approximately equal: the earth's magnetic field is thus nearly destroyed within the shell, and the magnet is free to obey the directive impulse of any current which may be sent round it. Such a shell acts as a **Magnetic Screen**; and such a screen, efficient as a protection from the influence of an external magnet, may be a sphere, an infinite or very large plane, or an equipotential surface of any form.

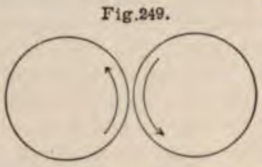
Fig. 248.



The **general problem of magnetic induction** is a problem of potential and lines of force, in which the body acted upon consists of perfectly-conducting molecules scattered through an absolutely non-conducting medium. This kind of problem involves difficulties of calculation, but is of the same nature as that of electrostatic induction through a heterogeneous dielectric, or that of conduction of heat or of electricity through a heterogeneous conductor, or that of the flow of a frictionless incompressible fluid through a heterogeneous porous material. A given amount of force within the magnetic field produces a certain amount of separation of magnetisms and a corresponding density of magnetic distribution; this may be regarded as an arrested flow, an accumulation, which is proportional to the continuous flow which is dealt with in problems of conduction: and the nature of the substance acted upon brings into the calculation a term, the Coefficient of Magnetic Induction, μ, which resembles the permeability of bodies to fluids, or the specific inductive capacity of electrostatic dielectrics.

As to the **nature of magnetism**, Ampère's theory is that every molecule of a magnetic substance is the seat of a separate current, circulating round it in a plane at right angles to the magnetic axis. This explanation meets most of the facts with great readiness; but in view of the doctrine of the Conservation of Energy we must postulate the entire absence of resistance to these molecular currents—a circumstance of which it is somewhat difficult to form a clear conception.

Fig. 249.



When all the molecules of a substance have their currents running in the same direction, and when all these currents are equal, the substance is uniformly magnetised, and in the interior any two contiguous molecules (Fig. 249) have currents in opposed directions whose effect on exterior particles is *nil*. The result of the whole is equivalent to a superficial sheet of electric current, the action of which may be approximately reduced by a kind of centre-of-gravity problem to the action of two Poles.

As to the **direction of the currents** within a magnet: a person standing on the Arctic Pole of the earth would, if those currents to which the earth's magnetism is supposed to be due were visible to him, see them, or rather their resultant, the current-sheet, travelling over the surface, circulating round him from east to west; those in front of him would therefore travel towards his right hand. The observer there situated would be at the negative end of the earth; the Positive pole is its Southern pole—that pole, namely, from which the positive or north-seeking end of the compass-needle is driven. An observer stationed at the positive or southern pole of the earth, the Antarctic Pole, would therefore see these currents pass round him, still from east to west, but apparently towards his **left**. These currents within a magnet are known as Ampère-currents.

Dimensions of Magnetic Measures.—Quantity of magnetism, *m*: force = $mm \div \text{distance}^2$; whence, just as in the case of electric quantity, p. 552, the dimensions of magnetic quantity are $[m] = [M^{\frac{1}{2}}L^{\frac{3}{2}}/T]$.

Strength or Intensity of Field, S: mechanical force acting on unit quantity of magnetism; this force is $\{(m \times \text{unity}) \div \text{distance}^2\}$; its dimensions are $[S] = [M^{\frac{1}{2}}L^{\frac{3}{2}}/T] \div [L^2] = [M^{\frac{1}{2}}/L^{\frac{1}{2}}T]$.

Magnetic Moment, *ml*: a magnetic quantity × a length; $[ml] = [M^{\frac{1}{2}}L^{\frac{5}{2}}/T]$.

Intensity of Magnetisation, I: magnetic moment per unit of volume; $[I] = [M^{\frac{1}{2}}L^{\frac{5}{2}}/T] \div [L^3] = [M^{\frac{1}{2}}/L^{\frac{1}{2}}T]$.

Magnetic Potential: work done in moving a quantity of magnetism; its dimensions are those of (Work done) ÷ (Quantity *m* moved); $[V] = [\text{Work}/m] = [ML^2/T^2] \div [M^{\frac{1}{2}}L^{\frac{3}{2}}/T] = [M^{\frac{1}{2}}L^{\frac{1}{2}}/T]$; the same dimensions as those of electric potential, electrostatically measured.

Magnetic Surface-density, σ: quantity of magnetism per unit of area; $[\sigma] = [m/\text{Area}] = [M^{\frac{1}{2}}L^{\frac{3}{2}}/T] \div [L^2] = [M^{\frac{1}{2}}/L^{\frac{1}{2}}T]$.

Strength of Shell, i: Surface-density × thickness; $[i] = [\sigma \times \text{thickness}] = [M^{\frac{1}{2}}/L^{\frac{1}{2}}T] \times [L] = [M^{\frac{1}{2}}L^{\frac{1}{2}}/T]$.

Coefficients of Induced Magnetisation, κ, **and of Magnetic Induction**, μ; numbers simply: dimensions $= [0]$.

ELECTROMAGNETIC INDUCTION.

When a closed current-bearing circuit is placed with its positive face (p. 617) facing a positive magnetic-pole, there is mutual repulsion, for the potential energy of the system tends to a minimum. The pole is repelled along the lines of force which trend positively from the positive face of the circuit; the pole therefore tends to travel repeatedly through the circuit along the closed lines of force. The potential energy of the system is thus found to have no fixed value, but to depend upon the number of times the pole passes through the circuit. This apparently anomalous result is due to the continuous supply of energy by the current itself.

If the current be brought nearer to the magnetic-pole, then, since work must be done in order to bring about this approach in the face of mutual repulsion, the potential energy of the system is increased by a fixed amount: a portion of this energy takes the form of a temporary increase of the current in the closed circuit; while the remainder may, by induction, produce an increased magnetic condition in the magnet.

Now, replace the magnet by an equivalent closed circuit ("circuit B"), the positive aspect of which faces the positive aspect of the original closed circuit ("circuit A"). These two circuits, again, repel one another: and if work be done in forcing them together, the energy appears in the form of a temporary increase in the intensities of both currents, and is presently converted into heat in the circuits.

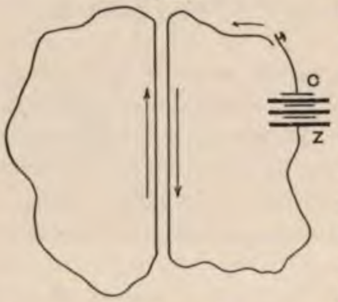
Conversely, when the magnet or the equivalent circuit is withdrawn, there is a corresponding temporary diminution in the corresponding current-intensities, and possibly in the corresponding magnet-strength.

These increases and diminutions in current-intensities are equivalent to the Induction of New Currents. The duration of these new induced currents is limited to the time spent in changing the relative positions of the mutually-inducing magnets or currents.

In the case of two circuits let the original intensity in one of the circuits be vanishingly small, approximately but not perfectly zero. This is the condition of most conductors in what we would call an undisturbed state; it is very improbable that the current within any conductor is ever absolutely *nil*. The result is that if a circuit bearing a current be brought towards a circuit capable of bearing a current, and if the former, the inducing current, have its positive face turned towards the circuit approached, there will be two effects produced: (1) an increase in the intensity of the inducing current, and (2) a new current developed by induction in the circuit approached, which had previously appeared to bear no current. This current has its positive face turned towards the approaching positive face of the inducing current, and is therefore opposed to it in its direction.

If two wires be laid alongside one another (Fig. 250), and if one of these wires be connected with the two poles of a battery, and thus form part of a Primary or Battery Circuit; while the other wire is merely a part of a complete metallic circuit, a so-called Secondary Circuit; then when contact is suddenly made in the primary circuit, a current of brief duration—a duration not exceeding in time the variable state of the primary current—is produced in the secondary circuit and is known as the **Secondary Current.** The primary current and the secondary current are, in the wires laid alongside one another, opposed in their direction.

Fig.250.



As long as the intensity of the primary current remains constant, the secondary circuit has in it no current; but any increase is accompanied by a brief opposed secondary current.

When the primary current is diminished, the primary circuit again presents a variable state; and as long as that variable state lasts there is again a current in the secondary circuit, which is on this occasion in the same direction as the waning primary current. When the primary current stops, there is a very abrupt secondary current, parallel to the ceasing current.

These secondary currents represent a definite amount of energy subtracted from the energy of the primary current,—an amount which depends only on the initial and final states or intensities of that current. Being of extremely short duration, they are of correspondingly great intensity. The secondary current produced on breaking the primary current is briefer, and therefore more intense than that produced on making it.

These statements may be generalised by saying that wherever a closed circuit, capable of bearing an electric current, lies wholly or in part in a Magnetic or Electromagnetic Field of Force, any disturbance in the Intensity of the Field of Force will induce a Current in the circuit; and the direction of the induced current is determined by the rule (**Lenz's Law**) that the new current will increase the already-existing resistances, or develop new resistance to that disturbance of the field which is the cause of induction.

A telephone circuit passing through a disturbed field of force will pick up signals: for example, at every lightning flash the instrument is heard to roar, and in order to prevent such effects of induction no part of the current is entrusted to earth, but the double wire necessary is coiled round itself so as to form a strand composed of two insulated wires. The effect of induction on one wire is then equal to the opposite effect of induction on the other wire. During thunderstorms military mine-fuses have been known to explode through induction in the wires controlling them.

We have seen that a closed current, A, whose positive aspect faces a positive magnetic pole or face of a magnetic shell or equivalent electric current, B, sends towards the latter, from within its own contour, a number of positive lines of force or of induction, which radiate from its positive face. If we change the standpoint and regard the current first mentioned—a current borne by circuit A—as placed within the magnetic field of the shell or the electromagnetic field of the equivalent circuit B, then the positive lines proceeding from the latter are, as regards the circuit A, negative, for they trend not from but towards its positive aspect.

Circuit A, as we have seen, tends to move by translation to a greater distance from circuit B. It will also tend to rotate until its negative aspect faces the positive side of B; it is then attracted towards B.

In the former case, as A moves away, the number of negative lines which pass within its contour diminishes. In the latter case—that of rotation—A tends, as it turns, first to set itself edge-on to B's lines of induction, and then so to place itself (its negative face opposite to B's positive face) that the lines of induction which emerge from B positively also emerge from A's positive face positively, and are positive with respect to A.

In either of these cases, translation or rotation, the number of negative lines embraced by the contour of A is diminished as far as possible, or the number of positive lines attains a maximum.

A little movable circuit may be made—De la Rive's floating battery—by thrusting a strip of copper and a strip of zinc through a cork, and connecting them by an arch of copper wire: when the whole is floated in water, the arch tends to lay itself at right angles to the magnetic meridian, copper to the west, zinc to the east; in this position the positive face of the arch is to the north, and the magnetic lines of force or induction which trend towards the north are embraced by the arch in the greatest possible number.

The general statement of the phenomenon is:—A movable circuit tends so to place itself as to have as few negative or as many positive lines of induction passing through it as possible ; a positive line being held to pass through a circuit when, after passing through, it emerges from the positive face in a positive direction ; a line passing through being held to be negative when its direction is towards the positive face. The position thus assumed is the position of least potential energy, that into which the whole system tends, as it were, to sink. A circuit in this position of least potential energy embraces as great a number as possible of positive lines of induction.

Mutual Attraction and Repulsion of Currents.—Suppose two currents in the plane of the paper, similar in their directions and having in consequence their nearer portions opposed in direction, as in Fig. 249. Let their directions be the same as those of the two currents in that figure. The left-hand current in that figure has lines of induction which ascend from the plane of the paper and descend through the contour of the right-hand circuit, meeting its ascending lines. These descending lines are therefore negative to the right-hand circuit, and that circuit tends to move away so as to embrace as few of them as possible. The portions of the currents which are nearest one another, running in opposite directions, thus seem to repel one another. The area in which downward lines meet upward lines is thus diminished as far as possible, and this enables us now to understand the propositions illustrated by Figs. 236, 237.

If a circuit embracing the greatest possible number of positive lines of induction, and therefore occupying the position of least potential energy, be pulled or turned into any other position, work must be done upon it ; and this work is done against mutual attractions. This doing of work is associated with diminution of the number of positive lines of induction embraced by the movable circuit. As the circuit moves in the field, lines of induction must be cut through by it. All cutting through lines of induction, when the number of lines enclosed by the circuit is diminished by the operation, is effected by the expenditure of work. The process attains its maximum when the movable circuit has been swung round through 180°. If it be still farther rotated, it comes to enclose fewer negative lines, then an increasing number of positive lines, until it regains its original position. At that moment the whole account shows that the energy spent upon the system has been equal to that given up by it, and there is, on the whole, no work done either by or upon the system.

The work done takes the form of the energy of induced currents, which always increase the resistance to the actual movement ; if A and B repel one another, their intensities are increased when they are urged together, diminished when they are drawn asunder ; if they attract one another these actions are reversed. This may be otherwise expressed by saying that when a circuit is made to enclose a greater number of negative lines of induction, or a smaller number of positive lines, its current is increased in intensity, or a new induced current set up in it ; while if it be made to enclose a smaller number of negative or a greater number of positive lines, the intensity of its current is diminished, or a new reverse current is set up in it. The result is the same whether it move so as to enclose more or fewer lines in an existing magnetic field, or whether the magnetic field itself vary so that its lines either open out and become fewer, or become more numerous and approach one another—a smaller or a greater number of them consequently passing through the given circuit.

If a part of a circuit of total resistance R be movable in a magnetic field of intensity N it will cut through all the lines in a certain area A in the course of time t; it will therefore cut through AN/t lines of induction per second. The mean E.M.D.P. set up is firstly, such, that if the movable part of the circuit *diminish* the area of the circuit as it moves in the terrestrial magnetic field, the current will run in the circuit (which is supposed to be set in a plane at right angles to the magnetic meridian) in a direction which seems from the standpoint of an observer stationed to the south to be the same as that of the hands of a watch: and, secondly, it is proportional to AN/t. In Electromagnetic measure, the units are so adjusted that the E.M.D.P. is **equal** to $-AN/t$. The mean Intensity is, then, AN/Rt in electromagnetic measure, and the Quantity passing is AN/R.

When a block of copper is whirled within a magnetic field, currents are set up in it, which produce resistance to the motion; the motion of the block very rapidly ceases, as if the magnetic field were highly viscous, and the block becomes hot. When a magnet-needle is suspended immediately above a copper plate, any oscillation in the magnet develops currents in the copper, and the magnet almost immediately comes to rest.

Self-Induction.—A current suddenly formed in a spiral wire is retarded by the mutual action of the different coils; it does not flow on, and its intensity is rendered less than it would have been in a straight wire; when suddenly broken it is prolonged and is as it were piled up, so that the so-called Extra-Current can force its way through greater resistances than a steady current can. In fact, a single Daniell cell can be made to decompose water by delivering a part of the energy of its current, at high potential, in the form of the so-called Extra-current.

These phenomena closely remind us of the phenomena of momentum in a water-pipe, already discussed under the Hydraulic Ram; and they can be explained as phenomena of momentum of the ether in the electromagnetic field.

If the current, thus suddenly made and broken, be a secondary current, its intensity is greatly increased, and it is rendered able to make a longer spark, by being sent through a spiral wire.

Two wires bearing currents in opposite directions, and twisted round one another, present no phenomena of self-induction; for which reason the wires leading to and from a galvanometer should be twisted together for some distance from the needle.

Coefficient of Mutual Induction of two Currents.—The Ether surrounding a pair of circuits of intensities $i_{,}$ and $i_{,,}$ must possess Energy, which Clerk Maxwell has shown to be proportional to squares and products of the intensities, and which may be written thus: $\{\frac{1}{2}Li_{,}^{2}+Mi_{,}i_{,,}+\frac{1}{2}Ni_{,,}^{2}\}$.

The second term vanishes when the currents are at an infinite distance from one another; it is at its greatest practical when the two circuits touch one another, its greatest theoretical when they absolutely coincide: at intermediate points it has intermediate values. It can be shown that $Mi_{,}i_{,,}$ is in any given position of A and B numerically equal (1) to the Mutual Potential

Energy of the two circuits and (2) to the Number of Lines of Induction which, being due to A, pass from A through B or, equally, being due to B, pass from B through A; and M is styled the **Coefficient of Mutual Induction.** M varies with the relative position of the two circuits.

The maximum value of M is its value when the two currents are made to run in the same circuit; let this be called M_0. The number of lines due to $i_{,}$ and the number due to $i_{,,}$ are to be added together for the conjoined current $(i_{,}+i_{,,})$; for they all pass through the same circuit; hence the number actually threading the circuit will be $2M_0 i_{,} i_{,,}$. In this case $M_0 i_{,} i_{,,}$ is equal to the part of the energy of the field which is due to the approximation of the currents $i_{,}$ and $i_{,,}$ from an infinite distance and their coincidence; and M_0 is equal, numerically, to half the number of lines of induction which pass through the circuit itself when $i_{,}$ and $i_{,,}$ are both unity, that is, when the conjoined current has an intensity $=2$.

Coefficient of Self-Induction.—We next see that $M_0 = L$. If the intensity of the second current $i_{,,}$ be 0, the energy of the field is $\frac{1}{2}Li_{,}^2$ only. A second current of the same intensity in a circuit of the same size, etc., at an infinite distance will have energy also equal to $\frac{1}{2}Li_{,}^2$. Together the energy will be $Li_{,}^2$. Now bring the two currents together and blend them; the energy is $\frac{1}{2}L(2i_{,})^2 = 2Li_{,}^2$. The system possesses energy equal to $Li_{,}^2$, due to the approximation; but this is also $M_0 i_{,}^2$ if both currents be equal to $i_{,}$; whence $L = M_0$. L is the Coefficient of Self-Induction; and the coefficient of self-induction of a circuit is equal, numerically, to the number of lines of induction which thread that circuit when it bears a current whose intensity is unity in electromagnetic measure.

If the energy of a current traversing a single circuit be derived from any external source, such as a battery, which is independent of induction, the energy supplied from that source during a very short time δt will be equal to $Ei.\delta t$, where E is the E.M.D.P. and i the current-intensity. (All our measurements in these paragraphs are supposed to be made in electromagnetic measures.) This energy is divided into three parts.

(1.) Heat in the circuit. This is equal to $Ri^2.\delta t$, where R is the resistance.

(2.) External work, mechanical, chemical or other. This we shall suppose $= 0$.

(3.) Work spent in imparting energy to the electromagnetic field. This is equal to $\frac{1}{2}L\left\{(i+\delta i)^2 - i^2\right\}$, where δi is the small change in the intensity produced during the time δt.

We thus have the equation

$$Ei.\delta t = Ri^2.\delta t + \tfrac{1}{2}L\left\{(i+\delta i)^2 - i^2\right\};$$

an equation which can be dealt with by integration, the effect being that we find the intensity, at any time t after the introduction of a new E.M.D.P. $=$ E into the circuit, to be

$$i_{,} = \frac{E}{R} - \frac{E}{R}\left(2{\cdot}718281^{-Rt/L}\right)$$

The intensity never comes fully up to the value E/R; but it approaches it indefinitely nearly as the time t increases. If, however, the coefficient L be large, as it is in a coil of wire, the second term on the right-hand side is not immeasurably small, and it represents what is equivalent to a reverse current lasting for an appreciable time, and delaying the development of a current of full intensity E/R. This reverse current is called the **Reverse Extra-Current** or the **Extra-Current of Closure** or of **Making.**

When a current is suddenly broken, the intensity at a time t after the current has been broken is $+(\mathrm{E}/\mathrm{R})\left(2{\cdot}718281^{-\mathrm{R}t/\mathrm{L}}\right)$. This indicates that there is still an onflow, a Direct Extra-Current or Extra-Current of Opening or Breaking; an onflow which results in a high potential at the broken extremity of the wire.

These extra-currents are thus associated with absorption of energy by the electromagnetic field while currents, the energy of which is derived from extraneous sources, are being produced or increased, and with liberation of energy by that field when such currents are broken or while they are being diminished.

They may also be looked at as phenomena of Induction. When the intensity of a current is increased, the circuit is made to embrace more lines of force: if it embrace N more lines of force in time δt an E.M.D.P. is set up equal in electromagnetic measure to $-\mathrm{N}/\delta t$. Where L is the coefficient of self-induction of a circuit, the establishment of current i in it causes the development of Li lines of force embraced by it; and this causes an E.M.D.P. $=\mathrm{E}_{,} = -\mathrm{L}i/\delta t$; whence $\mathrm{I}_{,}$, the mean intensity $= -\mathrm{L}i/\overline{\mathrm{R}.\delta t}$; this being the mean intensity of an induced current opposed in its direction to the originating current. When the main-current ceases, the induced current now produced on the disappearance of lines of force is direct, the direct extra-current.

The steady intensity $i = \mathrm{E}/\mathrm{R}$; whence the new "electromotive force" $\mathrm{E}_{,} = -\mathrm{LE}/\overline{\mathrm{R}.\delta t}$. Since δt is very small this may greatly exceed E; and it is greatest when the current passes through conductors of a spiral form in which the value of L is great, the conductor as a whole gathering many lines of force into its own axis. The extra-current may thus be able to spark across striking distances beyond the power of the main-current.

The quantity of electricity in the extra-current is in each case $\mathrm{Q}=\mathrm{I}_{,}.\delta t = \mathrm{EL}/\mathrm{R}^2$.

Induction Coils.—The effect of induction is multiplied when the two wires, that of the primary and that of the secondary circuit, though kept insulated from one another, are wound together round the same axis. The secondary current is then proportional in its intensity to the product of the number of turns in the two wires, provided that the resistances introduced by multiplying the coils, or the differences between the mutual distances of the different turns, be not too considerable. In Induction Coils the wires of the two circuits are wound round separate bobbins, which are then slipped the one over the other to a greater or less extent. On this extent depends the intensity of the secondary current. The primary current is made and broken with great frequency by means of a Contact-breaker.

This may be a mere mechanical contrivance, or it may be automatic.

In the latter case there lies a bar of soft iron in the axis of the inner, the primary, bobbin. When the primary current passes, this bar or core becomes an electromagnet. This electromagnet pulls towards itself an armature, a mass of soft iron, which is arranged near one of its extremities; this mass of soft iron is an integral part of the circuit of the primary current, and by its movement the primary current is broken. The electromagnet now loses its magnetic condition ; it ceases to attract the armature ; the latter, under the pressure of a spring, returns to its former position, and again completes the primary current ; the electromagnet is again made, and the armature again displaced. The soft iron armature is thus caused to oscillate and to impart to the primary current an intermittence, whose frequency depends upon the intensity of the current and, mainly, upon the pressure of the spring.

Magneto-Electric Machines.—When a magnet is thrust into the axis of a bobbin which forms part of a closed circuit, there is a current produced in that circuit. The current is opposed in direction to the magnetic molecular-currents, the Ampère-currents, of the pole which is introduced first. If a long magnet be drawn wholly through such a coil, there is at first a current in one direction as the one pole approaches; then as its midpoint passes the midpoint of the coil the current is *nil*, but is reversed as the opposite pole passes out. The current is at first opposed to the Ampère-currents of the approaching pole: it is ultimately the same in direction as those of the receding pole; and as all parts of a bar-magnet are seen, when looked at end-on, to have their currents in the same direction in space, the induced current changes in its direction as the magnet passes through.

This formation of induced currents, whenever there is a variation in the magnetic field surrounding a circuit, wholly or in part, is utilised in the various forms of Magneto-electric Machine.

Pixii's, the prototype of Clarke's (the very familiar machine used for medical purposes), consisted of a couple of parallel bobbins, each of which contained a soft iron core and was surrounded by a wire coil. Opposite the two cores rotated the two poles of a horse-shoe magnet: each of these poles glanced past the end first of the one and then of the other core in swift alternation. Each of the cores, being of soft iron, became instantly magnetic under the predominating induction of that pole which happened at any given instant to be the nearer. The cores therefore assumed in rapid alternation opposite magnetic characters, and in the coils surrounding them (and therefore in the circuit of which both coils were made to form part) there ran a rapid succession of currents of alternating direction.

It was found more convenient, as in the common medical apparatus, to rotate the bobbins with their cores in presence of the magnet, the principle being the same.

This kind of machine, by multiplication of the magnets and of the rotating coils, has been made (Stöhrer, Alliance machine, Lontin, de Méritens, Holmes, etc.) in very large dimensions, and can yield very powerful currents; but the currents are rapidly alternating in their direction, and they must for some purposes (electrometallurgy, transmission of power) be made to travel in one direction by means of a Commutator—a contrivance which, by mechanically changing at each half-revolution the disposition of the metallic path open to the current, changes the direction of each alternate current, and therefore renders the whole uniform in direction.

The nearer the rotating coil is to the magnets, and the more rapid the movement of the coil through the field, the more intense are the currents produced. Siemens's form of rotating-coil or Inductor is devised so as to combine these advantages.

The peculiar form of Inductor found in most Gramme machines, and originally due to Pacinotti, depends upon the following principles:—

1. A loop of wire passed over the north pole of a magnet has in it, while it approaches the centre of the magnet, currents opposed to the Ampère-currents of the magnet. As it recedes from the centre of the magnet it has in it currents similar in direction to the Ampère-currents. As it passes the midpoint there is no current; that point is a neutral point.

2. A ring of steel may be permanently, a ring of soft iron temporarily, magnetised; it is equivalent to two semicircular magnets whose north poles and whose south poles respectively face one another. In the two mutually-facing north poles, as well as in the two similarly-situated south poles, the currents must be in opposite directions in space. Such a ring-magnet produces no external effect on particles at a distance.

3. Let us suppose such a ring-magnet to be arranged in a vertical plane, its north pole uppermost: and let us divide it into a right and a left semicircular magnet, each of which has an upper north and a lower south pole.

The accompanying figure shows, by loops, *a*, *b*, *c*, *d*, the direction of the Ampère-currents in the magnet. The dotted lines show the directions of the currents induced in a metallic ring, which is slipped over the ring-magnet in the direction *abcd*. When passing *a* the loop is both receding from *d* and approaching a large district of the magnet in which the rotation is opposed to that at *d*; for both reasons its direction is the same as that of the Ampère-current at *d*. When midway between *a* and *b* everything is symmetrical: the loop approaches as many molecules as it leaves of the same kind: the total electromotive effect is *nil.* As the loop crosses from *b* to *c* the electromotive effect is again a maximum, but is of the reverse direction to that observed within the region *da*. There are thus two midpoints, at which the current in the circulating loop is *nil*, and the current is a maximum in one direction while the loop is being slipped over the north pole, a maximum in the reverse direction when over the south pole of the ring.

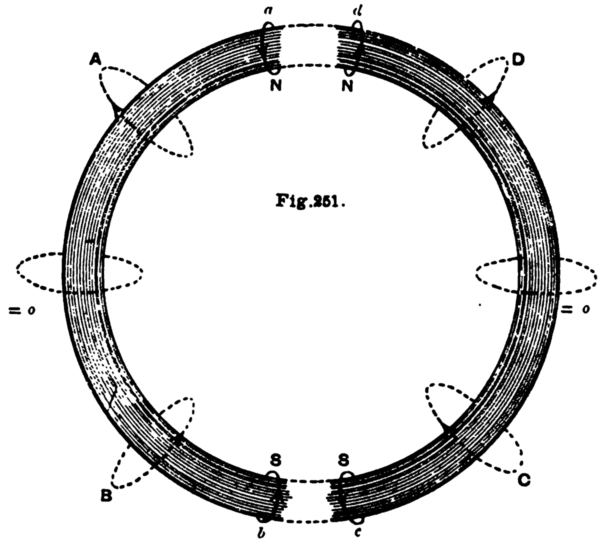


Fig. 251.

4. Replace the single movable loop by a closed spiral, covering the whole ring. The current in the upper two quadrants of the closed spiral is opposed to that in the lower two; both currents seem to be travelling towards one of the neutral points, and away from the other. There is, therefore, a difference of potential between the two neutral points of the spiral,—those points which are situated midway between the poles of the magnet; and if these neutral points be connected by a wire, a current will pass in that wire.

5. Instead of slipping the spiral over the ring-magnet, slip the ring-magnet continuously through the spiral coil. The result will be similar, but the neutral points will rotate round the coil.

6. Instead of shifting the ring-magnet, let the metal ring and the wire wound round it be immovably connected, but let the magnetism of the magnet shift its position and circulate round

the ring: the effect will again be similar, but the neutral points will shift round the spiral, being always 90° from the poles of the ring-magnet.

7. If a ring of very soft iron have well-insulated copper wire wound round it, and be continuously whirled round its own central axis between the fixed poles of an external magnet or of an electromagnet, it will present, in the neighbourhood of the external fixed magnetic poles, two induced poles, also fixed in their position. As the soft-iron ring rotates, we have a relative movement of the ring and of its magnetism, the converse of that explained in the preceding paragraph. We have, therefore, in the copper-wire coil two neutral points, always 90° distant from the positions of the external poles; and if these be connected by a wire, as long as the iron ring is rotated, so long will a continuous current run in that wire. The energy of this current is obtained from the work done in forcing the ring round through the resistant magnetic field.

Induced currents, as we know, last only as long as the variable state of the field; but with this form of inductor the field is continuously variable and the induced current is continuous.

The current produced in the spiral by the poles of the external magnet is, as it passes them, the same in direction as the main current in the spiral wire, so far as regards the side of the spiral next the external poles; the other side of the spiral, in which a reversed current would be induced by the poles, is farther from them, and is, besides, sheltered in great part by the mass of the iron ring.

The magnetic field—the field of the external magnet—within which the coil rotates may be that of a steel magnet, or it may be that of an electromagnet. In the latter case the electromagnet may be produced by an independent battery, or by the current generated in a small subsidiary magneto-electric machine, as in Wilde's machine; and the current produced by the electromagnet and coil may be utilised not in an exterior circuit, but in producing a still stronger electromagnet and a more intense electromagnetic field, whose induction, acting upon a second rotating coil, generates the current which traverses the exterior circuit.

In the modern so-called **Dynamo-electric machine** the magnetic field is produced by a soft-iron magnet which, when the machine is at rest, retains a mere trace of magnetisation or is slightly magnetised by induction within the terrestrial magnetic field. When the coil is set in rotation an extremely feeble current is generated in virtue of this feeble magnetisation of the soft iron; this current is not permitted at once to pass away through the exterior circuit, but is sent round the soft-iron magnet and increases its magnetic intensity. The soft-iron magnet, thus strengthened, reacts upon the intensity of the current produced, which presently attains a high maximum; this depends upon the speed of rotation.

Machines which bear no steel magnet, but depend for their initial action upon residual or terrestrial magnetism, are called dynamo-electric; those whose action depends directly or indirectly upon the induction of permanent steel-magnets are called magneto-electric machines.

Dynamo-electric machines are subject to the disadvantage that when the resistance increases the intensity falls, and the strength of the electromagnet falls also; but if the electromagnet be produced by the current of a subsidiary battery or machine this disadvantage disappears.

Dynamo-electric and magneto-electric machines have been applied to the purposes of electric lighting, of electrolysis,—in which their utility is limited, for a considerable amount of work corresponds only to a small amount of actual chemical decomposition,—to the production of heat at a distance—in which their usefulness is also small—to the production of the currents used in telegraphy, and to the transmission of energy to a distance.

The Transmission of Energy to a distance.—If a magnet or electromagnet be placed within a variable magnetic or electromagnetic field, it will tend to place its axis along the Lines of Induction. Hence a magnet surrounded by a coil of wire will, when a current is passed through the wire, tend to place itself at right angles to the plane of the current. (See Fig. 241.)

If the coil be placed vertically in the plane of the magnetic meridian, and if the needle be suspended horizontally at its centre, then, on the supposition that both poles of the magnet are at the centre of the coil, an ideal approximated to when the coil has an extremely large diameter or when the needle is extremely short, the deflection of the needle from the magnetic meridian is such that its tangent is proportional to the intensity of the current passing. Such an arrangement is called a Tangent Galvanometer. (See p. 661.)

If the coil be placed parallel to the deflected needle the sine of the deflection becomes proportional to the intensity: this is the Sine Galvanometer, in which instrument a long needle is used.

When a current is suddenly made and immediately stopped, the magnetic needle of a galvanometer receives an impulse and is thrown through a certain angle; it is continuously retarded during this throw by the directive force of the surrounding magnetic field, this field being that of the earth or of a neighbouring magnet.

The throw is such that the total quantity of electricity passing is proportional to the sine of half the angle of deflection (p. 662).

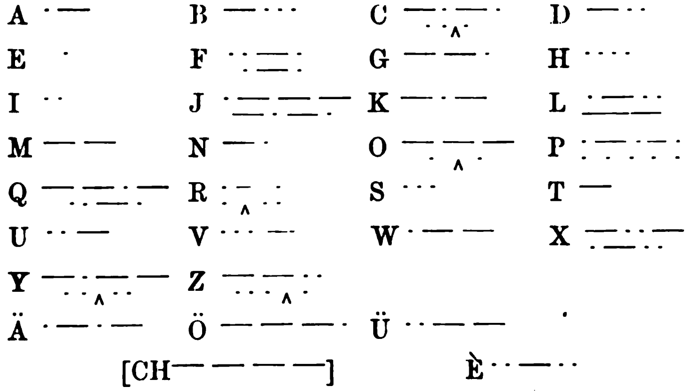
The twitch or throw of the needle renders manifest the passage of a very brief current; just as the position of equilibrium assumed by the needle, as it lies more or less completely across the current, indicates the persistence of a continuous current.

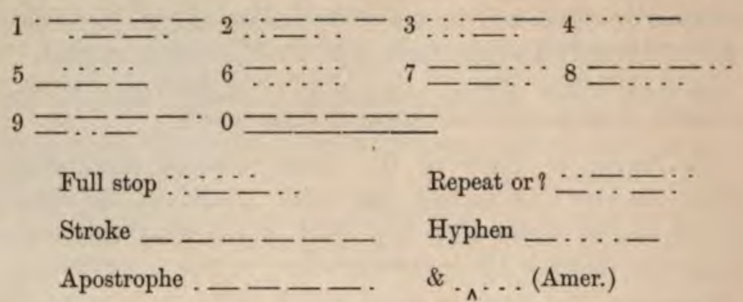
When the current ceases the magnet tends to oscillate for some time like a pendulum; but if it oscillate in a strong magnetic field of force—as, for example, in the neighbourhood of a strong magnet—its oscillations will be very rapid and of small amplitude. If masses of metal be so arranged that any oscillations of the magnet tend to produce retarding induced-currents in these masses, then, especially if it be a light needle, the oscillations of the magnet rapidly cease, as if it were immersed in a viscous medium, and the magnet is, without further oscillations, restored to its position of repose. A galvanometer arranged on this principle is a **Dead-beat Galvanometer.**

The same dead-beat effect is mechanically produced by making the magnet move in a small closed chamber of air which it nearly fills: it thus moves against air-resistance.

As often as a momentary-current is sent round the magnet of a galvanometer, so often will the twitch of the suspended magnet be repeated, and at intervals of time equal to those between the successive momentary-currents. This action—which is the simplest form of transmission of energy to a distance, for work is done in displacing the magnet within the field—is the basis of telegraphic signalling.

Longer and shorter currents produce longer throws and shorter twitches of the galvanometer-needle. These form the basis of a signal alphabet—the Morse code. The following is the alphabet, the upper line, where there are two, being the European, the lower the American form:—




1
2
3
4
5
6
7
8
9
0
Full stop
Repeat or ?
Stroke
Hyphen
Apostrophe
& (Amer.)

In some of the American forms it will be observed that a period of time, represented by ∧, intervenes in the midst of a set of signals representing one letter. The American form (U.S. and Canada) is the original, as devised by Prof. Morse: the European is an improved version.

In submarine telegraphy the signals used are not long and short, but right and left deflections—that is, positive and negative momentary currents.

When at the distant end of a circuit the conducting wire is passed round a soft-iron core, that soft-iron core becomes an electromagnet just as often, and remains an electromagnet just as long, as the circuit is or remains completed by a key at the home station. This electromagnet may govern the movements of a neighbouring mass of iron, and do work upon it: and the movements of this second mass may be utilised in an endless variety of ways for the repetition of movements similar to those executed at the home station by the hand of the operator, or by any mechanical contrivance adjusted so as to make and break contact in any pre-arranged manner. The mass of iron moved at the distant station may itself, by its movement, make and break a second electric circuit, and may thus control the movement of metallic masses at still more distant stations, as in the case of telegraphic relays.

Electromagnetic Interrupter for Tuning-Forks.—A tuning-fork of known pitch is set in vibration. As it vibrates it alternately makes and breaks a current which traverses the tuning-fork itself. This current is passed in its course round a little electromagnet, which is alternately made and unmade. This electromagnet is so arranged as alternately to attract and release one of the prongs of the tuning-fork, which is thus kept in continuous action. The intermittent current produced is sent round a second electromagnet, which rhythmically attracts and releases a second tuning-fork; this is thus kept vibrating in unison with the first, even although it be not precisely in tune with it.

As another example of this movement of distant masses we may take the Telephone in its simplest form. A plate of iron, P, is placed in the magnetic field of a magnet, M: the plate is caused, by being spoken at, to enter into certain vibrations; the vibrating plate P, by induction, acts upon the magnetism of the magnet M; the latter is alternately strengthened and weakened in accordance with the varying position of the vibrating plate: as M varies in strength it causes variations in the strength of a current passing through a coil, C, wound round its pole, or else, if there be no appreciable current passing in that wire, it causes a current to be formed in that wire whose intensity varies continuously on either side of zero-value, being now in the one direction, now in the other. This induced current reproduces in the mode of its variation the complex-harmonic curve which might have been recorded by a delicate writing-point attached to the vibrating plate. The variable current thus produced passes at the receiving station through the coil of a similar telephone. It there causes, by induction, variations in the strength of the magnet, which attracts the plate with varying degrees of force. That plate is either bent as a mass towards and from the magnet, or its molecules are disturbed by the varying induction: or these actions may be combined; in any case, the plate exerts varying pressure upon the surrounding air and produces in it Sound-Waves, which approximately reproduce in their complexity the sound-waves produced by the original voice.

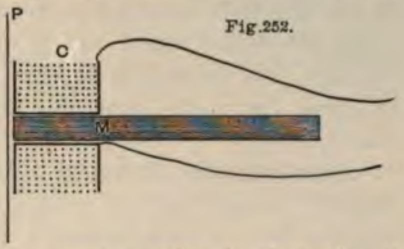


Fig.252.

It is a matter of indifference to the receiving telephone by what means the variations of current-intensity which it reveals have been produced. These may be due to variations of electromotive D.P. (vibrations of one of the plates of an electrostatic accumulator or oscillatory variations in its charge,—variations of the potential of a mass of mercury vibrating while in contact with water up and down a conical capillary tube), or to variations in the total resistance (length, cross-section, conductivity) of the conducting wire. The conductivity of the circuit may be caused to vary by squeezing the wire, by causing a certain length of it to vibrate; or again by interposing a certain length of a conductor whose conductivity varies with varying pressure (microphone) or with varying illumination (photophone).

According to Prof. Tait, the variations of current in an ordinary telephone are equivalent to actual currents whose intensity is one-thousand millionth part of the current ordinarily used in telegraphic work. This telegraphic current may, on long lines, be stated to be about one-sixtieth Ampère.

Page-Effect.—A telephone will work feebly even without any plate P; the varying constraint of the particles of the magnet M causes them to exert varying pressure upon the air. If a plate of any substance be connected with the extremity of M, that plate will act as a sounding-board, and will enhance the sound produced.

In the Differential Galvanometer two equal and separate wires are coiled round the same needle; through these wires currents may be sent in opposite directions: if the two currents be equal the needle remains at rest: if either predominate the needle moves.

Simultaneous Currents.—A current varying in a complex manner is equivalent to a number of co-existent currents, each varying in a simple-harmonic manner. Thus any number of currents may co-exist on the same circuit, and may be positive or negative. Of these some may be originated at one station in the circuit, others at others. Two currents in opposite directions and of equal strengths may thus pass through the same wire, and if led round a magnetic needle will produce no effect upon it.

By means of differential galvanometers two messages may be sent along the same telegraphic wire at the same time (Dúplex Telegraphy). Station A has a single wire leading from the positive pole of his battery: the current running in this he divides into two moieties, which he sends in opposite directions round the needle of his differential galvanometer: these two moieties are then sent on, the one to the distant station B, the other to A's own earth-plate. In the course of the one or the other branch-current the operator at A interposes resistances until the intensities of the opposed currents round his galvanometer-needle are equal; then, in whatever way A may make or break circuit, his galvanometer-needle will remain steady; but the needle at B will respond. Similarly, B sends signals to A, to which his own instrument is mute. The two stations may thus signal simultaneously, two operators being employed at each end, one to transmit, the other to receive; and the variations of electric condition produced in the single connecting-wire run through one another in a manner analogous to that in which waves meeting on a cord traverse one another,—but not identical, for electricity itself has no inertia. This is the Differential Method.

Bridge-Method in Duplex Telegraphy.—Suppose a triangle ABC; the current enters at A; B is connected with the distant station D; C is connected to earth through a resistance equal to that of the line BD; between B and C is the recording instrument of the home station. One moiety of the current which enters at A will run to earth, the other will travel to D. If the resistance in AB be so adjusted as to be equal to that in AC, B and C will be at equal potentials; no current will run through BC; the home instrument stands motionless. At the receiving station the apparatus may be precisely similar; it will then indicate the arrival of signals from A, but will be insensible to the movements of its own key.

If a current which has 300 maxima of intensity per second, and another of say 800 maxima per second, be sent along the same wire, the conjoined current will present variations of intensity such as might be represented by the curves of Fig. 45. Suppose a current presenting such variations of intensity to be passed round a soft-iron core near the end of which is a steel reed tuned to vibrate 300 times a second, and so adjusted as to be attracted in the sense of its vibrations when the soft-iron core attains its maximum intensity. The steel reed would, among other impulses to which it would not respond, receive a set of 300 maximum-attractions per second, which would set it in vibration. The same current may be also passed round a core, opposite the extremity of which is placed a reed tuned to 800 vibrations per second; that reed will pick out and will respond to the more rapid component of variation of intensity of the current, and will respond to it only. Further, suppose that the several components of variation are each not continuous, but interrupted: the corresponding vibrations of each reed will be similarly interrupted, and one telegraph clerk may be occupied with listening to each. A current whose variation of intensity is as complex as the sum of eight distinct S.H.M.'s may be practically resolved by as many distinct receiving-reeds into distinct signals; and since the duplex method of working may be applied to this plan, as many as sixteen distinct messages may travel along a single wire at the same time. This is the principle of Mr. Elisha Grey's Harmonic Telegraph.

Quadruplex Telegraphy.—A small current always runs in the circuit. There are two transmitting keys. The one reverses the direction of the current; this causes a needle within a magnetic field at the receiving station to swing to left or right; an effect which depends upon change of direction of the current within the circuit. The other key, when depressed, introduces a new battery into the circuit; the strength of the current is thereby increased, and the current is now enabled to make a certain soft-iron electromagnet move at the receiving station; an effect which depends upon the strength, but not upon the direction of the current in the circuit. The one receiving instrument thus records reversals, the other the enhancements of current-intensity. Two sets of signals may thus be sent in the same direction at the same time; and this arrangement when duplexed, preferably by the bridge-method above described, becomes quadruplex. This is the ground-principle of Prescott and Edison's system, which is described at length in Prescott's *Telephone*. The practical details are extremely ingenious; there may, for instance, be a critical instant at which the intensity-receiver is liable to be interfered with and to fail, through the current supplied to it fading away while being reversed by the reversing-key; a condenser then acts as a reservoir, and its discharge keeps up a current which tides over the critical instant; a result which is aided by a subsidiary local battery then brought into action by means of a relay.

The principle of **Reversed Action,** illustrated by the receiving telephone, comes also into play when a current is sent through the wire of a dynamo-electric or a magneto-electric machine. As rotation of a dynamo-electric machine produces a current in a certain sense, so an extraneous current sent through the machine in the same sense causes a reversed rotation of the inductor. In consequence of this, if we couple two dynamo-electric machines by connecting wires, so that both dynamos (so called for the sake of brevity) are on the same metallic circuit, and if we force the inductor of the one into rotation, the inductor of the other rotates in a reverse sense as soon as the current transmitted attains a certain intensity. The distant dynamo, which bears under such circumstances the name of **Electromotor,** may be of any size (for dynamo-electric machines up to 40 or 50 horse-power have been constructed), and the simple use of a key or commutator arrests or reverses its action at will. The intensity of the current passing round the circuit is diminished by the reversed rotation of the electromotor; this is equivalent to the production of a reverse current by the electromotor. The usefulness of the arrangement, the proportion of the energy absorbed by the electromotor in rotating against resistances to the total energy imparted by water-wheel or steam-engine to the driving dynamo, is equal to the ratio between the intensity of the virtual reverse-current produced by the electromotor and the intensity of the current produced by the dynamo. This Utility or **Efficiency** is not to be measured by the relative rapidities of rotation of the electromotor and dynamo, on account of the so-called dead turns; the rotation of the dynamo must exceed a certain speed before any current will be produced, and the current produced must exceed a certain strength before the electromotor will turn. The Activity of the arrangement (*i.e.*, the rate at which the electromotor can do external work, the amount of energy transmitted per second) is theoretically greatest when the virtual reverse-current is half that produced by the dynamo—that is, when the resistance in the exterior circuit is equal to that within the dynamo itself. In practice, however, this rule does not apply, because the phenomena of induction within a dynamo are exceedingly complicated, and the arrangement most advantageous for each machine must be found by experience. In general the exterior resistance should not exceed 42 % of the total resistance.

The theoretical activity is arrived at thus:—During each second the external work done is W ergs; where W also represents numerically the Activity in question; the energy converted into heat in the whole circuit is I^2R, and the energy provided by the dynamo is EI. Then $EI = I^2R + W$; a quadratic; whence $I = (E \pm \sqrt{E^2 - 4RW})/2R$. The quantity $(E^2 - 4RW)$ cannot have, physically, a negative sign, for its square root would then become an impossible quantity. $(E^2 - 4RW)$ cannot be less than zero: whence W cannot be greater than $E^2/4R$: when it is equal to $E^2/4R$, $I = E/2R$, and the intensity has been diminished from E/R to $E/2R$,—that is, to one half,—while the total resistance must have been doubled.

About forty per cent of the energy imparted to the dynamo can be recovered as work in the electromotor. Where energy is very cheap—as, for instance, that of water-power, windmill-power, or perhaps tidal power—or where coal is very dear, it may be of advantage to resort to this mode of transmission of energy.

As to the thickness of conducting wire necessary, there is no limit other than that imposed by the necessity of very good insulation. An ordinary telegraph wire could convey the whole energy of Niagara Falls, and convey it to any distance; but the wire would be at so high a potential that sparks would fly from it into the surrounding air. In the same way, if the amount of onflow of a fluid in a pipe were found to vary directly as the motive difference of pressure, any amount of energy might be transferred from one place to another by the smallest flow of water, for any water allowed to flow out of the pipe might be made to escape with any assignable velocity; provided always that the tube were strong enough at all points to sustain at all intermediate points the necessary pressure.

If a dynamo of resistance 5 Ohms, and producing a difference of potential of 1000 Volts, be the source, and a similar machine be the electromotor, while the connecting wire offers a resistance of r Ohms, the intensity of the current produced is $\left(\frac{1000}{5+5+r}\right)$ Ampères. If 500 such dynamos be coupled in file, their joint E.D.P. will be 500,000 Volts, and their resistances 2500 Ohms; if the receiving electromotors be also multiplied five-hundredfold, their resistances will be 2500 Ohms; if the connecting wire be unaltered, the intensity of the current passing will be $\frac{500{,}000}{2500+2500+r}$ Ampères; but if the connecting wire be also 500 times as long as at first, the intensity is $\frac{500{,}000}{2500+2500+500r} = \left(\frac{1000}{10+r}\right)$ Ampères, the same as in the former case. Though the intensity of the current passing is the same, the energy transmitted per second is not the same: it is 500 times as great. In the former case it is Intensity × E.D.P. $= \left(\frac{1000}{10+r}\right) \times 1000$ Ampère-Volts or Watts: in the latter it is $\frac{1000}{10+r}$ Ampères × 500,000 Volts $= \frac{500{,}000{,}000}{10+r}$ Watts.

When the total resistance is the internal resistance R_i, $I = \frac{E_,}{R_i}$; when it is $(R_i + R_e)$, $I_, = \frac{E_,}{R_i + R_e}$. These two distinct sets of circumstances are linked together by—(1) The criterion of maximum activity, $R_{i+e} = \frac{100}{58} R_i$; and (2) The energy imparted to the dynamos is a constant, $= E_, I_, = N$ ergs per second. From these equations we find $I_, = \sqrt{\frac{58}{100}\frac{N}{R_i}}$, and $E_, = R_{i+e} I_, = \left(\frac{100}{58} R_i\right) \left(\sqrt{\frac{58}{100}\frac{N}{R_i}}\right) = \sqrt{\frac{100}{58} R_i N} = \sqrt{R_{i+e} N}$. We must now choose numerical values for R_{i+e}, the total resistance internal and external, and N, the amount of energy imparted to the system. We shall use C.G.S. Electrostatic units.

Let the resistance be that of 4000 kilometres of copper wire of 1 sq. cm. in cross-section, and that of x dynamos and x electromotors. The dynamos are each supposed to generate an E.D.P. of 1000 Volts, or $3\frac{1}{3}$ C.G.S. Electrostatic units. Their number must be $(E_{,} \div 3\frac{1}{3})$.

Let the joint resistance of each dynamo and motor be 10 Ohms, or $\frac{10}{900,000,000000}$ C.G.S. Electrostatic unit of resistance. The resistance of the $(E_{,} \div 3\frac{1}{3})$ pairs of machines will be $\left\{(E_{,} \div 3\frac{1}{3}) \times \frac{10}{900,000,000000}\right\}$, or $\frac{E_{,}}{300,000,000000}$ C.G.S. Electrostatic units.

The wire (4000 kilometres) will offer a resistance of about 648 Ohms, or $\frac{216}{300,000,000000}$ Electrostatic C.G.S. units.

The total resistance $R_{i+e} = \frac{1}{300,000,000000}\left\{E_{,} + 216\right\}$ C.G.S.E.S. units.

Let N be the energy of the Falls of Niagara, measured also in C.G.S. units or ergs. About 100,000000,000000 grammes of water fall per hour through a height of about 4830 cm. The potential energy lost by the water is about 132,000,000000,000000 ergs per second = N.

The equation $E_{,} = \sqrt{R_{i+e}N}$ is now

$$E_{,} = \sqrt{\frac{1}{300,000,000000}\left\{E_{,} + 216\right\} \times 132,000,000000,000000},$$

a quadratic; whence $E_{,} = 440,020$ C.G.S.E.S. units or 132,006,000 Volts.

If iron telegraphic-wire 4 mm. in diar. were used, its resistance would be (the resistance of iron being $\frac{990}{164}$ that of copper) 31680 Ohms; the total resistance would be $\frac{1}{300,000,000000}\left\{E_{,} + 10560\right\}$; and $E_{,} = 450,320$ C.G.S.E.S. units, or 135,096,000 Volts.

No practicable insulation could be set up, adequate to sustain permanently so great a stress, which corresponds (Foster) to a striking distance of some half-a-dozen miles through air; and no dynamo-electric machines could be ranged in file to the number necessary, for the insulation of their coils would be broken down by sparks from the wire to the outer air.

It is practicable, however, with ordinary telegraphic wires insulated in the ordinary way, and with a 16-horse-power dynamo, to drive a 6-horse-power electromotor at a distance of 30 miles.

The wire must also, by possessing sufficient thickness, offer so little resistance that it is not so far heated as to deteriorate in conductivity.

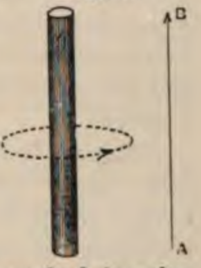
THE NATURE OF THE MAGNETIC FIELD.

Magnetic Rotatory Polarisation.—If a plane-polarised beam of light or radiant heat be sent through a magnetic field occupied by a transparent medium, its plane will, by the retardation or acceleration in phase of one of its circular components, be rotated. The sense of this rotation depends upon the direction of the lines of force and upon the nature of the medium; its amount depends upon the thickness and the nature of the medium and upon the intensity of the field (resolved in the direction of the ray).

This has, in the hands of Becquerel and Lord Rayleigh, been made the basis of a new method of measurement of the intensity of a current; the current passed through a coil produces within the coil an electromagnetic field, the intensity of which is proportional to the intensity of the current; a plane-polarised beam sent along the axis of the coil is rotated to an extent proportionate to the current-intensity.

In diamagnetic substances, and in flint glass and a few magnetic substances, the direction of rotation of the plane of polarisation is that shown by Fig. 253, in which AB represents a line of force, and the arrowed circle represents the direction of rotation of the plane. Whether the ray travel in the direction AB or in the direction BA, the absolute rotation imparted to it on its transmission through the magnetic field is the same; whence, if it be reflected from a mirror and sent back through the field, the rotation of its plane will be doubled.

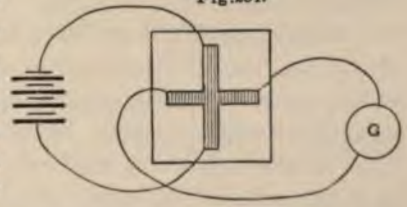
Fig.253.



In most magnetic media the direction above mentioned is reversed. The rotation is better marked in solutions of nickel than of iron, and in uniaxial crystals it is most marked along the axis.

Hall's Experiment.—A thin cross of gold leaf on a piece of glass. A current from a battery passes through two arms of the cross, but does not affect a galvanometer connected with the other two arms. When the cross is made to face the lines of force of a strong magnetic field, a constant current is indicated by the galvanometer. The current of the pile is, as it were, pushed into the galvanometer circuit.

Fig.254.



Kerr's Experiment.—Polarised light reflected from the polished face of a magnet undergoes rotation of the plane of its polarisation: when reflected from the north-seeking pole it is rotated from left to right.

Maxwell's Theory.—These phenomena can be explained and deduced from one another by supposing that there is some kind of rotation round the lines of force; and this rotation may be regarded, with extreme probability, as rotation within extremely small whirlpools, whose common axes are arranged in the direction of the lines of force.

Some forms of Hall's experiment can, however, as Mr. Shelford Bidwell has shown (*Phil. Mag.*, April 1884), be explained as examples of Thomson and Peltier effects without invoking the aid of rotations in the magnetic field.

This condition in the magnetic field is not assumed instantaneously, but is propagated through the non-conductor surrounding the current with a definite velocity, v. While the disturbance is being propagated, there is a period of adjustment, and during this period the phenomena of the Variable State and of Induction appear: these are due to the forces called into play when the velocity of the whirlpools or vortices is changing (Clerk Maxwell).

During these changes the mechanism connecting the vortices, whatever be the nature of that mechanism, is exposed to Stress which is equivalent to Electromotive Difference of Potential; if that mechanism yield elastically there is Electric Displacement, and the return to the normal condition gives rise to an Electric Current.

Maxwell's Theory of Light.—The same kind of stress has been shown by Clerk Maxwell to be perfectly competent to explain the phenomena of Light. The condition of the medium occupying the path of a beam of light is equivalent to that which the same region would assume if exposed to the induction of rapidly-alternating currents.

In a non-conductor there is, under the action of such stresses, an electric displacement, and energy is stored up in this displacement; but in a conductor any disturbance is propagated as an electric current. If Light be an electromagnetic phenomenon two consequences follow. The first of these is: Since in non-conductors the displacement produces a restitution-force which varies as the displacement—a criterion of vibratory movement propagated with a definite velocity; but in conductors no such force is manifested, and the energy of electric disturbance must soon be converted into heat: That light-vibrations are not possible in conductors, and conductors should be always opaque, while non-conductors ought, if homogeneous, to be transparent. With few exceptions this is the rule. The second consequence is that v, the velocity of propagation of an electromagnetic disturbance in a non-conductor, ought to be the same as that of light. This constant, v, is otherwise proved to be the same as the ratio of the electrostatic to the electromagnetic unit of electrical intensity or quantity presently to be explained; and this ratio is experimentally found to be such as to give v the mean value of 30,000,000000 cm. per sec., which coincides sufficiently with the velocity of light.

If an electrostatically-charged body could be whirled round a magnetic needle at the rate of 30,000,000000 cm. per second, it ought to act upon it in the same way as a circulating electric-current. At very high speeds, such as are physically within our reach, the same effect should be observed in small degree, and Prof. Rowland of Baltimore has succeeded in making it manifest.

A consequence of these conclusions is (Maxwell) that the sp. ind. cap. of a dielectric ought to be equal to μ^2, where μ is its refractive index for waves of infinite length.[*] In some substances this is the case; it is so in sulphur (Romich, Nowak, Boltzmann), turpentine, petroleum and benzol (Silow); but in vegetable and animal oils (Hopkinson) and in glass, Iceland-spar, fluor-spar, and quartz (Romich and Nowak), the sp. ind. cap. is too great. We probably ought, however, not to expect any more than a general agreement: the most momentary charge and discharge with which we can affect a condenser is thousands of millions of times slower than those rapid alternations of electric condition with which the phenomena of light are held to be identical.

Electromagnetic Measure.—A current of given intensity, I in electrostatic units, must be represented by smaller numbers when electromagnetic units are used: a current of $I = 60000,000000$ is a current of $i = 2$; for the C.G.S. electromagnetic unit of intensity is 30,000,000000 times as great as the C.G.S. electrostatic unit.

The basis of the Electromagnetic System of Measurement is the identity of effect between a closed current of electromagnetic intensity i, and a magnetic shell of the same contour, whose magnetic strength is numerically the same, being reckoned as also equal to i.

If $i = 1$, the current is equivalent in magnetic effect to a magnetic shell whose strength is unity and whose area and outline are the same as that of the circuit; and this is the **Electromagnetic C.G.S. Unit of Current-Intensity** (**Definition i.**).

If we suppose a wire bearing a current and one cm. in length to be bent into a circular arc whose radius is one cm., and if we suppose a unit magnetic pole to be placed at the centre of the circle of which the circular arc forms a part; and if we further suppose that the force exerted by the current upon the unit magnet-pole is equal to one dyne;—then the current is one whose intensity is in electromagnetic measure equal to unity. In such a case $i = 1$; and this may be taken as **Definition ii.** of the Electromagnetic C.G.S. Unit of Intensity. The general formula is $f = m.il/r^2$, where f is the force exerted upon a magnet-pole placed at the centre of such an arc, m the strength of that pole, i the intensity of the current, l the length and r the radius of the circular arc into which the wire is bent.

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[*] If μ, $\mu_{,}$ be the refractive indices corresponding to the respective wave-lengths λ and $\lambda_{,}$, we know that to a rough approximation $\mu = A + B/\lambda^2$ and $\mu_{,} = A + B/\lambda_{,}^2$, where A and B are constants, found by experiment. From these, knowing the numerical values of μ, $\mu_{,}$, λ, $\lambda_{,}$, we can find that of A, which is the approximate value of μ when $\lambda = \infty$.

The potential differs in this case from the force exerted in having r instead of r^2 as its divisor (comp. p. 180). When $l = 2\pi r$, a whole circumference, the potential is $2\pi i.m$, the same as the mutual potential of a magnetic shell of strength i and a magnet-pole m placed very near its surface.

If a current $= i$ in electromagnetic units pass along a wire through a uniform field of magnetic force, of strength N, the wire is acted upon by a force equal to Ni dynes for every cm. of the length of the wire; and this gives us another definition (**Definition iii.**) of the Electromagnetic C.G.S. Unit of Intensity. This also enables us to measure the intensity of an intense magnetic or electromagnetic field; a current is led through it; the wire is forced in one or other direction; this force can be balanced by a known weight. If the current be sent through a column of mercury in a known magnetic field there is a difference between the manometric pressures at the two sides of the column (Lippmann).

Related to these definitions we have still a fourth (**Definition iv.**); a current of intensity i, traversing a straight wire of indefinite length, acts upon a unit magnetic-pole at a distance d from the wire with an electromagnetic force equal to $2i/d$. This conclusion is similar to that of prop. 7, p. 177: here i the intensity replaces σ the density there. A unit current would therefore act upon a pole m with a force $2m/d$.

The Intensity of a Current, electromagnetically measured, and the Magnetic Strength of a shell must accordingly have the same **dimensions.** The Magnetic Strength of a magnetic shell is (p. 625) numerically equal to the product of the Magnetic Quantity per unit of area into the Thickness of the shell; its dimensions must hence be those of Magnetic Quantity, divided by an Area, and multiplied by a linear Thickness; but the dimensions of Magnetic Quantity must (since the imaginary magnetic matter obeys laws resembling those of attracting or repelling electric matter, similarly imaginary) be like those of electric quantity, $[M^{\frac{1}{2}}L^{\frac{3}{2}}/T]$; the dimensions of Magnet-Strength are accordingly $[M^{\frac{1}{2}}L^{\frac{3}{2}}/T] \div [L^2] \times [L] = [M^{\frac{1}{2}}L^{\frac{1}{2}}/T]$; the Electromagnetic Unit of Intensity has the same dimensions, and therefore differs from the electrostatic unit, whose dimensions are $[M^{\frac{1}{2}}L^{\frac{3}{2}}/T^2]$, by the term $[L/T]$, which represents a Velocity: the numerical value of this velocity must be found by experiment, which shows it to be 30,000,000000 cm.

Measurement of v.—The ratios between the electromagnetic and electrostatic units may be determined by several methods, of which two may be taken as examples.

Weber's method.—Charge a Leyden jar with a known quantity of electricity, Q; discharge the jar through a wire, which passes round a galvanometer-needle. The quantity of Electricity passing through the galvanometer may be measured in terms of the deviation undergone by the needle in consequence of being thrown by the instantaneous current. This gives the quantity, q, in electromagnetic measure. These separate measurements of the numerical values, Q and q, of the one quantity of electricity give the ratio between the electrostatic and the electromagnetic unit.

Sir William Thomson's method.—The two ends of a wire of great resistance, R, are kept at a constant potential-difference, E; a constant current runs through the wire; this current is found to have an intensity i C.G.S. electromagnetic units; the difference of potential, electromagnetically measured, is (by Ohm's law) $e = i\mathrm{R}$. E and e bear to one another the relation of $1 : v$; whence v may be found numerically.

Dimensions.—Intensity, i. A current of electricity whose intensity is electrostatically represented by I will be represented electromagnetically by only (I/v) units; whence a current of I C.G.S. electrostatic units' intensity, which has dimensions $[\mathrm{M}^{\frac{1}{2}}\mathrm{L}^{\frac{3}{2}}/\mathrm{T}^2]$, will have in electromagnetic measure dimensions, $[i] = \{[\mathrm{M}^{\frac{1}{2}}\mathrm{L}^{\frac{3}{2}}/\mathrm{T}^2] \div [\mathrm{L}/\mathrm{T}]\} = [\mathrm{M}^{\frac{1}{2}}\mathrm{L}^{\frac{1}{2}}/\mathrm{T}]$.

Quantity, q, = Intensity × Time; $[q] = \{[\mathrm{M}^{\frac{1}{2}}\mathrm{L}^{\frac{1}{2}}/\mathrm{T}] \times [\mathrm{T}]\} = [\mathrm{M}^{\frac{1}{2}}\mathrm{L}^{\frac{1}{2}}]$.

Potential, or Difference of Potential, e, = work done ÷ quantity of electricity upon which work is done; $[e] = \{[\mathrm{ML}^2/\mathrm{T}^2] \div [\mathrm{M}^{\frac{1}{2}}\mathrm{L}^{\frac{1}{2}}]\} = [\mathrm{M}^{\frac{1}{2}}\mathrm{L}^{\frac{3}{2}}/\mathrm{T}^2]$.

Electrical Force, the mechanical force acting on electromagnetic unit of quantity. Its dimensions are those of mechanical force ÷ quantity; $[\mathfrak{E}] = [f/q] = \{[\mathrm{ML}/\mathrm{T}^2] \div [\mathrm{M}^{\frac{1}{2}}\mathrm{L}^{\frac{1}{2}}]\} = [\mathrm{M}^{\frac{1}{2}}\mathrm{L}^{\frac{1}{2}}/\mathrm{T}^2]$.

Resistance = difference of potential ÷ current-intensity; $\{[\mathrm{M}^{\frac{1}{2}}\mathrm{L}^{\frac{3}{2}}/\mathrm{T}^2] \div [\mathrm{M}^{\frac{1}{2}}\mathrm{L}^{\frac{1}{2}}/\mathrm{T}]\} = [\mathrm{L}/\mathrm{T}]$.

Capacity is quantity of electricity stored up per unit potential-difference produced by it; its dimensions are $\{[\mathrm{M}^{\frac{1}{2}}\mathrm{L}^{\frac{1}{2}}] \div [\mathrm{M}^{\frac{1}{2}}\mathrm{L}^{\frac{3}{2}}/\mathrm{T}^2]\} = [\mathrm{T}^2/\mathrm{L}]$.

Specific Conductivity: the intensity of current passing across unit area under the action of unit electrical force. Its dimensions are those of current-intensity ÷ (electrical force × area), viz., $\{[\mathrm{M}^{\frac{1}{2}}\mathrm{L}^{\frac{1}{2}}/\mathrm{T}] \div [(\mathrm{M}^{\frac{1}{2}}\mathrm{L}^{\frac{1}{2}}/\mathrm{T}^2 \times \mathrm{L}^2)]\} = [\mathrm{T}/\mathrm{L}^2]$.

Specific Resistance, the reciprocal of the specific conductivity; $[\mathrm{L}^2/\mathrm{T}]$.

Coefficients of Self-Induction and Mutual Induction of Currents: Ratios between E.M.D.P. produced and the rate of change of current-intensity producing it: the dimensions of the former are $[\mathrm{M}^{\frac{1}{2}}\mathrm{L}^{\frac{3}{2}}/\mathrm{T}^2]$; those of the latter (Intensity ÷ Time) $= [\mathrm{M}^{\frac{1}{2}}\mathrm{L}^{\frac{1}{2}}/\mathrm{T}] \div [\mathrm{T}] = [\mathrm{M}^{\frac{1}{2}}\mathrm{L}^{\frac{1}{2}}/\mathrm{T}^2]$; the ratio therefore has the dimensions $[\mathrm{M}^{\frac{1}{2}}\mathrm{L}^{\frac{3}{2}}/\mathrm{T}^2] \div [\mathrm{M}^{\frac{1}{2}}\mathrm{L}^{\frac{1}{2}}/\mathrm{T}^2] = [\mathrm{L}]$.

From these dimensions we find that that which is measured electrostatically as a current of intensity I, is electromagnetically a current of intensity $(\mathrm{I}/v) = i$. Similarly, by comparison with the electrostatic measures, we find that electrostatic quantity, Q, is numerically expressible as (Q/v) electromagnetic units; potential-difference, E in electrostatic measure, as $(\mathrm{E}v)$ in electromagnetic; resistance, R electrostatic units, as $(\mathrm{R}v^2)$ electromagnetic units; capacity, C, as (C/v^2) electromagnetic units; specific conductivity and resistance, equal to c and r electrostatic units, are respectively equal to (cv^2) and (r/v^2) electromagnetic units.

Practical Units.—Some of the units of the C.G.S. Electromagnetic System are inconveniently large or small. It is therefore the practice not to use the C.G.S. electromagnetic units of electrical quantity, intensity, resistance, etc., but to build up an electromagnetic system based on new units of length, l, and of mass, m. These are respectively 1000,000000 cm. and the 100,000,000000th part of a gramme. The unit of current-intensity is then $[m^{\frac{1}{2}}l^{\frac{1}{2}}/\mathrm{T}] = [\,(\mathrm{M}/100{,}000{,}000000)^{\frac{1}{2}}\,.\,(\mathrm{L} \times 1000{,}000000)^{\frac{1}{2}}/\mathrm{T}] = \frac{1}{10}\,[\mathrm{M}^{\frac{1}{2}}\mathrm{L}^{\frac{1}{2}}/\mathrm{T}]$. The new unit of intensity, the **Ampère**, is thus equal to $\frac{1}{10}$ C.G.S. Electromagnetic Unit. In the same way we find the unit of resistance, the **Ohm**, $= 10^9$ C.G.S. Electromagnetic Units. The Megohm = 1 million Ohms; the Microhm = one-millionth Ohm. The unit of difference of potential, the **Volt**, $= 10^8$ C.G.S. Electromagnetic Units; the Megavolt = 1 million Volts; the Microvolt = one-millionth Volt. The unit of capacity $[\mathrm{T}^2/l] = [\mathrm{T}^2/1000{,}000000\mathrm{L}] = \{[\mathrm{T}^2/\mathrm{L}] \div 1000{,}000000\} = 10^{-9}$ C.G.S. Electromagnetic Unit = 1 **Farad.** The Farad $= 10^{-9} \times$ one C.G.S. Electromagnetic Unit of Capacity; but the latter unit is equal to the electrostatic unit $\times\, v^2$, or to 9×10^{20} Electrostatic Units; the Farad is therefore equal to $10^{-9} \times (9 \times 10^{20}) = (9 \times 10^{11})$ Electrostatic Units of Capacity. The electrostatic capacity of a sphere is equal to its radius; a Farad is therefore the electrostatic capacity of a sphere of (9×10^{11}) cm. radius; and for convenience the standard in use is the Microfarad, the millionth of a farad.

Practical Units.	Number of C.G.S. Electrostatic Units.	Number of C.G.S. Electromagnetic Units.
Intensity—Ampère .	3000,000000	1/10
Quantity—Coulomb .	3000,000000	1/10
Potential—Volt . .	1/300	100,000000
Resistance—Ohm .	1/900,000,000000	1000,000000
Capacity—Farad .	900,000,000000	1/1000,000000

In electric lighting a certain unit is frequently made use of as a conventional basis for estimating the sum due by the consumer. This unit represents 1000 Ampère-Volt-Hours, and is equivalent to the Energy conveyed by a current of one Ampère intensity, passing down a fall of potential of one Volt, and sustained for 1000 hours. This amount of Energy = 1 Ampère-Volt or Watt × 3,600,000 sec., and is therefore equal to $\{10{,}000000$ Ergs per sec. $\times\ 3{,}600000$ sec.$\} = 36{,}000000{,}000000$ Ergs or 2,654,340 foot-pounds, or about 865,000 *ca*, an amount of heat which would convert 2·95 lbs. of ice-cold water into steam at 100° C.; and the commercial unit of current is a current of any intensity continued until this quantity of energy has been transmitted through the consumer's apparatus.

Electromagnetic Measurement of Intensity.—The electromagnetic units of measurement have all been derived in theory from the electromagnetic measurement of intensity of a current: the electromagnetic measurement of a current is therefore a fundamental measurement. It is effected by the use of **galvanometers** and **electrodynamometers.**

In a Tangent-galvanometer in which the coil consists of only one turn, the force acting upon a pole m very near the centre is $Fm = mil/r^2 = mi\,.\,2\pi r/r^2 = mi\,.\,2\pi/r$, when i is measured in C.G.S. Electromagnetic Units, and where F is the force acting on a unit-pole. The force exerted by the horizontal component of the earth's magnetism is H on a unit-pole, Hm on a pole m. The deflection of the needle is θ. The magnetic couple is $H\,.\,2ml\,.\sin\theta$ if the moment of the magnet be $2l\,.\,m$. From the "Equilibrium of Couples," page 159, prop. 2, we learn that $Fm : Hm :: \tan\theta : 1$, or $F = H \tan\theta$. Therefore $i\,.\,2\pi/r = H\tan\theta$ or $i = H\tan\theta\,.\,r/2\pi$. H can be found as on page 623, or turned up in observational tables of local magnetic intensities; θ can be observed; r can be measured; whence i can be found numerically in electromagnetic C.G.S. measure.

If the coil consist of n turns, whose mean radius is $r_{,}$, the force acting on unit-pole, the intensity of the electromagnetic field, at the centre is $i\,.\,n\,.\,2\pi/r_{,}$. If it be a coil of rectangular section with inner and outer radii $r_{,}$ and $r_{,,}$, and of length l cm., with n turns in it on the whole, the intensity is equal to

$$\left\{ i\,.\,n\,.\,\frac{2\pi}{r_{,}-r_{,,}}\,.\log\frac{r_{,,}+\sqrt{r_{,,}^{2}+(l/_{2})^2}}{r_{,}+\sqrt{r_{,}^{2}+(l/_{2})^2}} \right\}.$$

It may be observed that within a single **solenoid**, very long as compared with its breadth, presenting n turns per cm. and a current-intensity i, the intensity of the field at any axial point is $4\pi\,.\,ni$. This corresponds to the $4\pi I$ of p. 629, the force acting on a unit magnetic-particle within a magnet.

In Galvanometers, in which a passing current produces a magnetic field in which a magnetic needle is deflected, the amount of this deflection indicates the strength of the current. It is well in all cases to produce as uniform a field of force as possible. This is effected by arranging a number of coils so as to surround the field, not wholly but in outline. In Helmholtz's Galvanometer, for example, there are two parallel coils, between which the needle is placed at a mean distance from each equal to half the mean radius of either. For sensitiveness, each winding of the wire is made to come as near the magnet as is practicable.

Galvanometer-Constant.—When a current passes through the wire of a galvanometer, the needle is in a magnetic field of a certain intensity or strength, measured, as usual, by F, the force locally acting upon a *unit* magnetic-pole. If $i = 1$, F has a certain numerical value which involves only measurements derived from the construction of the galvanometer itself: it is known as the Galvanometer-Constant, and gives the numerical value of the strength of the field when the current traversing the instrument is of unit intensity. It is distinctively represented by the symbol Γ, and the force acting on a unit-pole when the intensity of the current is i, is equal to Γi; acting on a pole m it is equal to Γim. For example, in a tangent-galvanometer of one turn the force acting is, as above, $mi \,.\, 2\pi/r$; whence $\Gamma = 2\pi/r$.

Ballistic Galvanometer.—If a tangent-galvanometer be constructed with a short heavy needle of length $2l$, and if a very brief current, enduring only for the exceedingly small interval $\delta\tau$, be passed through it, the needle will receive a twitch and after the current has passed will swing through an angle θ. The last equations under Ballistic Pendulum (p. 199) were $\frac{1}{2}\mathrm{I}\omega^2 = (\mathrm{M}+m)gh.2\sin^2\overline{\theta/2}$ and $\omega = 2\sqrt{(\mathrm{M}+m)\,gh/\mathrm{I}}\,.\sin\overline{\theta/2}$. The problem here corresponds closely, but instead of $(\mathrm{M}+m)\,h$ we have the magnetic moment of the needle $(=2l.m)$ which we write as M; instead of g the local intensity of gravity (*i.e.*, the force acting upon a unit-mass), we have H the effective component of the local intensity of the field within which the needle moves after the current has passed,—that is, of the terrestrial magnetic field. The equations of that paragraph therefore become for our present purposes

$$\tfrac{1}{2}\mathrm{I}\omega^2 = \mathrm{MH}\,.\,2\sin^2\overline{\theta/2}$$

which represents the energy imparted to the needle, and

$$\omega = 2\sqrt{\mathrm{MH}/\mathrm{I}}\,.\sin\overline{\theta/2}$$

which represents the initial angular velocity imparted to it.

But again, we can in other terms express the work done by the needle during the brief period of its action; as the product, namely, of the twisting moment into half the angle of twist imparted during that period. The twisting moment is $\Gamma im \times 2l$, the virtual length of the magnet; the angle of twist is an exceedingly small angle $\alpha = \frac{1}{2}\omega \times \delta t$ where ω is the angular velocity initially imparted to the needle. The work done is therefore $\frac{1}{2}\Gamma im\,.\,2l\,.\,\frac{1}{2}\omega\delta t = \frac{1}{2}\Gamma\,.\,2ml\,.\,i\delta t\,.\,\frac{1}{2}\omega = \frac{1}{2}\Gamma\mathrm{MQ}\,.\,\frac{1}{2}\omega$ where Q is the whole quantity of electricity passing in time δt. This is equal to $\frac{1}{2}\mathrm{I}\omega^2$.

Now equating these two values of $\frac{1}{2}\mathrm{I}\omega^2$ the energy, we have

$$\begin{aligned}\tfrac{1}{4}\Gamma\mathrm{MQ}\,.\,\omega &= \mathrm{MH}\,.\,2\sin^2\overline{\theta/2}\\ \mathrm{Q} &= \frac{4\mathrm{H}}{\Gamma}\cdot\frac{2\sin^2\overline{\theta/2}}{\omega}\\ &= \left(\frac{4\mathrm{H}}{\Gamma}\,.\,2\sin^2\overline{\theta/2}\right)\div\left(2\sqrt{\mathrm{MH}/\mathrm{I}}\,.\sin\overline{\theta/2}\right)\\ &= \frac{4\mathrm{H}}{\Gamma}\cdot\sqrt{\frac{\mathrm{I}}{\mathrm{MH}}}\,.\,\sin\overline{\theta/2}.\end{aligned}$$

But T, the time of a *complete* oscillation of a needle swinging freely in the terrestrial magnetic field, is $\mathrm{T} = 2\,\pi\sqrt{\dfrac{\mathrm{I}}{\mathrm{MH}}}$;

whence $$\mathrm{Q} = \frac{2\mathrm{H}}{\Gamma}\cdot\frac{\mathrm{T}}{\pi}\,.\,\sin\overline{\theta/2}.$$

In Electrodynamometers the current is passed through a coil which is suspended within a strong and uniform magnetic field, such as that produced by powerful electromagnets actuated by a second current, or again by fixed coils through which a second current is passing. The deflection of the suspended coil depends upon the strength of the current passing through it, and also upon the strength of the magnetic field surrounding it. If the same current traverse both the fixed and the suspended coils, the rotating couple is proportional to i^2, and therefore to the energy of the current.

The two coils have the respective mean radii r and $r_{,}$, r the greater, $r_{,}$ the less; the respective numbers of turns are n and $n_{,}$; when the plane of the suspended coil makes with the plane of the larger coil an angle θ, the couple tending to bring the two coils into the same plane with their currents opposed is $i^2 . 2\pi^2 . r_{,}^2 . nn_{,} . \sin\theta . /r$. If l and $l_{,}$ be the lengths of wire in the two coils respectively, this expression may be written as $\frac{1}{2}i^2 . ll_{,} . r_{,} . \sin\theta . /r^2$.

If the current to be measured be a rapidly alternating one, the result is the same as if it were constant; it is reversed in both coils at the same time, and the algebraic sign of i^2 is always positive.

If the movable coil be free to slip up and down the axis of a fixed coil in which a current is passing, the inner coil may be sucked in or repelled with a force which may be balanced and measured by known weights: or if the current in the suspended coil be variable, the tension tending to draw it in to the fixed coil may be made to act against a spring, and graphically to record its own variations upon a uniformly-moving piece of paper.

The principle of the differential galvanometer may be here applied, as in Prof. Langley's Thermic Balance. The suspended coil is composed of two separate wires wound together, but insulated from one another: a single current is divided into two equal moieties which run in opposite directions through the two wires of the coil; there is no effect. The least variation in one of these moieties, as when the conductivity of its path is affected by the local application of heat, causes imperfect compensation, and practically a small uncompensated current passes: however feeble this may be, it can be rendered manifest and measurable by increasing the strength of the magnetic field within which the double coil is suspended.

The part of the divided circuit to which heat may be locally applied may be an exceedingly thin strip of platinum. This may be moved up and down say in the dark region of the spectrum. In some places it is heated, in others—dark lines—it is not. By thus groping in the dark it discovers the dark lines and the specially "bright" lines of the heat-spectrum. An instrument of this kind is sensitive to differences of temperature of $\frac{1}{14000}$ F°.

Electromagnetic Measurement of Resistance.—If a circle of wire, radius r, area πr^2, stand at right angles to the magnetic meridian it will embrace H lines of terrestrial magnetic induction per sq. cm. or $H . \pi r^2$ lines over its whole area; if it be turned round a vertical axis through 180° it will come to embrace $H . \pi r^2$ lines oppositely directed with reference to it: the number of lines of force or induction passing through it has therefore been increased or decreased by $2H . \pi r^2$. The circle of wire thus rotated (Earth-inductor) becomes the seat of a current whose E.M.D.P. is numerically equal to $2H . \pi r^2$ in electromagnetic units, and whose intensity is $i = 2H . \pi r^2/Rt$, where R is the resistance (also measured in electromagnetic units) and t the time occupied in the rotation through 180°. If a small needle be suspended at the centre of this rotating circle, that needle will be deflected; the rotating circle acts somewhat as if it were its own Tangent-galvanometer; but instead of a deflection θ such that $\tan\theta = i\,.\,2\pi.n/r\text{H}$ where n is the number of coils and r their mean radius, we have (approximately) $\tan 2\theta = i\,.\,\pi^2\,.\,n^2/r\text{H}$. But $i = 2\text{H}\,.\,\pi r^2/\text{R}t$; whence $\tan 2\theta = 2\pi^3 n^2 r/\text{R}t$. If t be the 2Nth part of a second, the coil will make N complete turns per second and $\tan 2\theta = 4\text{N}\pi^3 n^2 r/\text{R}$. Therefore $\text{R} = 4\text{N}\pi^3 n^2 r/\tan 2\theta$. Of these quantities, n the number of coils and r their mean radius are obtained by measurement; $\tan 2\theta$ is the ratio between the scale-reading (straight scale) and the distance of the scale from the mirror fixed to the centre of the deflecting needle; N can be read off on a speed-indicator. When the resistance is so adjusted that to a speed N there corresponds a deflection θ such that the product above (with due corrections) is numerically equal to 1000,000000, the resistance employed is equal to one Ohm. This is the principle of the method by which the British Association Committee on Electrical Standards constructed the original standard Ohm.

Measurement of the Capacity of a Conductor or Condenser.—The capacity is $\text{C} = \text{Q}/\text{V}$, and therefore we can find the value of C if we find, in terms of units of the same system, the quantity Q with which a body is charged, and the potential V to which this charge raises it. This potential V may be the difference between the potential of the body charged and that of the earth, or it may be the difference between the potentials of the opposed plates of a condenser.

This is, however, not a convenient method; and the practical method is first to construct standard condensers of known capacity, and then to compare the capacity of the body examined with these standards.

Standard Condensers.—Suppose a conductor of capacity C and bearing a charge Q to be discharged through a known resistance which includes a galvanometer; the resistance being so considerable that the discharge is far from instantaneous. The initial potential of the conductor is $\text{V} = \text{Q}/\text{C}$. A current will pass through the galvanometer but will continuously diminish in intensity. At the end of time t let the potential have sunk to v which is the nth part of V, and the charge to q; and at the end of a very small further interval δt, let these have further sunk by the amounts $\dot{v}$ and $\dot{q}$ respectively. Then the quantity which has escaped in time δt is $\dot{q}$, which is necessarily equal to $\text{C}\dot{v}$; it is also equal to the instantaneous intensity i multiplied by the time δt, and this product is equal, by Ohm's law, to $(v/\text{R}) \times \delta t$. Hence $(v/\text{R}).\,\delta t = \text{C}\dot{v}$, or $\delta t = \text{CR}.\,\dot{v}/v$. From this we find, by means of the Integral Calculus, that the time which must have elapsed between the initial instant at which the potential had been V and that instant at which the potential had sunk to v is equal to $\text{CR}\log(\text{V}/v)$. But by our supposition this time is t, and the ratio V/v is equal to n. Whence $t = \text{CR}\log n$, or $\text{C} = t/\overline{\text{R}\,.\log n}$. If we observe the successive values of the current intensity at equal intervals of time we can find the value of n, and then, knowing the value of R in electrostatic, in electromagnetic, or in practical units, we can find the corresponding value of C, the capacity in units of the corresponding system.

Comparison of Capacities.—1. De Sauty's Bridge-method, applicable to small capacities. Two condensers of capacities $\text{C}_{,}$ and $\text{C}_{,,}$, if charged to the same potential V, must be charged with the respective quantities $\text{C}_{,}\text{V}$

and $C_{\prime\prime}V$. If one of these be discharged through a resistance $R_{\prime}$ for time δt, the current which is set up is of mean intensity $i_{\prime}$, and the quantity passing in time δt is $i_{\prime} . \delta t$. But this is equal to the fall in the value of Q during the time δt; that is, to $\dot{q}$. But $\dot{q} = C\dot{v}_{\prime}$. Whence $i_{\prime} = C\dot{v}_{\prime}/\delta t$, and $R_{\prime}$, which varies inversely as $i_{\prime}$, is proportional to $\delta t/C\dot{v}$. Similarly, if $C_{\prime\prime}$ be discharged by a current of intensity $i_{\prime\prime}$ through a resistance $R_{\prime\prime}$, that resistance must bear the same proportion to $\delta t/C_{\prime\prime}\dot{v}_{\prime\prime}$; and if $\dot{v}_{\prime\prime}$, the fall of potential, be the same as in condenser $C_{\prime}$, that is, if $\dot{v}_{\prime\prime} = \dot{v}_{\prime}$; then the equation

$$R_{\prime}/R_{\prime\prime} = \frac{\delta t/C_{\prime}\dot{v}_{\prime}}{\delta t/C_{\prime\prime}\dot{v}_{\prime}} = C_{\prime\prime}/C_{\prime}$$

shows that the Resistances through which the two condensers charged to equal potentials must be discharged, in order that the potentials of the two condensers may fall concurrently and remain persistently equal to one another, must be inversely proportional to the respective Capacities of the bodies discharged through them.

This being postulated, the arrangement of the apparatus is indicated by Fig. 255. A, a battery ; K, a key with which the wire B may be at will connected either with the battery A or directly with the earth, or else, as in the figure, isolated from these. When the battery is connected with B, the condensers $C_{\prime}$ and $C_{\prime\prime}$ are charged through the resistances $R_{\prime}$ and $R_{\prime\prime}$. Connect, then, A with B for a certain time ; disconnect. The two condensers if not already at equal potentials soon become so, for an equalising current traverses EGF ; when equalisation is complete, the needle of the galvanometer G returns to rest. Now put B to earth. The charges of $C_{\prime}$ and $C_{\prime\prime}$ escape through $R_{\prime}$ and $R_{\prime\prime}$ respectively. If the resistance $R_{\prime}$ be disproportionately great, the outflow through it is disproportionately small, and the potential at F sinks faster than that at E ; a current therefore passes from E to F, and the galvanometer-needle in G is deflected. If $R_{\prime} : R_{\prime\prime} :: C_{\prime\prime} : C_{\prime}$, the galvanometer-needle remains at rest, for the potentials at E and F as they sink, sink together and are concurrently equal to one another. If therefore we adjust the resistances $R_{\prime}$ and $R_{\prime\prime}$ until we find that on effecting the three operations—(1) connecting B with the battery A ; (2) isolating B from A until the galvanometer-needle comes to rest ; (3) putting B to earth—the last of these is followed by no deflection of the galvanometer-needle, then, since we know the relative values of the resistances $R_{\prime}$ and $R_{\prime\prime}$, we know, inversely, the relative values of the capacities $C_{\prime\prime}$ and $C_{\prime}$; and as one of these capacities is a standard, we are thus enabled to state absolutely the actual value of the capacity to be measured.

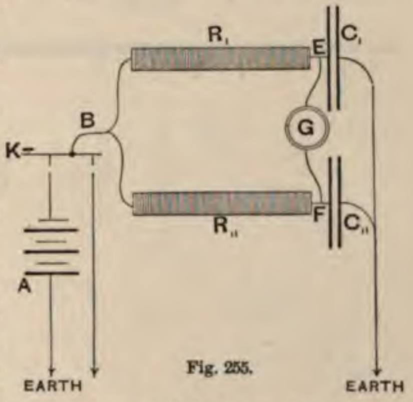

Fig. 255.

2. Compensation-method. If a wire Cu Zn (Fig. 256) connecting two poles of a battery be connected at any one point with the earth, the potential of that point must become equal to zero ; but the difference of potential between the extremities of the wire remains unaffected. The positive potential

at Cu (Fig. 256) bears to the negative potential at Zn a numerical ratio, the same as that between CuO and OZn; for obviously CuA : ZnB :: CuO : OZn.

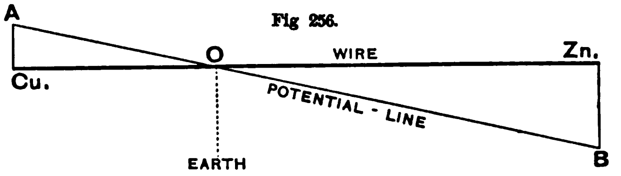

Fig 256.

Let now between the points Cu and O a resistance of reduced length $R_\prime$ be placed, and between the points O and Zn a resistance of reduced length $R_{\prime\prime}$. The potential at a point just between $R_\prime$ and Cu and the potential at a point just between $R_{\prime\prime}$ and Zn bear to one another the ratio of $R_\prime : R_{\prime\prime}$. If these potentials be $+V_\prime$ and $-V_{\prime\prime}$ respectively, we have $V_\prime : V_{\prime\prime} :: R_\prime : R_{\prime\prime}$. Let these two points, at potentials $V_\prime$ and $-V_{\prime\prime}$ respectively, be connected with two condensers of which the one has standard capacity $C_\prime$, the other the capacity $C_{\prime\prime}$ to be determined. The two condensers will become charged to the respective potentials $V_\prime$ and $V_{\prime\prime}$; but the aim is so to adjust the potentials that these condensers shall become charged with equal but opposite quantities of electricity. Suppose this adjustment to have been effected. Then Fig. 257 illustrates the successive operations.

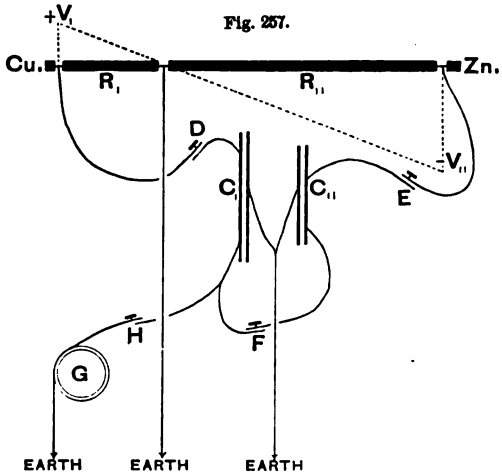

Fig. 257.

(i.) Connect at D and E. The condensers become charged to potentials $+V_\prime$ and $-V_{\prime\prime}$ respectively. They are therefore charged with quantities $+C_\prime V_\prime$ and $-C_{\prime\prime}V_{\prime\prime}$. Disconnect at D and E.

(ii.) Connect at F. The two charges $+C_\prime V_\prime$ and $-C_{\prime\prime}V_{\prime\prime}$ blend, and there remains in the conjoined condensers a residual charge of $(C_\prime V_\prime - C_{\prime\prime}V_{\prime\prime})$, which, if $C_\prime V_\prime = C_{\prime\prime}V_{\prime\prime}$, is equal to zero.

(iii.) Connect at H. The residual charge, if any, runs to earth and deflects the needle of the galvanometer; if none, there is no deflection. There is no deflection when $C_\prime V_\prime - C_{\prime\prime}V_{\prime\prime}$. But $V_\prime : V_{\prime\prime} :: R_\prime : R_{\prime\prime}$. Therefore, when there is no deflection on making contact at H, $C_\prime R_\prime = C_{\prime\prime}R_{\prime\prime}$ and the capacities $C_\prime$ and $C_{\prime\prime}$ are inversely as the corresponding resistances.

Adjust therefore the resistances $R_\prime$ and $R_{\prime\prime}$ until there comes to be no deflection of the galvanometer-needle after operation (iii.), and from the known ratio of the resistances we find that of the capacities, for $R_\prime/R_{\prime\prime} = C_{\prime\prime}/C_\prime$.

It will be kept in mind that all our statements as to positive and negative quantities and currents are based upon the purely arbitrary convention that vitreous electricity is positive, and resinous electricity negative. This convention happens to harmonise with that which regards the north-seeking end of a magnet as its positive pole. We do not really know, however, in what direction an electric current travels, or whether it may not be—as there seems some reason for believing that it is—the sum of two opposed currents, propagated simultaneously from the two poles of a battery.

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